Some Novel Developments in Quarkonium Electromagnetic Transitions

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Outline

★ Examine whether the X(3872) can carry $J^{pc} = 2^{-+}$ by studying the following EM transition process

$$\eta_{c2}\,(\,^1D_2\,) \rightarrow J/\psi\;(\psi')\;+\;\gamma$$

★ Develop a *"hard-scattering" mechanism* to deal with some strongly-hindered EM transition processes, e.g.,

$$\Upsilon(3S) \rightarrow \eta_b + \gamma \quad \text{and} \quad \psi(2S) \rightarrow \eta_c + \gamma$$

Why EM transitions are interesting to the theorists?

Quarkonium is known to have a complicated hierarchy of scales, and enjoys rich dynamics.

The typical velocity of quark is small,

$$v << 1,$$

This implies $m >> m v >> m v^2$

Additional scale is Λ_{QCD}

EM transition: probes internal structure and interplay between different dynamical scales

Dynamical scales in quarkonium

Labelle (1997), Pineda and Soto (1998)
Beneke and Smirnov (1998), and among many others

★ hard: $p^{\mu} \sim O(m)$

★ semi-hard (soft): $p^{\mu} \sim O(m\, v)$

★ potential: $p^{0} \sim O(m\, v^{2})$, $p^{i} \sim O(m\, v)$

★ ultra-soft: $p^{\mu} \sim O(m\, v^{2})$

Basics (What are EM transitions)

★ Very old subject, first starting from late 1920s since Paul Dirac first creates the Quantum theory that couples radiation and matter

★ Standard treatment is within NR potential quark model + QED, employing the idea of *multipole expansion*

EM transitions easily identifiable from the Nonrelativistic QED (NRQED)

NRQED Lagrangrian

$$\mathcal{L}_{\mathrm{NRQCD}} = \psi^\dagger \left\{ iD_0 + \frac{\mathrm{D}^2}{2m} + c_F\, g \frac{\boldsymbol{\sigma}\cdot \mathrm{B}}{2m} + c_D\, g \frac{[\mathrm{D}\cdot, \mathrm{E}]}{8m^2} + i c_S\, g \frac{\boldsymbol{\sigma}\cdot [\mathrm{D}\times, \mathrm{E}]}{8m^2} + \ldots \right\} \psi$$

$$\mathbf{j}\cdot \mathbf{A}_{\mathrm{em}} = e_Q \psi^\dagger \left\{ \frac{\{\mathbf{D}\cdot, \mathbf{A}_{\mathrm{em}}\}}{2m} + (1+\kappa_Q)\, \frac{\boldsymbol{\sigma}\cdot \mathbf{B}_{\mathrm{em}}}{2m} + \ldots \right\} \psi$$

E1

M1

Multipole expansion (long-wave length approximation)

Underlying physics behind multipole expansion:

photon momentum is **ultrasoft** (k ~ m v^2), much smaller than the typical 3-momentum scale of quarkonium (p~ **m v**), thus photon cannot resolve fine structure of quarkonium

photon wave length >> radius of quarkonium

***equivalently,* k r << 1**

One can multipole-expand the electromagnetic field

$$e^{\mathbf{i\,k\,r}} = 1 + I\,k\,r + \ldots$$

Two leading EM transitions—most familiar

- ★ E1: $\Delta S=0$, $\Delta L=1$ parity-changing, spin conserving $\psi' \rightarrow \chi_{cJ}+\gamma$

- ★ M1: $\Delta S=1$, $\Delta L=0$ spin-flipping, parity conserving $J/\psi \rightarrow \eta_c+\gamma$

Old-fashioned way of dealing with EM transitions are complicated and not easy to follow

★ Old way of dealing with EM transition is by combining quantum mechanics and the quantized radiation field: cumbersome and not systematic

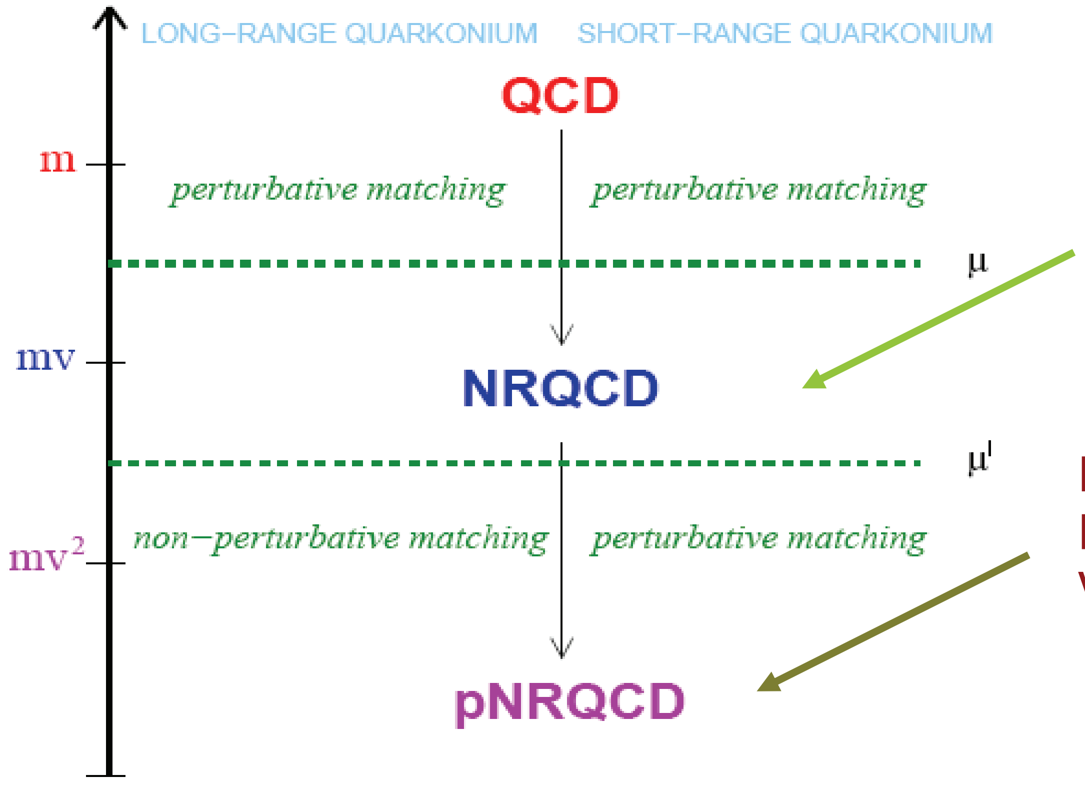
Grotch; Sebastian; Sucher; Rosner (1970-80s)

★ Modern way of treating EM transition is by invoking the effective field theory approach

★ **potential NRQCD** (pNRQCD) is the ideal framework to deal with EM transitions: incorporating long-wavelength expansion

Effective field theory machinery: useful to tackle EM transitions

Modern method, desired to deal with problem with widely separated scales



Caswell, Lepage (1986)

Pineda, Soto (1998); Brambilla, Pineda, Soto, Vairo (2000)

What EFT achieves for dealing with M1 transition

Matching NRQCD(QED) onto pNRQCD

Brambilla, Jia and Vairo (2005)

Starting from following **NRQED** lagragian:

$$
\begin{aligned}
\mathscr{L}_{\mathrm{NR}} &= \psi^\dagger\left(iD_0+\frac{\mathbf{D}^2}{2m}\right)\psi+\frac{c_F e e_Q}{2m}\psi^\dagger\sigma\cdot\mathbf{B}^{\mathrm{em}}\psi+\frac{ic_S e e_Q}{8m^2}\psi^\dagger\sigma\cdot[\nabla\times,\mathbf{E}^{\mathrm{em}}]\psi \\
&+ \frac{c_{W1} e e_Q}{8m^3}\psi^\dagger\{\nabla^2,\sigma\cdot\mathbf{B}^{\mathrm{em}}\}\psi-\frac{c_{W2} e e_Q}{4m^3}\psi^\dagger\nabla_i\sigma\cdot\mathbf{B}^{\mathrm{em}}\nabla_i\psi \\
&- \frac{c_{p'p} e e_Q}{8m^3}\left[\nabla\psi^\dagger\cdot\sigma\,\mathbf{B}^{\mathrm{em}}\cdot\nabla\psi+\nabla\psi^\dagger\cdot\mathbf{B}^{\mathrm{em}}\sigma\cdot\nabla\psi\right]+(\psi\to i\sigma^2\chi^*)\,,
\end{aligned}
$$

Multipole expansion automatically embedded within pNRQCD

pNRQCD Lagrangian density with
SU(3)$_c$xU(1)$_{em}$ gauge group

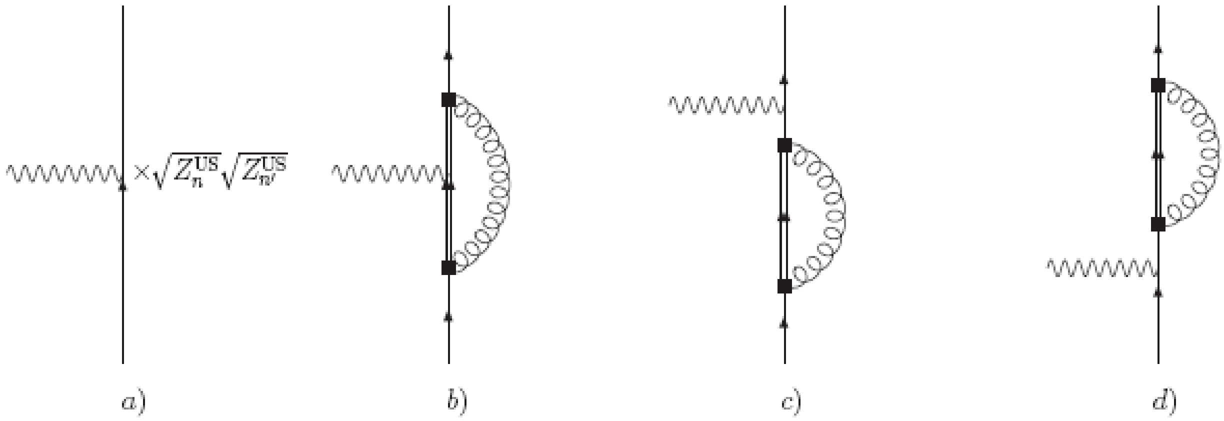
Brambilla, Jia and Vairo (2005)

$$\mathcal{L}_{\mathrm{pNRQCD}} = \int d^3r \,\mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_S^{(0)}(r) \right) \mathrm{S} \right\} + \mathcal{L}_{\mathrm{light}} + \mathcal{L}_{\gamma\,\mathrm{pNRQCD}}$$

$$\begin{aligned}\mathcal{L}_{\gamma\,\mathrm{pNRQCD}} = \int d^3r \,\mathrm{Tr}\Bigg\{ & ee_Q \mathrm{S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\mathrm{em}}\mathrm{S} + \frac{c_F^{\mathrm{em}} ee_Q}{2m} \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}}\right\}\mathrm{S} \\ & + \frac{c_F^{\mathrm{em}} ee_Q}{16m} \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot(\mathbf{r}\cdot\boldsymbol{\nabla}_R)^2\mathbf{B}^{\mathrm{em}}\right\}\mathrm{S} + \frac{ee_Q}{8m^2} r V_S^{(0)\prime} \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\mathrm{em}})\right\}\mathrm{S} \\ & - \frac{c_S^{\mathrm{em}} ee_Q}{16m^2} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}_R\times, \mathbf{E}^{\mathrm{em}}]\right]\mathrm{S} - \frac{c_S^{\mathrm{em}} ee_Q}{16m^2} \left[\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}_R^i\mathbf{E}^{\mathrm{em}})]\right]\mathrm{S} \\ & + \frac{c_{W12}^{\mathrm{em}} ee_Q}{4m^3} \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\mathrm{em}}\right\}\boldsymbol{\nabla}_r^2\mathrm{S} + \frac{c_{p'p}^{\mathrm{em}} ee_Q}{4m^3} \left\{\mathrm{S}^\dagger, \boldsymbol{\sigma}^i\,\mathrm{B}^{\mathrm{em}\,j}\right\}\boldsymbol{\nabla}_r^i\boldsymbol{\nabla}_r^j\mathrm{S} \Bigg\}\end{aligned}$$

Neat thing pNRQCD can do

- ★ Integrating out soft and potential modes
- ★ Easily investigate effect of *ultra-soft mode (Lamb-shift like effect*)



Color-octet effect vanishes

★ Easily understandable from time-independent perturbation theory of quantum mechanics

★ Because M1 operator $\sigma \cdot \mathbf{B}^{\mathrm{em}}$ is the ***unit operator*** in coordinate space

$$|N\rangle = \sqrt{Z_n^{\mathrm{us}}}|Q\overline{Q}_1(n)\rangle + |Q\overline{Q}_8 g\rangle + \sum_{m \neq n} |Q\overline{Q}_1(m)\rangle \cdots ,$$

EFT prediction to $J/\psi \rightarrow \eta_c + \gamma$

Reproduce Standard M1 transition width (1st -order relativistic correction included) Eichten's review in CERN-QWG-Yellow Book

$$
\begin{aligned}
I_1 &= \left\langle n'0 \left| (1+\kappa_Q)\left(1-\frac{k_\gamma^2 r^2}{24}\right) + (1+2\kappa_Q)\frac{k_\gamma}{4m} \right| n0 \right\rangle \\
I_2 &= -\left\langle n'0 \left| (1+\kappa_Q)\frac{\mathbf{p}^2}{2m^2} + \frac{\mathbf{p}^2}{3m^2} \right| n0 \right\rangle , \\
I_3 &= \left\langle n'0 \left| \frac{\kappa_Q}{6m} r V_0' \right| n0 \right\rangle , \\
I_4 &= \pm \frac{4}{E_{n0}^{(0)} - E_{n'0}^{(0)}} \left\langle n'0 \left| (1+\kappa_Q)\frac{V_{ss}}{m^2} \right| n0 \right\rangle , \\
\xcancel{I_5} &\xcancel{= \left\langle n'0 \left| -\frac{\eta}{m} V_S \right| n0 \right\rangle ,}
\end{aligned}
$$

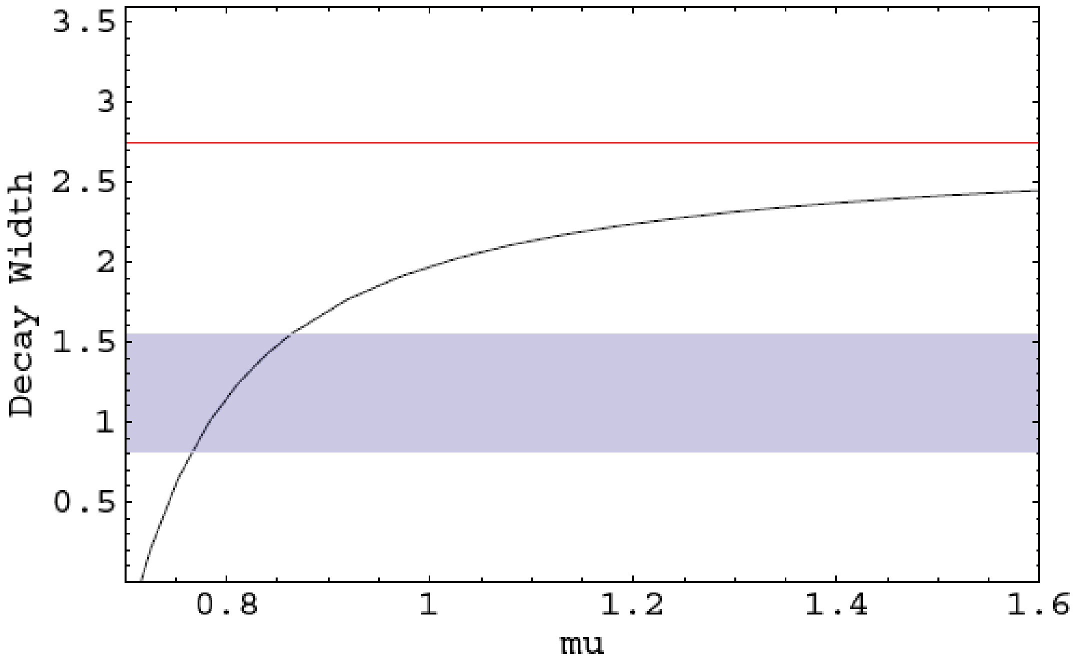
Insights gained from pNRQCD approach with respect to phenomenological potential model

- ★ No large nonperturbatively-induced anomalous magnetic dipole moment of bound heavy quark
- ★ No scalar-confinement piece contribution (I_5 term should not be present)
- ★ Leading nonperturbative ultrasoft effects vanishes

EFT prediction to $J/\psi \rightarrow \eta_c + \gamma$

★ weakly-coupled (***Coulombic***) J/ψ

$$\Gamma[J/\psi \to \eta_c\gamma] \approx \frac{4\alpha_{\rm em} e_c^2}{3\hat{m}_c^2} k_\gamma^3 \left[1 + 2\kappa_c - \frac{2C_F^2\alpha_s^2(\mu)}{3}\right]$$



★ Examine whether the X(3872) quantum number can be 2^{-+} or not

Jia, Sang and Xu, arXiv:1007.4541,
submitted to Phys. ReV. D

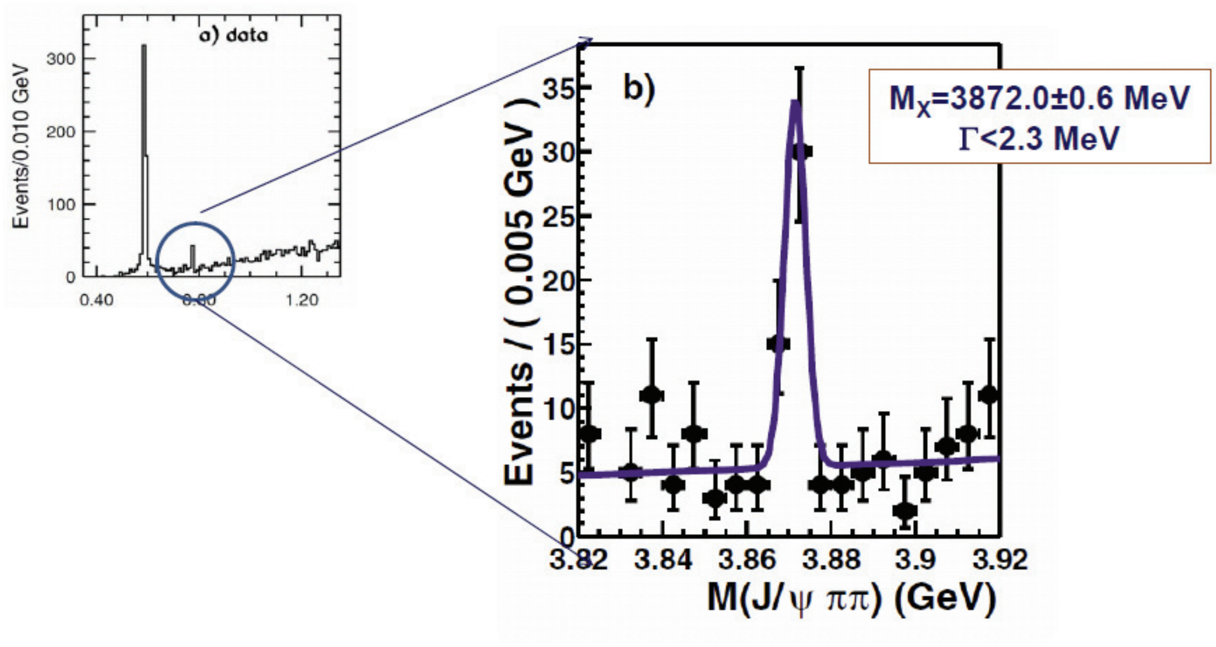
★ The first systematic work investigating the radiative transition processes

$$\eta_{c2}({}^1D_2) \rightarrow J/\psi(\psi') + \gamma$$

Combine pNRQCD with potential models

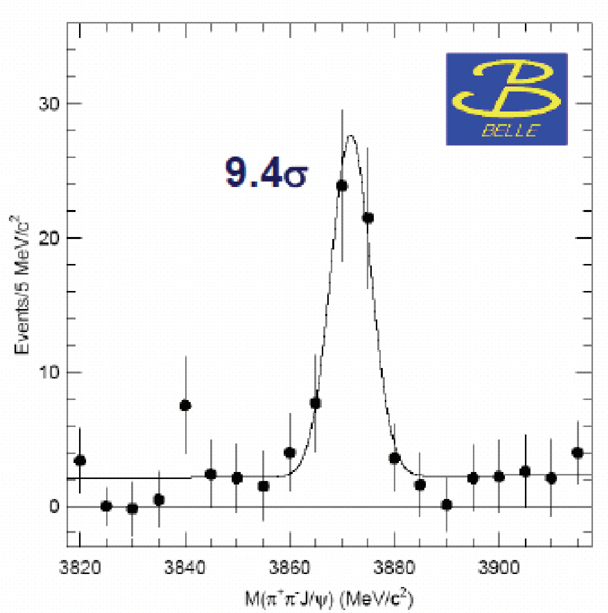
X(3872): The first exotic charmonium above threshold observed since 2003

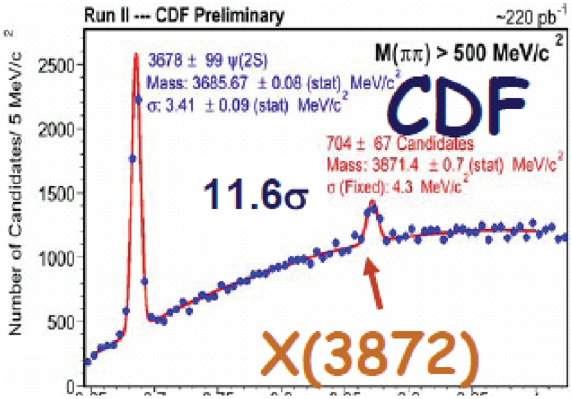
X(3872)

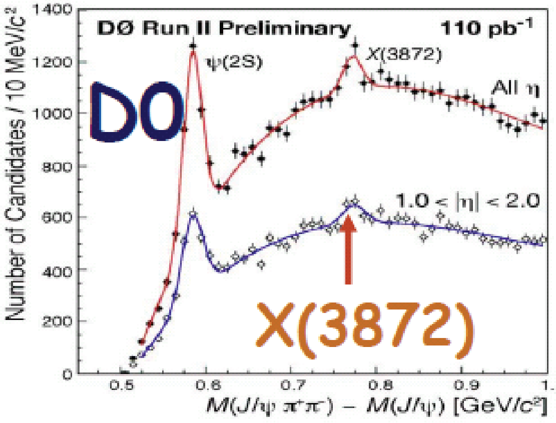


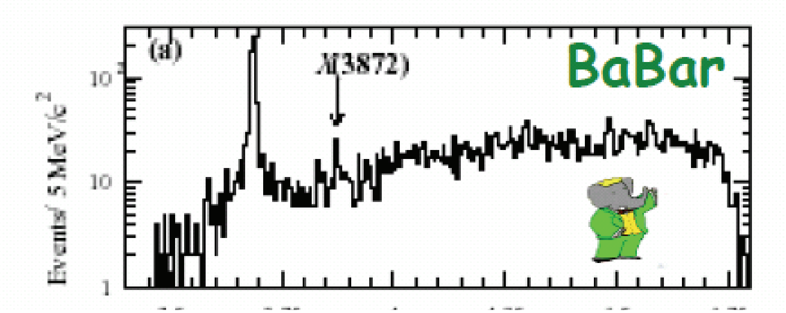
Belle PRL **91**, 262001 (2003)

X_{3872} is well established
seen in 4 experiments









The popular interpretation of X(3872)

Difficult to understand for the X particle to a conventional charonium state χ_c **(2P)**

Popular exotic explanation:

★ S-wave DD* molecule

Tornqvist, Close, Page, Voloshin, Braaten

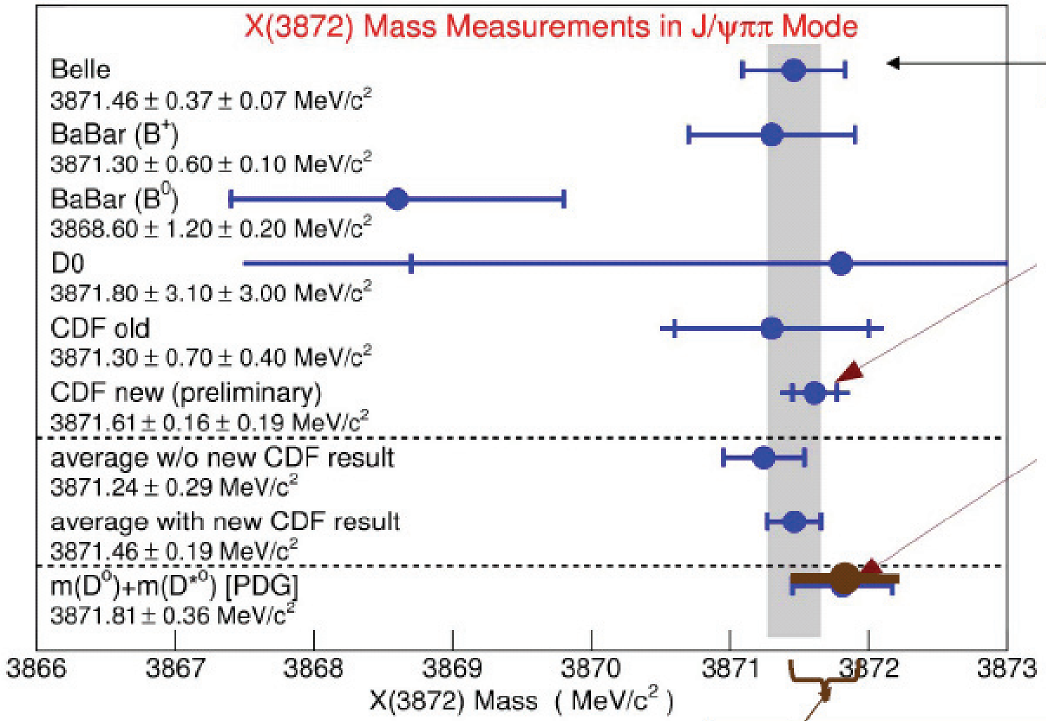
★ Tetraquark state

Miani, Polosa et al.

Other explanation: Cusp? Hybrid? …

$M_{X(3872)} \approx M_{D^0} + M_{D^{*0}}$

$<M_X> = 3871.46 \pm 0.19$ MeV



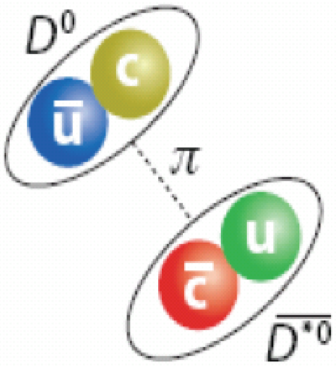
new Belle meas.

new CDF meas.

$M_{D0} + M_{D^*0}$
3871.8±0.4 MeV

$\delta m = -0.35 \pm 0.41$ MeV

X(3872) looks like a $D^0\overline{D}^{*0}$ "molecule"



N. Tornqvist
Z. Phys C 61, 525 (1994)

Composite	J^{PC}	Deuson
$D\bar{D}^*$	0^{-+}	η_c ($\approx$ 3 870)
$D\bar{D}^*$	1^{++}	χ_{c1}($\approx$ 3 870)
$D^*\bar{D}^*$	0^{++}	χ_{c0}($\approx$ 4 015)
$D^*\bar{D}^*$	0^{-+}	η_c ($\approx$ 4 015)
$D^*\bar{D}^*$	1^{+-}	h_{c0}($\approx$ 4 015)
$D^*\bar{D}^*$	2^{++}	χ_{c2}($\approx$ 4 015)

predicted by Tornqvist in 1994, requires: $J^{PC} = 1^{++}$ or 0^{-+}

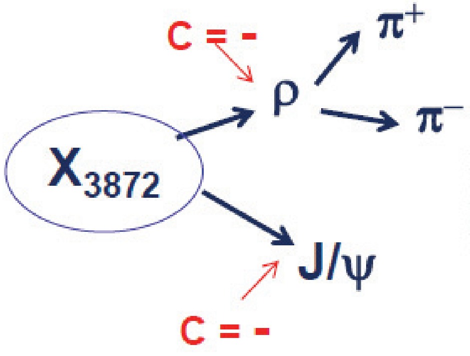
$C(X_{3872}) = +$

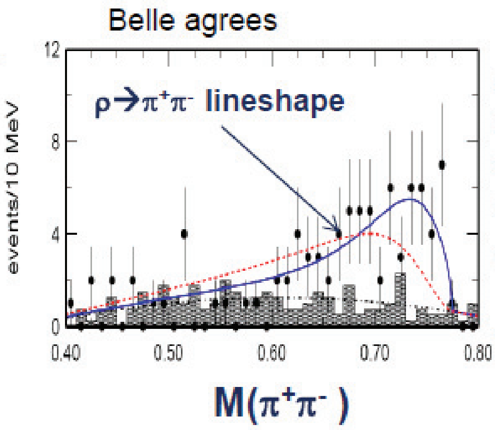
$X_{3872} \rightarrow \gamma\ J/\psi$ ➡ $C(X_{3872})$ must be (-)×(-) = +

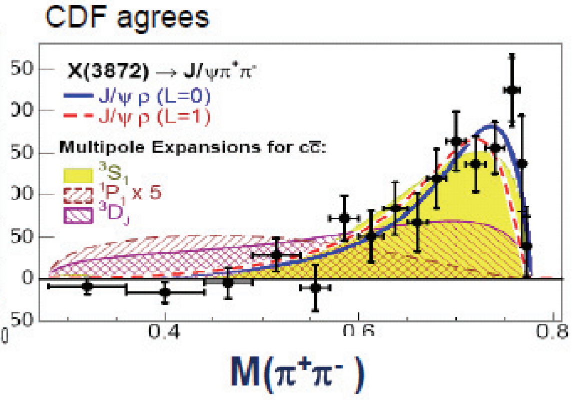
C = -   C = -

if $C(X_{3872})$ is + :

$\pi^+\pi^-$ system in $X_{3872} \rightarrow \pi^+\pi^-\ J/\psi$ must come from $\rho \rightarrow \pi^+\pi^-$

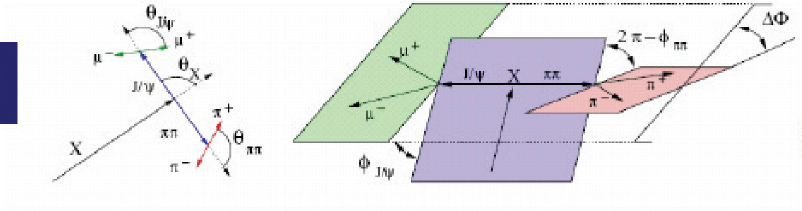



Belle agrees


CDF agrees

CDF angular correlation analysis

Only 1^{++} or 2^{-+} fit data



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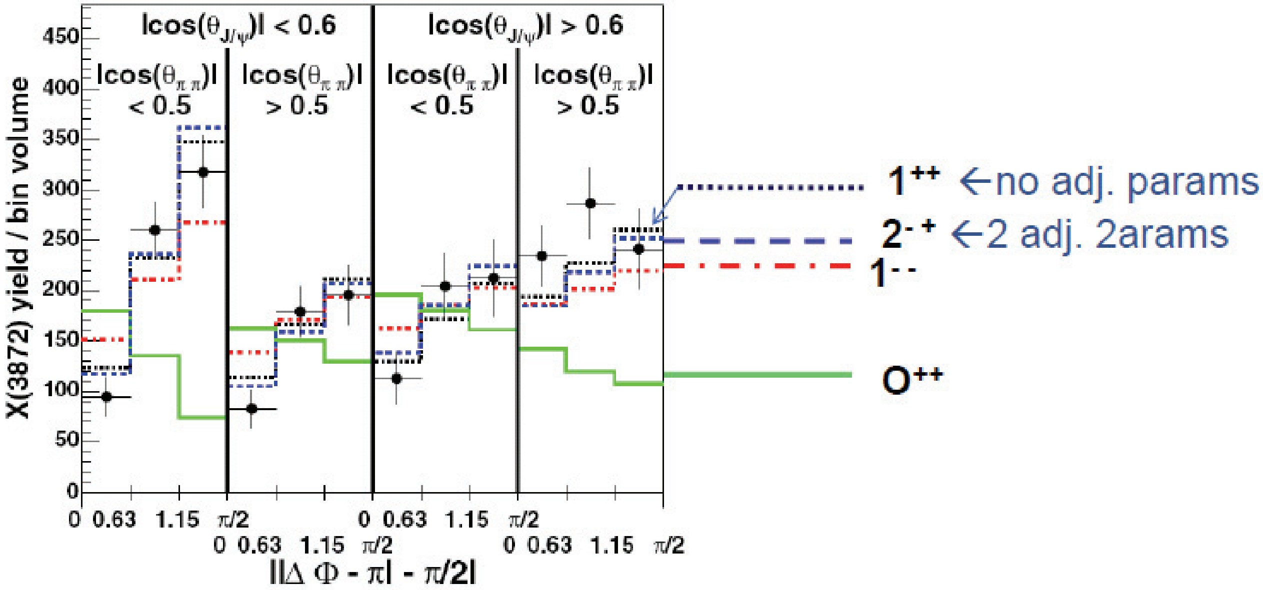


1^{++} fits well with no adjustable parameters

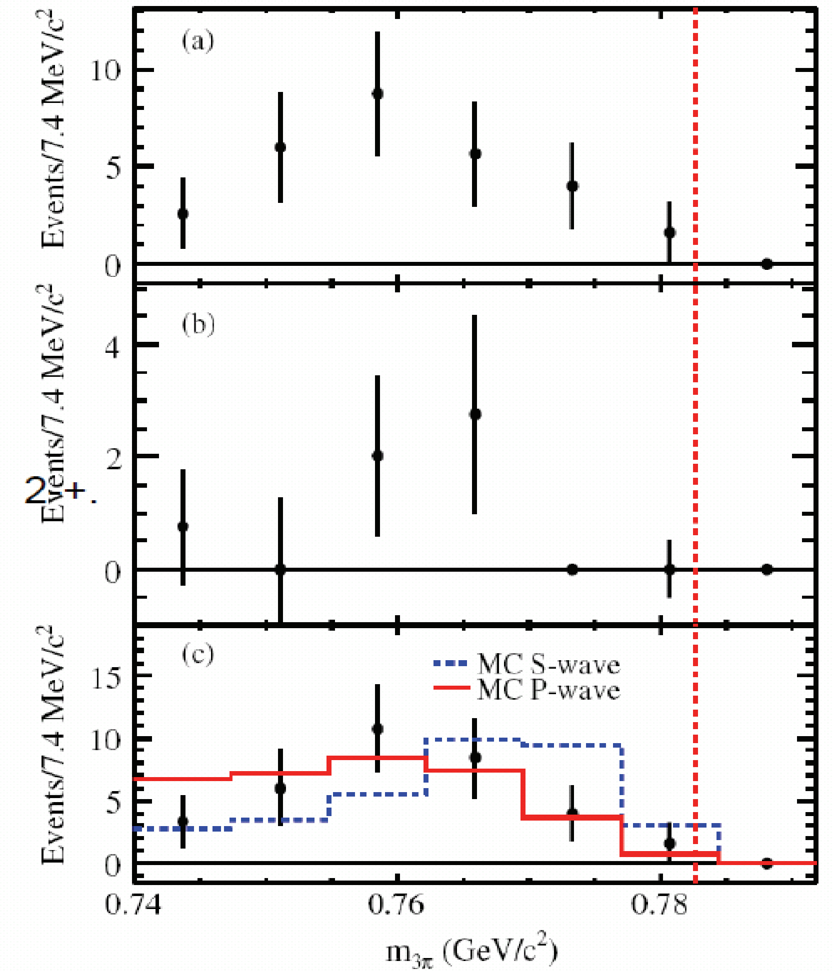
2^{-+} looks like 1^{++} for $\alpha \approx 0.6 e^{i20^o}$, at least with current statistics

New surprise this year by BaBar!

However, very recently experiment analysis by BABAR indicates that the J^{PC} of X(3872) would more favor 2^{-+} rather than 1^{++} .

BABAR PRD (2010)

$$X \to J/\psi\omega$$



$J^{PC} = 1^{++}$ or 2^{-+} ?

Our strategy:

Assuming the *X(3872)* is η_{c2} ($^1D_2$) , studying the EM transition processes:

$$\eta_{c2} \rightarrow J/\psi(\psi') + \gamma$$

Then compare with BaBar and Belle data

Identify those spin-dependent γ-QQbar interactions from pNRQCD lagrangian

pNRQCD Lagrangian density with SU(3)$_c$xU(1)$_{em}$ gauge group

Brambilla, Jia and Vairo (2005)

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, {\rm Tr} \left\{ {\rm S}^\dagger \left(i\partial_0 + \frac{\boldsymbol{\nabla}_R^2}{4m} + \frac{\boldsymbol{\nabla}_r^2}{m} - V_S^{(0)}(r) \right) {\rm S} \right\} + \mathcal{L}_{\rm light} + \mathcal{L}_{\gamma\,\rm pNRQCD}$$

$$\begin{aligned}\mathcal{L}_{\gamma\,\rm pNRQCD} = \int d^3r \, {\rm Tr} \Bigg\{ & ee_Q {\rm S}^\dagger \mathbf{r}\cdot\mathbf{E}^{\rm em}{\rm S} + \frac{c_F^{\rm em} ee_Q}{2m} \, \{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\rm em}\}\,{\rm S} \\ & + \frac{c_F^{\rm em} ee_Q}{16m} \, \{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot(\mathbf{r}\cdot\boldsymbol{\nabla}_R)^2\mathbf{B}^{\rm em}\}\,{\rm S} + \frac{ee_Q}{8m^2}\, rV_S^{(0)\prime} \, \{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\times(\hat{\mathbf{r}}\times\mathbf{B}^{\rm em})\}\,{\rm S} \\ & - \frac{c_S^{\rm em} ee_Q}{16m^2} \, [{\rm S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}_R\times, \mathbf{E}^{\rm em}]]\,{\rm S} - \frac{c_S^{\rm em} ee_Q}{16m^2} \, [{\rm S}^\dagger, \boldsymbol{\sigma}\cdot[-i\boldsymbol{\nabla}_r\times, \mathbf{r}^i(\boldsymbol{\nabla}_R^i\mathbf{E}^{\rm em})]]\,{\rm S} \\ & + \frac{c_{W12}^{\rm em} ee_Q}{4m^3} \, \{{\rm S}^\dagger, \boldsymbol{\sigma}\cdot\mathbf{B}^{\rm em}\}\, \boldsymbol{\nabla}_r^2{\rm S} + \frac{c_{p'p}^{\rm em} ee_Q}{4m^3} \, \{{\rm S}^\dagger, \boldsymbol{\sigma}^i\,\mathbf{B}^{{\rm em}\,j}\}\, \boldsymbol{\nabla}_r^i\boldsymbol{\nabla}_r^j{\rm S} \Bigg\}\end{aligned}$$

Some technicalities

Consider the S-D-wave mixing for ψ'

$$\boxed{|\psi'\rangle = \cos\phi|2^3S_1\rangle - \sin\phi|1^3D_1\rangle,}$$

$$\langle 0|S(\mathbf{R},\mathbf{r})|n^3S_1(\mathbf{P},\lambda)\rangle = \frac{1}{\sqrt{4\pi}} R_{n0}(r) \frac{\boldsymbol{\sigma}\cdot \mathrm{e}_{n^3S_1}(\lambda)}{\sqrt{2}} e^{i\mathbf{P}\cdot\mathbf{R}},$$

$$\langle 0|S(\mathbf{R},\mathbf{r})|n^1P_1(\mathbf{P},\lambda)\rangle = \sqrt{\frac{3}{4\pi}} R_{n1}(r) \frac{\mathrm{e}_{n^1P_1}(\lambda)\cdot\hat{\mathbf{r}}}{\sqrt{2}} e^{i\mathbf{P}\cdot\mathbf{R}},$$

$$\langle 0|S(\mathbf{R},\mathbf{r})|n^1D_2(\mathbf{P},\lambda)\rangle = \sqrt{\frac{15}{8\pi}} R_{n2}(r) \frac{\hat{r}^i\, h^{ij}_{n^1D_2}(\lambda)\,\hat{r}^j}{\sqrt{2}} e^{i\mathbf{P}\cdot\mathbf{R}},$$

$$\langle 0|S(\mathbf{R},\mathbf{r})|n^3D_1(\mathbf{P},\lambda)\rangle = \frac{3}{\sqrt{8\pi}} R_{n2}(r) \frac{\mathrm{e}(\lambda)\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} - \frac{1}{3}\,\mathrm{e}(\lambda)\cdot\boldsymbol{\sigma}}{\sqrt{2}} e^{i\mathbf{P}\cdot\mathbf{R}}.$$

Transition amplitude

$$\mathcal{M}[^1D_2 \to {}^3S_1 + \gamma] = \frac{ee_Q}{2m_Q}\sqrt{\frac{2}{15}}\left\{\frac{\mathbf{e}^* \cdot \mathbf{k} \times \varepsilon_\gamma^*\, k^i h^{ij} k^j}{|\mathbf{k}|^2} c_F^{\mathrm{em}} J_1 + k^i h^{ij} \left(\mathbf{e}^* \times \varepsilon_\gamma^*\right)^j \left(c_S^{\mathrm{em}} - 1\right) J_2 \right.$$
$$\left. + \, e^{*i} h^{ij} \left(\mathbf{k} \times \varepsilon_\gamma^*\right)^j \left(J_2 + J_4 - c_{p'p}^{\mathrm{em}} J_3\right) \right\}$$

$$\mathcal{M}[^1D_2 \to {}^3D_1 + \gamma] = \frac{ee_Q}{2m_Q}\frac{6}{\sqrt{15}} e^{*i} h^{ij} \left(\mathbf{k} \times \varepsilon_\gamma^*\right)^j c_F^{\mathrm{em}} J_0$$

$$J_0 = \int_0^\infty dr\, R_{^3D_1}(r) R_{^1D_2}(r)\, r^2,$$
$$J_1 = -\frac{|\mathbf{k}|^2}{4}\int_0^\infty dr\, R_{n^3S_1}(r) R_{^1D_2}(r)\, r^4,$$
$$J_2 = \frac{|\mathbf{k}|}{2m_Q}\int_0^\infty dr\, R'_{n^3S_1}(r) R_{^1D_2}(r)\, r^3,$$
$$J_3 = -\frac{1}{m_Q^2}\int_0^\infty dr\, \left(R''_{n^3S_1}(r) - \frac{R'_{n^3S_1}(r)}{r}\right) R_{^1D_2}(r)\, r^2,$$
$$J_4 = \frac{1}{2m_Q}\int_0^\infty dr\, R_{n^3S_1}(r) V_S^{(0)\prime}(r) R_{^1D_2}(r)\, r^3.$$

$$R'' + \frac{2}{r}R' + \left[m(E - V(r)) - \frac{l(l+1)}{r^2}\right]R = 0$$

R is the solution of radial schroedinger equation. And *V(r)* is potential determined by potential model.

Helicity and multipole amplitudes

$$A_0 \equiv A_{1,1} = -A_{-1,-1} = \frac{2c_F^{\rm em} J_1 - (2c_S - 1) J_2 + c_{p'p}^{\rm em} J_3 - J_4 + 3\sqrt{2} c_F^{\rm em} J_0 \sin\phi}{\sqrt{6}},$$

$$A_1 \equiv A_{0,1} = -A_{0,-1} = \frac{-c_S^{\rm em} J_2 + c_{p'p}^{\rm em} J_3 - J_4 + 3\sqrt{2} c_F^{\rm em} J_0 \sin\phi}{\sqrt{2}},$$

$$A_2 \equiv A_{-1,1} = -A_{1,-1} = -J_2 + c_{p'p}^{\rm em} J_3 - J_4 + 3\sqrt{2} c_F^{\rm em} J_0 \sin\phi,$$

The connection between helicity amplitude and multipole amplitude:

$$A_\nu = \sum_{J_\gamma} \sqrt{\frac{2J_\gamma + 1}{2J_{\eta_{c2}} + 1}}\, a_{J_\gamma} \langle J_\gamma, 1; 1, \nu - 1 | J_{\eta_{c2}}, \nu \rangle$$

Karl, Meshkov and Rosner: PRL (1980)

One can easily get the orthogonal transformation between helicity and multipole amplitudes

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \sqrt{\frac{3}{10}} & \sqrt{\frac{3}{5}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{5}} & -2\sqrt{\frac{2}{15}} & \frac{1}{\sqrt{15}} \end{pmatrix} \begin{pmatrix} A_0 \\ A_1 \\ A_2 \end{pmatrix},$$

And

$$\begin{aligned} a_1 &= \frac{2c_F^{\rm em} J_1 - 5(1 + c_S^{\rm em}) J_2 + 10 c_{p'p}^{\rm em} J_3 - 10 J_4 + 30\sqrt{2} c_F^{\rm em} J_0 \sin\phi}{2\sqrt{15}}, \\ a_2 &= \frac{2c_F^{\rm em} J_1 - 3(c_S^{\rm em} - 1) J_2}{2\sqrt{3}}, \\ a_3 &= \frac{2c_F^{\rm em} J_1}{\sqrt{15}}. \end{aligned}$$

a1, a2, a3 means M1, E2, M3 multipole amplitude respectively.

Decay Width

It's easy to get the decay width expression for the orthogonality between different helicity (multipole) amplitudes.

$$\Gamma[\eta_{c2} \to J/\psi(\psi') + \gamma] = \frac{2\alpha e_Q^2 |\mathbf{k}|^3}{75 m_c^2} \left(|A_0|^2 + |A_1|^2 + |A_2|^2 \right)$$

One also can express the decay width via multipole amplitudes:

$$\Gamma[\eta_{c2} \to J/\psi(\psi') + \gamma] = \frac{2\alpha e_Q^2 |\mathbf{k}|^3}{75 m_c^2} \left(|a_1|^2 + |a_2|^2 + |a_3|^2 \right)$$

Different Potential Models

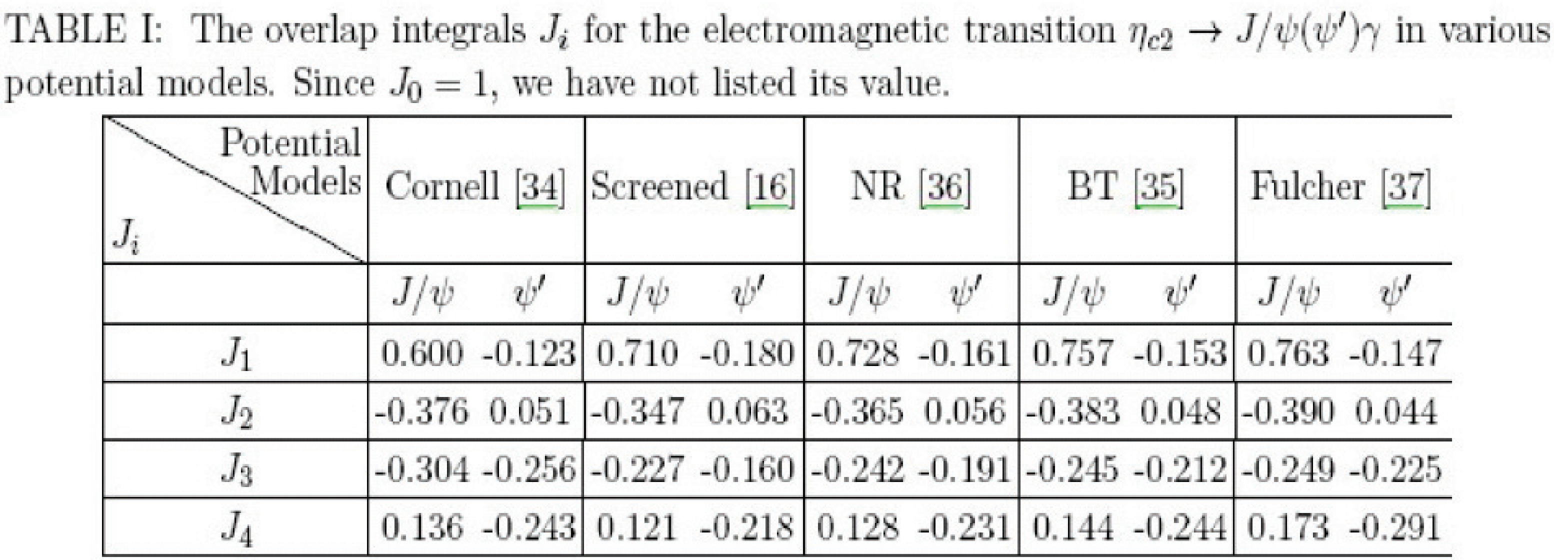


TABLE I: The overlap integrals J_i for the electromagnetic transition $\eta_{c2} \to J/\psi(\psi')\gamma$ in various potential models. Since $J_0 = 1$, we have not listed its value.

J_i \ Potential Models	Cornell [34]		Screened [16]		NR [36]		BT [35]		Fulcher [37]	
	J/ψ	ψ'	J/ψ	ψ'	J/ψ	ψ'	J/ψ	ψ'	J/ψ	ψ'
J_1	0.600	-0.123	0.710	-0.180	0.728	-0.161	0.757	-0.153	0.763	-0.147
J_2	-0.376	0.051	-0.347	0.063	-0.365	0.056	-0.383	0.048	-0.390	0.044
J_3	-0.304	-0.256	-0.227	-0.160	-0.242	-0.191	-0.245	-0.212	-0.249	-0.225
J_4	0.136	-0.243	0.121	-0.218	0.128	-0.231	0.144	-0.244	0.173	-0.291

Cornell Potential: E.Eichten *et al.* (1978) (1980)

Screened Potential: B. Q. Li and K. T. Chao (2009)

NR Potential: T. Barnes, S. Godfrey and E.S. Wanson (1980)

BT Potential: W. Buchmuller and S. H. H. Tye (1981)

Fulcher Potential: L. P. Fulcher (1991)

Physical Predictions

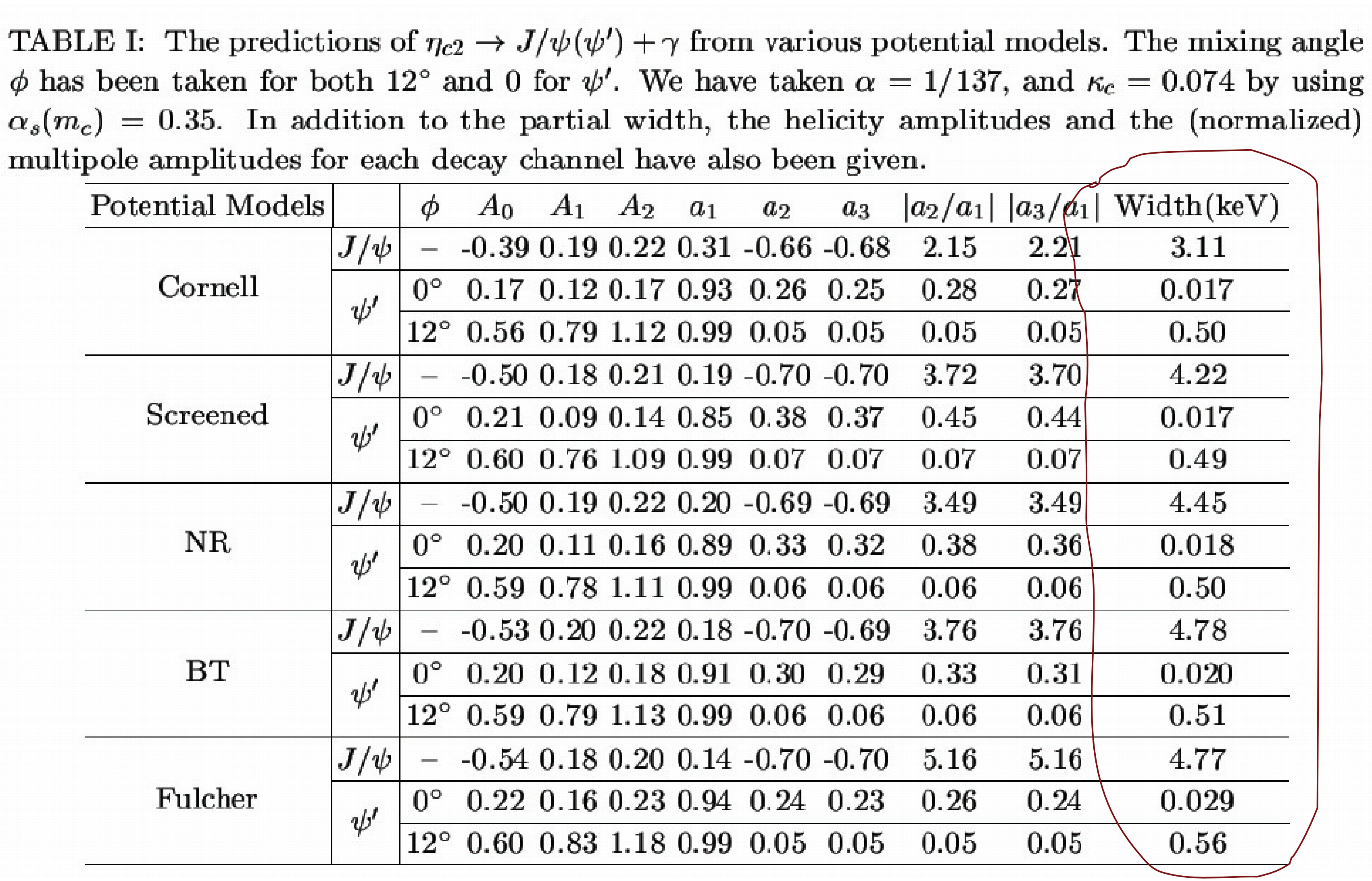


TABLE I: The predictions of $\eta_{c2} \to J/\psi(\psi') + \gamma$ from various potential models. The mixing angle ϕ has been taken for both 12° and 0 for ψ'. We have taken $\alpha = 1/137$, and $\kappa_c = 0.074$ by using $\alpha_s(m_c) = 0.35$. In addition to the partial width, the helicity amplitudes and the (normalized) multipole amplitudes for each decay channel have also been given.

Potential Models		ϕ	A_0	A_1	A_2	a_1	a_2	a_3	$\|a_2/a_1\|$	$\|a_3/a_1\|$	Width(keV)
Cornell	J/ψ	–	-0.39	0.19	0.22	0.31	-0.66	-0.68	2.15	2.21	3.11
	ψ'	0°	0.17	0.12	0.17	0.93	0.26	0.25	0.28	0.27	0.017
		12°	0.56	0.79	1.12	0.99	0.05	0.05	0.05	0.05	0.50
Screened	J/ψ	–	-0.50	0.18	0.21	0.19	-0.70	-0.70	3.72	3.70	4.22
	ψ'	0°	0.21	0.09	0.14	0.85	0.38	0.37	0.45	0.44	0.017
		12°	0.60	0.76	1.09	0.99	0.07	0.07	0.07	0.07	0.49
NR	J/ψ	–	-0.50	0.19	0.22	0.20	-0.69	-0.69	3.49	3.49	4.45
	ψ'	0°	0.20	0.11	0.16	0.89	0.33	0.32	0.38	0.36	0.018
		12°	0.59	0.78	1.11	0.99	0.06	0.06	0.06	0.06	0.50
BT	J/ψ	–	-0.53	0.20	0.22	0.18	-0.70	-0.69	3.76	3.76	4.78
	ψ'	0°	0.20	0.12	0.18	0.91	0.30	0.29	0.33	0.31	0.020
		12°	0.59	0.79	1.13	0.99	0.06	0.06	0.06	0.06	0.51
Fulcher	J/ψ	–	-0.54	0.18	0.20	0.14	-0.70	-0.70	5.16	5.16	4.77
	ψ'	0°	0.22	0.16	0.23	0.94	0.24	0.23	0.26	0.24	0.029
		12°	0.60	0.83	1.18	0.99	0.05	0.05	0.05	0.05	0.56

Discussion

Surveying the predicted partial widths from various potential models, one can place the following upper limit for the ratio of the two branching fractions:

$$\frac{\mathcal{B}[\eta_{c2} \to \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \to J/\psi + \gamma]} < 0.16, (Cornell) \qquad \phi = 12^\circ$$

$$\frac{\mathcal{B}[\eta_{c2} \to \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \to J/\psi + \gamma]} < 6.1 \times 10^{-3}.(Fulcher) \qquad \phi = 0^\circ$$

Contradictions

The ratio between two channel contradiction with the BABAR measurement no matter considering the mixing angle or not.

Theoretical:

$$\frac{\mathcal{B}[\eta_{c2} \to \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \to J/\psi + \gamma]} < 0.16, (Cornell) \qquad \phi = 12^\circ$$

$$\frac{\mathcal{B}[\eta_{c2} \to \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \to J/\psi + \gamma]} < 6.1 \times 10^{-3}.(Fulcher) \qquad \phi = 0^\circ$$

Experiment:

$$\frac{\mathcal{B}[X(3872) \to \psi' + \gamma]}{\mathcal{B}[X(3872) \to J/\psi + \gamma]} = 3.4 \pm 1.4.$$

The absolute branching fraction for each channel

- The total width of X(3872) in PDG(2008):

$$\Gamma = 3.0^{+2.1}_{-1.7}\ \mathrm{MeV}$$

For a conservative estimate we use the lower end of the PDG data: $\Gamma_X = $ 1.3 MeV, and largest predicted patial width for each two channel, then we estimate:

$$\mathcal{B}[\eta_{c2} \to J/\psi + \gamma] < 3.7 \times 10^{-3}. \quad (\mathrm{BT\ potential})$$

$$\mathcal{B}[\eta_{c2} \to \psi' + \gamma] < 4.3 \times 10^{-4}, \ (\mathrm{Fulcher}) \qquad \phi = 12^\circ$$

$$\mathcal{B}[\eta_{c2} \to \psi' + \gamma] < 2.2 \times 10^{-5}. \ (\mathrm{Fulcher}) \qquad \phi = 0^\circ$$

Compare our predictions with the data

Theoretical:

$$\mathcal{B}[\eta_{c2} \to J/\psi + \gamma] < 3.7 \times 10^{-3}. \quad \text{(BT potential)}$$

$$\mathcal{B}[\eta_{c2} \to \psi' + \gamma] < 4.3 \times 10^{-4}, \text{(Fulcher)} \qquad \phi = 12^\circ$$

$$\mathcal{B}[\eta_{c2} \to \psi' + \gamma] < 2.2 \times 10^{-5}. \text{(Fulcher)} \qquad \phi = 0^\circ$$

Experiment:

$$\mathcal{B}[X(3872) \to J/\psi + \gamma] > 5.9 \times 10^{-3}, \quad \mathcal{B}[X(3872) \to \psi' + \gamma] > 1.9 \times 10^{-2},$$

Contradiction!

★ New mechansim to treat strongly hindered EM transitions

Jia, Xu, Zhang, PRD82: 014008, 2010

(arXiv:0901.4021)

What is the proper theoretical framework when photon becomes hard so that multipole expansion breaks down?

Example: $\Upsilon(3S) \to \eta_b + \gamma$ (hindered M1 transition)

$\psi(2S) \to \eta_c + \gamma$

hard-scattering mechanism

$\Upsilon(3S) \rightarrow \eta_b + \gamma$: the discover channel of $\eta_b(1S)$

- ★ $\Upsilon(1S)$ **was initially found in 1977,** only 3 years later than J/ψ

- ★ It takes 30 years to find this elusive particle

- ★ In 2008, we finally had a definite answer

BABAR, PRL 2008

What if Multipole expansion becomes invalid?

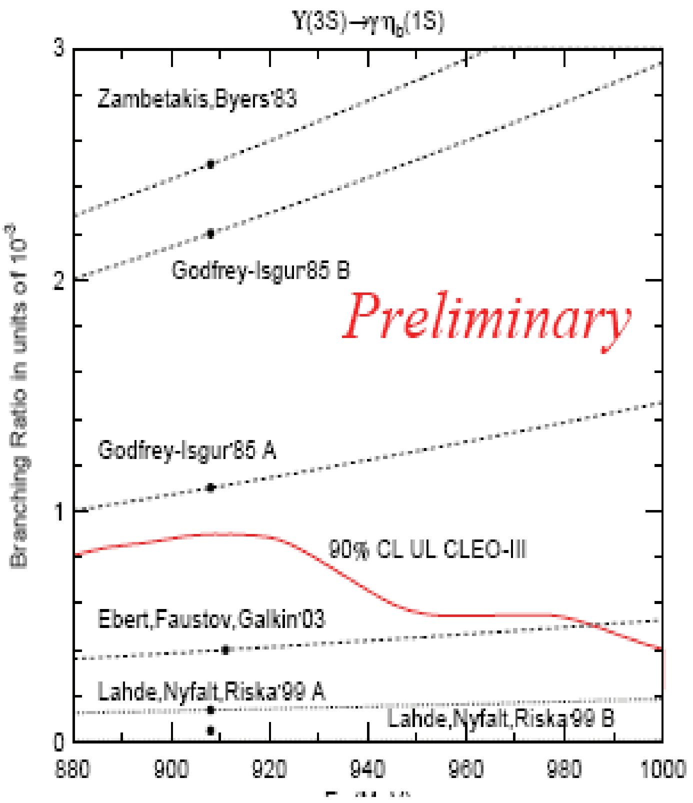
For hindered M1 transition $\Upsilon(3S) \rightarrow \eta_b + \gamma$,

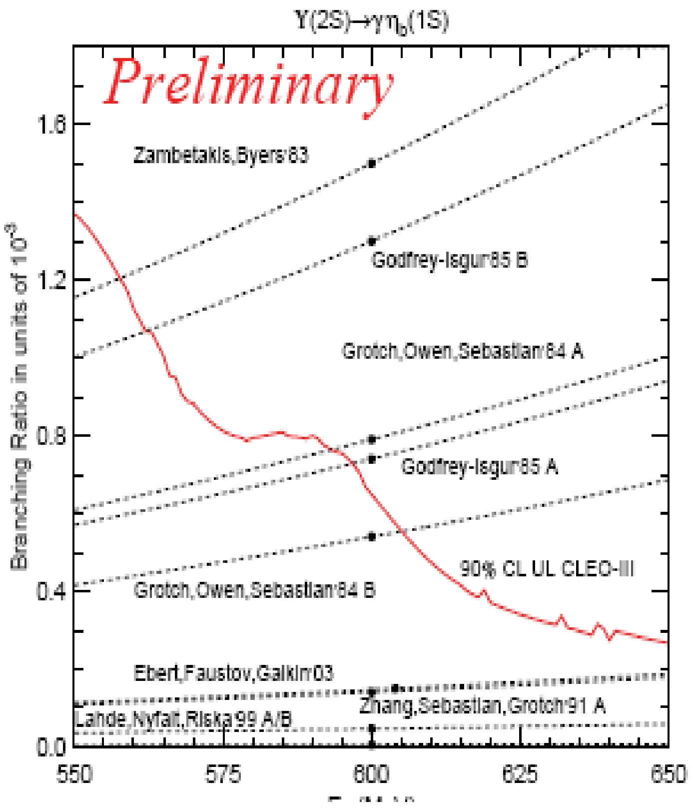
The photon carries momentum~ 1 GeV, not natural to be regarded as ultrasoft

In other words, $k\,r \sim 1$, ME breaks down:

$e^{i\mathbf{k}\,\mathbf{r}} = 1 + i\,k\,r + \ldots$ ***can not be justified!***

A variety of predictions have been made (CLEO-III 2004)





No satisfactory theory for strongly-hindered M1 transition exists yet!

- ★ **Lattice QCD**: difficult to measure the strongly-hindered M1 transition,

  difficult to prepare the highly excited states

  (Dudek et al. 2006)

- ★ **Hadron-loop model**: to model coupled channel effects for $\psi' \rightarrow \eta_c + \gamma$

  (Li and Zhao, 2008)

  Requires some free input parameters and not very predictive

Our goal

- ★ It is desirable to understand the Babar measured result: (BABAR, PRL 2008)

$$B[\Upsilon(3S) \rightarrow \eta_b + \gamma] = (4.8 \pm 1.3) \times 10^{-4}$$

- ★ **Motivation:** *Is that possible to reproduce this value in a* ***simple*** *scenario and a* ***predictive*** *framework?*

The strongly-hindered M1 problem

The key assumption:

Assuming the photon to be semi-hard (soft): $k^\mu \sim O(m v)$, rather than ultrasoft $O(m v^2)$

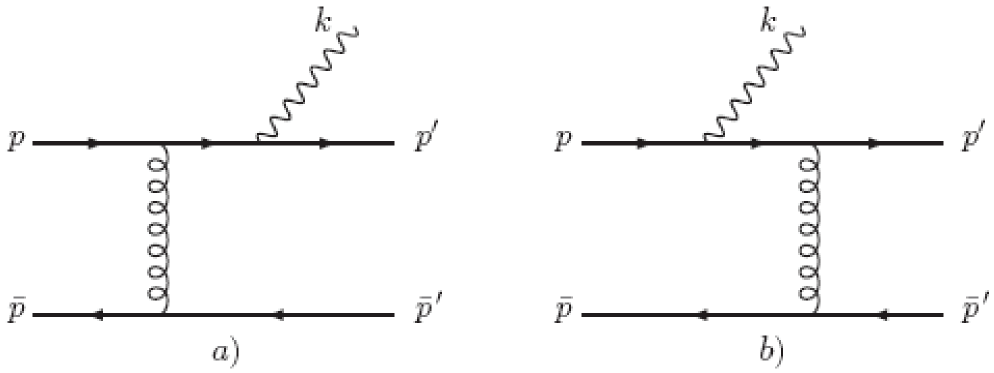
★ We give up **M.E. (long-wavelength approximation)** in the very beginning

If a potential quark emits a semi-hard photon, it must have virtuality greater than $m v^2$, so cannot emerge in an external state.

Elucidating Analogy: π EM form factor at large momentum transfer

- ★ Hard-scattering picture is more plausible than Feynman/soft mechanism.

Lepage and Brodsky (1980)



Exchange of a hard gluon is essential!

Factorization: the key idea

- ★ Exchange of hard gluon is mandatory
- ★ Integrating out **hard** quanta to achieve factorization

$$F_\pi(Q^2) = \int_0^1 \int_0^1 dx dy \, \phi_\pi(x) T(x, y, Q) \phi_\pi(y) + \cdots$$

Factorization: the key idea for the applicability of perturbative QCD

- ★ Only because of the nontrivial trait of ***factorization***, ***pQCD*** then has predictive power

- ★ Otherwise we can not do much, like $\rho \rightarrow \pi\gamma$, one has to entirely resort to nonperturbative tools such as **Lattice simulation** or **QCD sum rules**

Structure of M1 amplitude: dictated by parity and Lorentz invariance

One define Lorentz scalar reduced amplitude $\mathscr{A}$

$$\mathcal{M}[n\,{}^3S_1(P) \to n'\,{}^1S_0(P') + \gamma(k)]$$
$$= \mathscr{A}\, \epsilon_{\mu\nu\alpha\beta}\, P^\mu\, \varepsilon^{\nu}_{[n\,{}^3S_1]}\, k^\alpha\, \varepsilon^{*\beta}_{\gamma},$$

Simple manipulations (the game about separating scales)

Expand **quark propagator** in relative velocity ***v***:

$$\frac{1}{(p'+k)^2-m^2}=\frac{1}{k\cdot P'+2k\cdot q'}$$
$$\approx\frac{1}{k\cdot P}+\frac{2\,\mathbf{k}\cdot\mathbf{q}'}{(k\cdot P)^2}+\cdots,$$

Manipulate **gluon propagator** as follows:

$$\frac{1}{(\frac{k}{2}+q'-q)^2+i\epsilon}\approx\frac{-1}{(\mathbf{q}'-\mathbf{q})^2+\mathbf{k}\cdot(\mathbf{q}'-\mathbf{q})-i\epsilon}.$$

Note: Cannot further expand the denominator!

Factorization formula in momentum space

$$\mathscr{A} = 2\frac{4ee_Q g_s^2 C_F}{(k\cdot P)^2}\iint\frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3}\,\phi^*_{n'0}(\mathbf{q}') \times T(\mathbf{q}'-\mathbf{q})\,\phi_{n0}(\mathbf{q}),$$

$$T(\mathbf{q}) = -\frac{\mathbf{k}\cdot\mathbf{q}}{\mathbf{q}^2+\mathbf{k}\cdot\mathbf{q}-i\epsilon}.$$

T – semi-hard-scattering kernel pert.

ϕ – momentum-space wave function of quarkonium nonpert.

Factorization formula in coordinate space

- ★ ***Fourier transform*** is straightforward

$$\mathscr{A} = \frac{4ee_Q C_F \alpha_s}{M_n} \mathscr{E}_{nn'},$$

$$\mathscr{E}_{nn'} = \int_0^\infty dr\, r^2\, R^*_{n'0}(r)\, \mathscr{T}(r)\, R_{n0}(r),$$

- ★ Where [*contour integral*, note iϵ prescription]

$$\boxed{\mathscr{T}(r) = \frac{e^{\frac{i}{2}kr}}{M_n r}\left[j_0\left(\frac{kr}{2}\right) - \frac{2}{kr} j_1\left(\frac{kr}{2}\right) + i j_1\left(\frac{kr}{2}\right)\right]}$$

$j_{0,1}$ are spherical Bessel functions

Our factorization formula is very simple

- ★ $R_{n0}(r)$: radial Schrödinger wave function of S-wave quarkonium

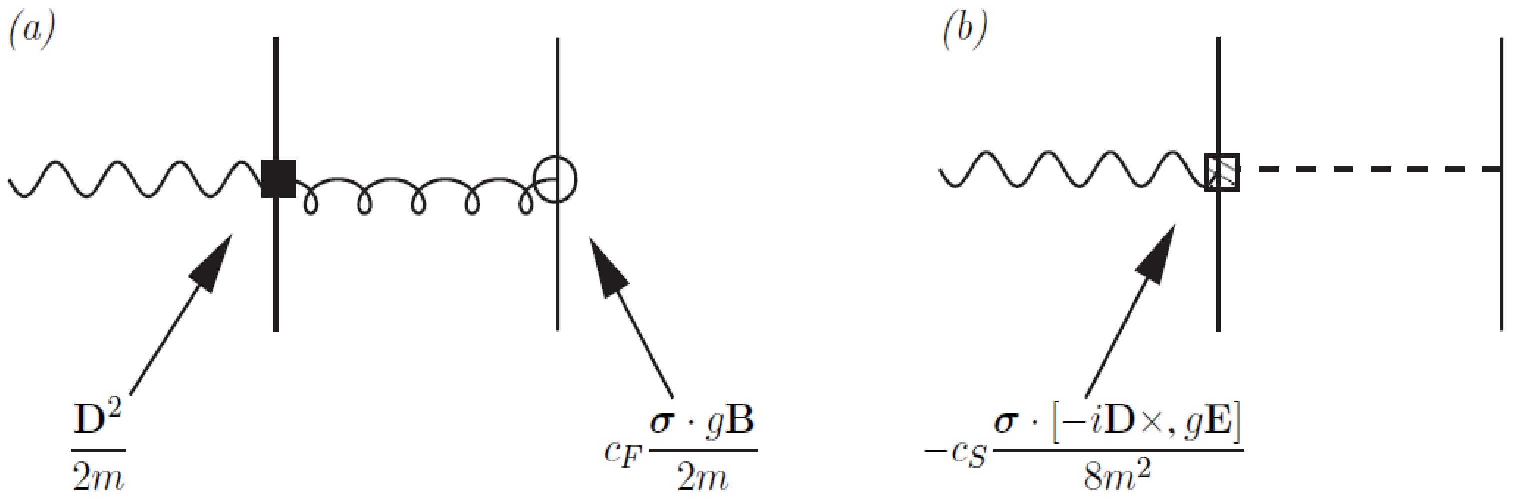
- ★ $j_l(kr/2)$: Spherical Bessel function takes into account so-called finite-size effect

- ★ One peculiarity: the “hard-scattering” kernel becomes complex: gluon can stay on-shell!

Examining the long-wave length limit of photon

Our result can be identified with pNRQCD calculation:

Brambilla, Jia and Vairo (2005)



Transition width from *“hard-scattering” approach*

$$\Gamma[n\,{}^3S_1 \to n'\,{}^1S_0 + \gamma] = \frac{k^3}{12\pi}\,|\mathscr{A}|^2$$

$$= \frac{16}{3}\,\alpha e_Q^2\,\frac{k^3}{M_n^2}\,C_F^2\alpha_s^2\,|\mathscr{E}_{nn'}|^2\ ,$$

No freely adjustable parameters in our "hard-scattering" mechanism

- ★ Parameters tuned to reproduce the spectra of bottomonium and charmonium family using famous **Cornell** or **Buchmuller-Tye** potential models

- ★ Strong coupling constant $\alpha_s(\mu)$ is not arbitrary, $\mu \sim m\, v$, typical quark 3-momentum scale in quarkonium (semi-hard scale)

We fix $\mu \sim 1.2$ GeV for b-bbar pair

$\mu \sim 0.9$ GeV for c-cbar pair

Our final predictions

- ★ Reasonable agreement, excellent for $\Upsilon(3S) \rightarrow \eta_b + \gamma$, even okay for $\psi(2S) \rightarrow \eta_c + \gamma$

TABLE I: Measured and predicted branching fractions of various hindered $M1$ transition processes $n\,^3S_1 \to n'\,^1S_0 + \gamma$ for bottomonium and charmonium. The photon momentum k is determined by physical kinematics. All the quarkonium masses are taken from the PDG08 compilation [14], except η_b mass is taken to be 9389 MeV [2], and $\eta_b(2S)$ mass taken as 9997 MeV [3]. For $\Upsilon(2S)\to\gamma\eta_b$, we use the preliminary BaBar result [15]; for $\psi(2S)\to\gamma\eta_c$, we quote the latest Cleo measurement [16], instead of the world average value given in [14]. We have taken $\alpha_s(\mu) = 0.43$ and 0.59 for $\mu = 1.2$ and 0.9 GeV, respectively.

Decay modes	k (MeV)	$\mathcal{B}$ (Exp.)	α_s	$\mathscr{E}_{nn'}(\times 10^{-2})$		$\mathcal{B}$ (Our prediction)	
				Cornell	BT	Cornell	BT
$\Upsilon(2S)\to\gamma\eta_b$	614	$(4.2 \pm 1.4) \times 10^{-4}$	0.43	$3.7e^{i2.0^\circ}$	$3.2e^{i2.7^\circ}$	1.4×10^{-4}	1.1×10^{-4}
$\Upsilon(3S)\to\gamma\eta_b$	921	$(4.8 \pm 1.3) \times 10^{-4}$	0.43	$2.7e^{i2.6^\circ}$	$2.5e^{i3.3^\circ}$	3.7×10^{-4}	3.3×10^{-4}
$\Upsilon(4S)\to\gamma\eta_b$	1123	–	0.43	$2.2e^{i2.8^\circ}$	$1.9e^{i3.7^\circ}$	4.3×10^{-7}	3.2×10^{-7}
$\Upsilon(4S)\to\gamma\eta_b(2S)$	566	–	0.43	$1.7e^{i2.2^\circ}$	$1.6e^{i2.7^\circ}$	3.2×10^{-8}	2.7×10^{-8}
$\psi(2S)\to\gamma\eta_c$	638	$(4.3 \pm 0.6) \times 10^{-3}$	0.59	$6.4e^{i9.7^\circ}$	$5.7e^{i12.9^\circ}$	2.7×10^{-3}	2.1×10^{-3}

Are we on the right track?

We feel encouraged that we have captured some correct and relevant pieces of physics, especially concerning the absence of any freely tunable input parameters

Summary

- ★ We carry out a detailed study to $\eta_{c2} \to J/\psi(\psi') + \gamma$. If BaBar measurement is correct, then the 2^{-+} assignment for X(3872) is highly unlikely.
- ★ We develop a ***hard-scattering mechanism*** and we argue that it is more plausible than the traditional multipole-expansion when the photon becomes too energetic ($>$ 1 GeV ?).
- ★ Our formalism should also be applicable to **E1** transition, or perhaps, even **hadronic transitions**. And why not?

Thanks for your attention!