The Polyakov loop and correlator of Polyakov loops at NNLO

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based on

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Outline

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Motivations

The Polyakov loop and the correlator of two Polyakov loops are two related and relevant quantities for the dynamics of one or two static sources in a thermal bath at temperature T.

- The Polyakov loop average in a thermal ensemble at a temperature T is
  $\langle L_R \rangle \equiv \frac{1}{d_R} \langle \mathrm{Tr\ L_R} \rangle$ (R $\equiv$ color representation)
  $d_A = N^2 - 1$, $d_F = N$ and $L_R(\mathbf{x}) = \mathrm{P}\exp\left(ig \int_0^{1/T} d\tau A^0(\mathbf{x}, \tau) \right)$.
- The (connected part of a) Polyakov loop correlator is
  $C_{\mathrm{PL}}(r, T) \equiv \frac{1}{N^2} \langle \mathrm{Tr} L_F^\dagger(\mathbf{0}) \mathrm{Tr} L_F(\mathbf{r}) \rangle - \langle L_F \rangle^2$.

Their relevance comes from the fact that they are gauge invariant quantities well known from lattice calculation.
∘ `e.g. Petreczky EPJC 43 (2005) 51`

Moreover, we have that

$$\langle \mathrm{Tr\ L_F^\dagger}(\mathbf{0}) \mathrm{Tr\ L_F}(\mathbf{r}) \rangle = \sum \mathrm{e}^{-\mathrm{E_n}/\mathrm{T}}$$

∘ `Lüscher Weisz JHEP 0207 (2002) 049, Jahn Philipsen PRD 70 (2004) 074504`

Motivations

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Despite the relevance, not much is known about the correlator in perturbation theory. The correlator is known at LO since long.

○ `McLerran Svetitsky PRD 24 (1981) 450`
`Gross Pisarski Yaffe RMP 53 (1981) 43`

Beyond leading order, it was computed only for $1/r \sim m_D$ (m_D is the Debye mass or inverse electric screening length).

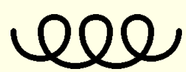
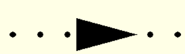
○ `Nadkarni PRD 33 (1986) 3738`

Static and non-static modes

It is convenient to perform the calculation in static gauge $\partial_0 A^0(x) = 0$:

$$L(\mathbf{x}) = \exp\left(\frac{igA^0(\mathbf{x})}{T}\right)$$

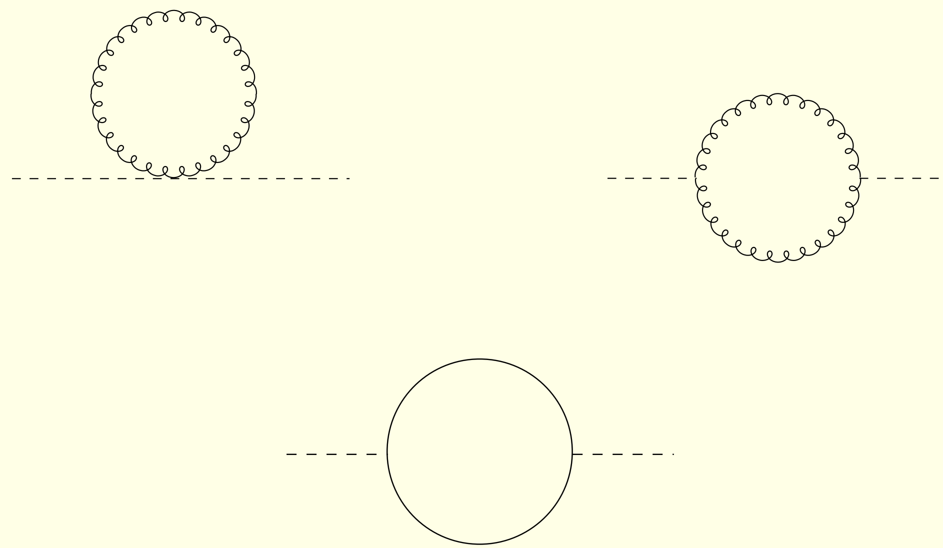
Propagators may be split into a static and a non-static component:

$D_{00}(\omega_n, \mathbf{k}) =$		$= \dfrac{\delta_{n0}}{\mathbf{k}^2}$
$D_{ij}(\omega_n \neq 0, \mathbf{k}) =$		$= \dfrac{1}{\omega_n^2 + \mathbf{k}^2}\left(\delta_{ij} + \dfrac{k_i k_j}{\omega_n^2}\right)(1 - \delta_{n0})$
$D_{ij}(\omega_n = 0, \mathbf{k}) =$		$= \dfrac{1}{\mathbf{k}^2}\left(\delta_{ij} - (1-\xi)\dfrac{k_i k_j}{\mathbf{k}^2}\right)\delta_{n0}$
$D_{\text{ghost}}(\omega_n, \mathbf{k}) =$		$= \dfrac{\delta_{n0}}{\mathbf{k}^2}$

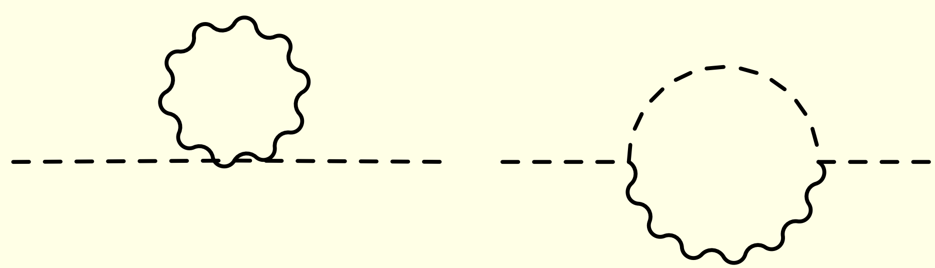
$\omega_n \equiv 2\pi n T$ are the Matsubara frequencies.

Π_{00} at one loop

The temporal component of the gluon self-energy gets non-static



and static contributions



- The calculation is performed in dimensional regularization: $d = 3 - 2\epsilon$.
- $\Pi_{00}(|\mathbf{k}| \ll T) = m_D^2 + \ldots$ where m_D is the Debye mass:

$$m_D^2 \equiv \frac{g^2 T^2}{3} \left(N + \frac{n_f}{2} \right).$$

- We keep order ϵ corrections of the type

$$T |\mathbf{k}|^{1-2\epsilon} \epsilon$$

because the Fourier transform of $|\mathbf{k}|^{1-2\epsilon}/|\mathbf{k}|^4$, coming from a self-energy insertion in a temporal-gluon propagator, is divergent.

- Static loops contribute only through the scale m_D.

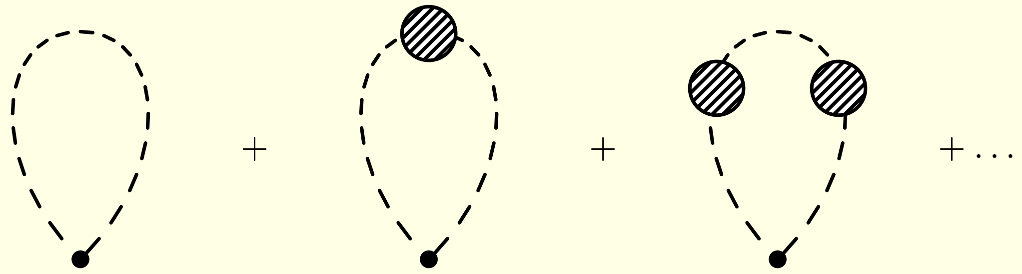
◦ Curci Menotti ZPC 21 (1984) 281
Heinz Kajantie Toimela AP 176 (1987) 218
Rebhan PRD 48 (1993) 3967, NPB 430 (1994) 319

The Polyakov loop at NNLO

We assume the following hierarchy of scales:

$$T \gg m_D$$

Up to NNLO the contributing diagrams are



giving

$$\langle L_R \rangle = 1 + \frac{C_R \alpha_{\rm s}}{2} \frac{m_D}{T} + \frac{C_R \alpha_{\rm s}^2}{2} \left[C_A \left(\ln \frac{m_D^2}{T^2} + \frac{1}{2} \right) - n_f \ln 2 \right] + \mathcal{O}(g^5)$$

- At the scale m_D, the gluon-self energies get resummed in the propagator

$$\frac{1}{\mathbf{k}^2 + m_D^2}$$

- The logarithm, $\ln m_D^2/T^2$, signals that an infrared divergence at the scale T has canceled against an ultraviolet divergence at the scale m_D.

Comparison with the literature

In 1981, Gava and Jengo obtained:

$$\langle L_R \rangle_{\rm GJ} = 1 + \frac{C_R \alpha_{\rm s}}{2} \frac{m_D}{T} + \frac{C_R C_A \alpha_{\rm s}^2}{2} \left(\ln \frac{m_D^2}{T^2} - 2 \ln 2 + \frac{3}{2} \right) + \mathcal{O}(g^5)$$

This result disagrees with ours. The origin of the disagreement has been traced back to not having resummed the Debye mass in the temporal gluons contributing to the static gluon self energy.

○ `Gava Jengo PLB 105 (1981) 285`

Our result agrees with a recent determination of Burnier, Laine and Vepsäläinen, who use a dimensionally reduced EFT framework in a covariant or Coulomb gauge.

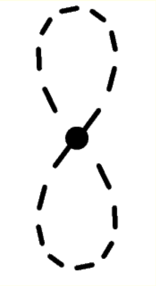
○ `Burnier Laine Vepsäläinen JHEP 1001 (2010) 054`

The Polyakov loop: some higher order terms

- Non-static modes at the scale m_D:

$$\delta\langle L_R\rangle_{\mathrm{NS},\,m_D} = \frac{3g^4 C_R}{4(4\pi)^3}\frac{m_D}{T}\left[\beta_0 \ln\left(\frac{\mu}{4\pi T}\right)^2 + 2\beta_0\gamma_E + \frac{11}{3}C_A - \frac{2}{3}n_f\,(4\ln 2 - 1)\right]$$

  This contribution fixes the renormalization scale of g^3 in the LO term to $\mu \sim 4\pi T$.

- 

$$\delta\langle L_{\mathrm{R}}\rangle = \left(3C_R^2 - \frac{C_R C_A}{2}\right)\frac{\alpha_{\mathrm{s}}^2}{24}\left(\frac{m_D}{T}\right)^2$$

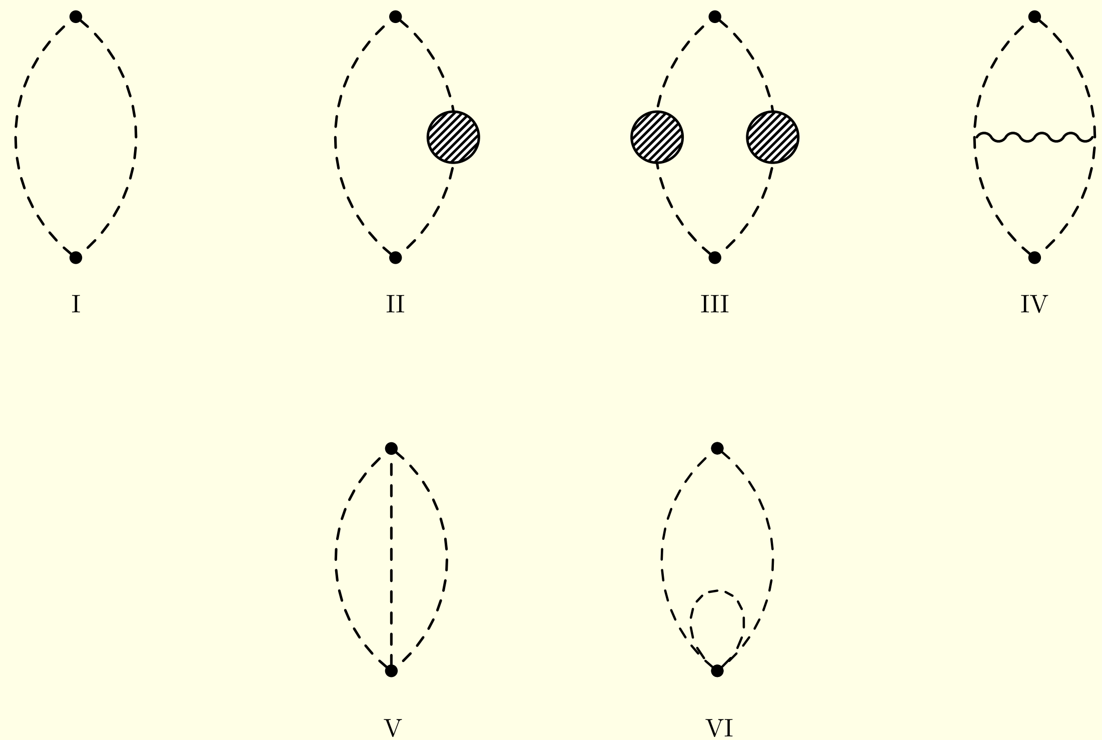
  This contribution comes from the scale m_D: it is the leading contribution whose color structure is non linear in C_R

The Polyakov loop correlator at NNLO

We assume the following hierarchy of scales:

$$\frac{1}{r} \gg T \gg m_D \gg \frac{g^2}{r}.$$

We calculate the Polyakov loop correlator up to order $g^6 (rT)^0$:



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We calculate the Polyakov loop correlator up to order $g^6 (rT)^0$:

$$\begin{aligned}
C_{\rm PL}(r,T) \quad = \quad & \frac{N^2-1}{8N^2} \left\{ \frac{\alpha_{\rm s}(1/r)^2}{(rT)^2} - 2\frac{\alpha_{\rm s}^2}{rT}\frac{m_D}{T} \right. \\
& + \frac{\alpha_{\rm s}^3}{(rT)^3}\frac{N^2-2}{6N} + \frac{1}{2\pi}\frac{\alpha_{\rm s}^3}{(rT)^2}\left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right) \\
& + \frac{\alpha_{\rm s}^3}{rT}\left[C_A\left(-2\ln\frac{m_D^2}{T^2} + 2 - \frac{\pi^2}{4}\right) + 2n_f\ln 2\right] \\
& \left. + \alpha_{\rm s}^2\frac{m_D^2}{T^2} - \frac{2}{9}\pi\alpha_{\rm s}^3 C_A \right\} + \mathcal{O}\left(g^6(rT), \frac{g^7}{(rT)^2}\right)
\end{aligned}$$

Comparison with the literature

In 1986, Nadkarni calculated the Polyakov loop correlator at NNLO assuming the hierarchy:

$$T \gg 1/r \sim m_D$$

Whenever the previous results do not involve the hierarchy $rT \ll 1$, they agree with Nadkarni's ones, expanded for $m_D r \ll 1$.

○ `Nadkarni PRD 33 (1986) 3738`

The Polyakov loop correlator in an EFT language

Integrating out $1/r$ from static QCD leads to pNRQCD:

$$\begin{aligned}
\mathcal{S}_{\rm pNRQCD} \quad = \quad & \int_0^{1/T} d\tau \int d^3x \int d^3r \, {\rm Tr}\Big\{ {\rm S}^\dagger (\partial_0 + V_s){\rm S} + {\rm O}^\dagger (D_0 + V_o){\rm O} \\
& \qquad -iV_A \left({\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm O} + {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm S} \right) \\
& \qquad -\frac{i}{2} V_B \left({\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E} \right) \\
& \qquad +\frac{i}{8} V_C \left(r^i r^j {\rm O}^\dagger D^i g E^j {\rm O} - r^i r^j {\rm O}^\dagger {\rm O} D^i g E^j \right) + \delta\mathcal{L}_{\rm pNRQCD} \Big\} \\
& + \int_0^{1/T} d\tau \int d^3x \, \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{l=1}^{n_f} \bar{q}_l \not{D} q_l \right)
\end{aligned}$$

where S and O$= \sqrt{2}\, O^a T^a$ are the quark-antiquark color singlet and octet fields.

- Pineda Soto NPB PS 64 (1998) 428
  Brambilla Pineda Soto Vairo NPB 566 (2000) 275
  Brambilla Eiras Pineda Soto Vairo PRD 67 (2003) 034018

pNRQCD potentials

$$V_s(r) = -C_F \frac{\alpha_s(1/r)}{r}\left[1 + \left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right)\frac{\alpha_s}{4\pi} + \mathcal{O}(\alpha_s^2)\right],$$

$$V_o(r) = \frac{1}{2N}\frac{\alpha_s(1/r)}{r}\left[1 + \left(\frac{31}{9}C_A - \frac{10}{9}n_f + 2\gamma_E\beta_0\right)\frac{\alpha_s}{4\pi} + \mathcal{O}(\alpha_s^2)\right],$$

$$(N^2-1)V_o(r) + V_s(r) = \frac{N(N^2-1)}{8}\frac{\alpha_s^3}{r}\left(\frac{\pi^2}{4} - 3\right)\left[1 + \mathcal{O}(\alpha_s)\right].$$

- Fischler NPB 129 (1977) 157, Billoire PLB 92 (1980) 343
  Kniehl Penin Schröder Smirnov Steinhauser PLB 607 (2005) 96

For our accuracy, $V_A = V_B = V_C = 1$.

The Polyakov loop correlator in pNRQCD

$$C_{\rm PL}(r,T) = \frac{1}{N^2}\Bigg[Z_s\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle + Z_o\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle + \mathcal{O}\left(\alpha_{\rm s}^3(rT)^4\right)\Bigg] - \langle L_F\rangle^2.$$

- Integrating out the scale $1/r$ and matching to the previous determination of $C_{\rm PL}(r,T)$, we get:

$$Z_s = Z_o = 1$$
$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = e^{-V_s(r)/T}$$
$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle|_{1/r} = (N^2-1)e^{-V_o(r)/T}$$

  This is consistent with the spectral decomposition.

- If we assume instead the spectral decomposition, then the matching provides a non-trivial verification of the two-loop octet potential.

Integrating out T and m_D

$$\langle S(\mathbf{r},\mathbf{0},1/T)S^\dagger(\mathbf{r},\mathbf{0},0)\rangle = e^{-V_s(r)/T}(1+\delta_s) \equiv e^{-f_s/T}$$

$$\langle O^a(\mathbf{r},\mathbf{0},1/T)O^{a\,\dagger}(\mathbf{r},\mathbf{0},0)\rangle = e^{-V_o(r)/T}\left[(N^2-1)\,\langle L_A\rangle + \delta_o\right] \equiv (N^2-1)e^{-f_o/T}$$

- δ_s and δ_o stand for loop corrections to the singlet and octet correlators:

$$\delta_s = \delta_{s,T} + \delta_{s,m_D} \qquad \delta_o = \delta_{o,T} + \delta_{o,m_D}$$

- $\langle L_A\rangle$ comes from the covariant derivative D_0 acting on the octet field in the pNRQCD Lagrangian:

$$\langle L_A\rangle = \langle L_A\rangle_T + \langle L_A\rangle_{m_D}$$

  The adjoint Polyakov loop $\langle L_A\rangle$ factorizes the contribution coming from the gluons in the thermal bath that bind with the color-octet quark-antiquark states to form part of the spectrum appearing in the spectral decomposition of the Polyakov loop correlator.

- For $T \gtrsim g^2/r$, the octet correlator is not suppressed with respect to the singlet one, while for $T \ll g^2/r$, the Polyakov loop correlator is dominated by the singlet contribution.

Singlet and octet free energies

f_s and f_o can be interpreted as the singlet and octet free energies in pNRQCD.

$$
\begin{aligned}
f_s(r,T,m_D) \quad = \quad & V_s(r) \\
& + \frac{2}{9}\pi N C_F \alpha_s^2 r T^2 \left[1 + \sum c_n^{\rm NS} (rT)^{2n+2}\right] - \frac{\pi}{36} N^2 C_F \alpha_s^3 T \\
& - \left(\frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi}(r m_D)^2 T - \frac{2}{3}\zeta(3) N C_F \alpha_s^2 r^2 T^3\right)\left[1 + \sum c_n^{\rm S} (rT)^{2n+2}\right] \\
& + C_F \frac{\alpha_s}{6} r^2 m_D^3 + T\mathcal{O}\left(g^6 (rT), \frac{g^8}{rT}\right)
\end{aligned}
$$

Singlet and octet free energies

f_s and f_o can be interpreted as the singlet and octet free energies in pNRQCD.

$$
\begin{aligned}
f_o(r,T,m_D) \quad = \quad & V_o(r) \\
& -\frac{C_A\alpha_{\rm s}}{2}m_D + \frac{1}{48}C_A^2\alpha_{\rm s}^2\frac{m_D^2}{T} \\
& -\frac{C_A\alpha_{\rm s}^2}{2}T\left[C_A\left(-\ln\frac{T^2}{m_D^2}+\frac{1}{2}\right) - n_f\ln 2 + b_1\,g + b_2\,g^2 + a\,\alpha_{\rm s}\right] \\
& -\frac{\pi}{9}\alpha_{\rm s}^2 rT^2\left[1+\sum c_n^{\rm NS}(rT)^{2n+2}\right] - \frac{\pi}{72}N\alpha_{\rm s}^3 T \\
& +\left(\frac{3}{4N}\zeta(3)\frac{\alpha_{\rm s}}{\pi}(rm_D)^2T - \frac{1}{3}\zeta(3)\alpha_{\rm s}^2 r^2T^3\right)\left[1+\sum c_n^{\rm S}(rT)^{2n+2}\right] \\
& -\frac{1}{N}\frac{\alpha_{\rm s}}{12}r^2m_D^3 + T\mathcal{O}\left(g^6(rT),\frac{g^8}{rT}\right)
\end{aligned}
$$

Note that higher multipole contributions in the singlet and octet sector are related.

In the Polyakov loop correlator, $C_{\rm PL}(r,T)$, strong cancellations occur between the singlet energy, octet energy and Polyakov loop

$$\langle L_R\rangle = 1 + \frac{C_R\alpha_{\rm s}}{2}\frac{m_D}{T} + \frac{C_R\alpha_{\rm s}^2}{2}\left[C_A\left(\ln\frac{m_D^2}{T^2}+\frac{1}{2}\right) - n_f\ln 2 + a\,\alpha_{\rm s} + b_1\,g + b_2\,g^2\right] + \left(3C_R^2 - \frac{C_RC_A}{2}\right)\frac{\alpha_{\rm s}^2}{24}\left(\frac{m_D}{T}\right)^2 + \mathcal{O}\left(g^7\right).$$

They lead, up to order $g^6(rT)^0$, to the previous result.

Comparison with the literature and discussion I

EFT approaches for the calculation of the correlator of Polyakov loops for the situation $m_D \gtrsim 1/r$ and $T \gg 1/r$ were developed in the past. In that situation, the scale $1/r$ was not integrated out, and the Polyakov-loop correlator was described in terms of dimensionally reduced effective field theories of QCD, while the complexity of the bound-state dynamics remained implicit in the correlator.

Those descriptions are valid for largely separated Polyakov loops when the correlator is either screened by the Debye mass, for $m_D r \sim 1$, or the mass of the lowest-lying glueball, for $m_D r \gg 1$.

- `Braaten Nieto PRL 74 (1995) 3530`
  `Nadkarni PRD 33 (1986) 3738`

Comparison with the literature and discussion II

The color-singlet quark-antiquark potential has been calculated in real-time formalism in the same thermodynamical situation considered here.

- The real part of the real-time potential differs from $f_s(r, T, m_D)$ by

$$\frac{1}{9}\pi N C_F \alpha_{\rm s}^2 r T^2 - \frac{\pi}{36} N^2 C_F \alpha_{\rm s}^3 T$$

  The origin of the difference has been traced back to terms that would vanish for large real times.

- The real-time potential has also an imaginary part that is absent in the free energy.
- Performing the calculation of $\langle S(\mathbf{r}, \mathbf{0}, \tau) S^\dagger(\mathbf{r}, \mathbf{0}, 0)\rangle$ for an imaginary time $\tau \leq 1/T$ and then continuing analytically τ to large real times, one gets back both the real and the imaginary parts of the real-time color-singlet potential.

∘ Brambilla Ghiglieri Petreczky Vairo PRD 78 (2008) 014017

Comparison with the literature and discussion III

Jahn and Philipsen have analyzed the gauge structure of the allowed intermediate states in the correlator of Polyakov loops: the quark-antiquark component, φ, of an intermediate state made of a quark located in $\mathbf{x}_1$ and an antiquark located in $\mathbf{x}_2$ should transform as

$$\varphi(\mathbf{x}_1, \mathbf{x}_2) \to g(\mathbf{x}_1)\varphi(\mathbf{x}_1, \mathbf{x}_2)g^\dagger(\mathbf{x}_2)$$

under a gauge transformation g.

- The pNRQCD decomposition of the Polyakov loop correlator in terms of a color singlet and a color octet correlator is in accordance with that result for, in pNRQCD, both the singlet field S and the octet field O transform in that way.
- We remark, however, a difference in language: in pNRQCD, singlet and octet refer to the gauge transformation properties of the quark-antiquark fields, while, in Jahn and Philipsen, they refer to the gauge transformation properties of the physical states. In that last sense, of course, octet states cannot exist as intermediate states in the correlator of Polyakov loops.

◦ Jahn Philipsen PRD 70 (2004) 074504

Comparison with the literature and discussion IV

Burnier, Laine and Vepsäläinen have recently performed a weak-coupling calculation of the untraced Polyakov-loop correlator in Coulomb gauge and of the cyclic Wilson loop up to order g^4.

Both these objects may be seen as contributing to the correlator of two Polyakov loops. It is expected that large cancellations occur between these correlators and their octet counterparts in order to reproduce the Polyakov-loop correlator. Such large cancellations should occur at the level of the scales $1/r$, T and m_D. In the case of the untraced Polyakov-loop correlator, the octet contribution should restore gauge invariance and, in the case of the cyclic Wilson loop, the octet contribution should cancel the divergences observed at order g^4.

∘ `Burnier Laine Vepsäläinen JHEP 1001 (2010) 054`