Hadron Interactions from Lattice QCD

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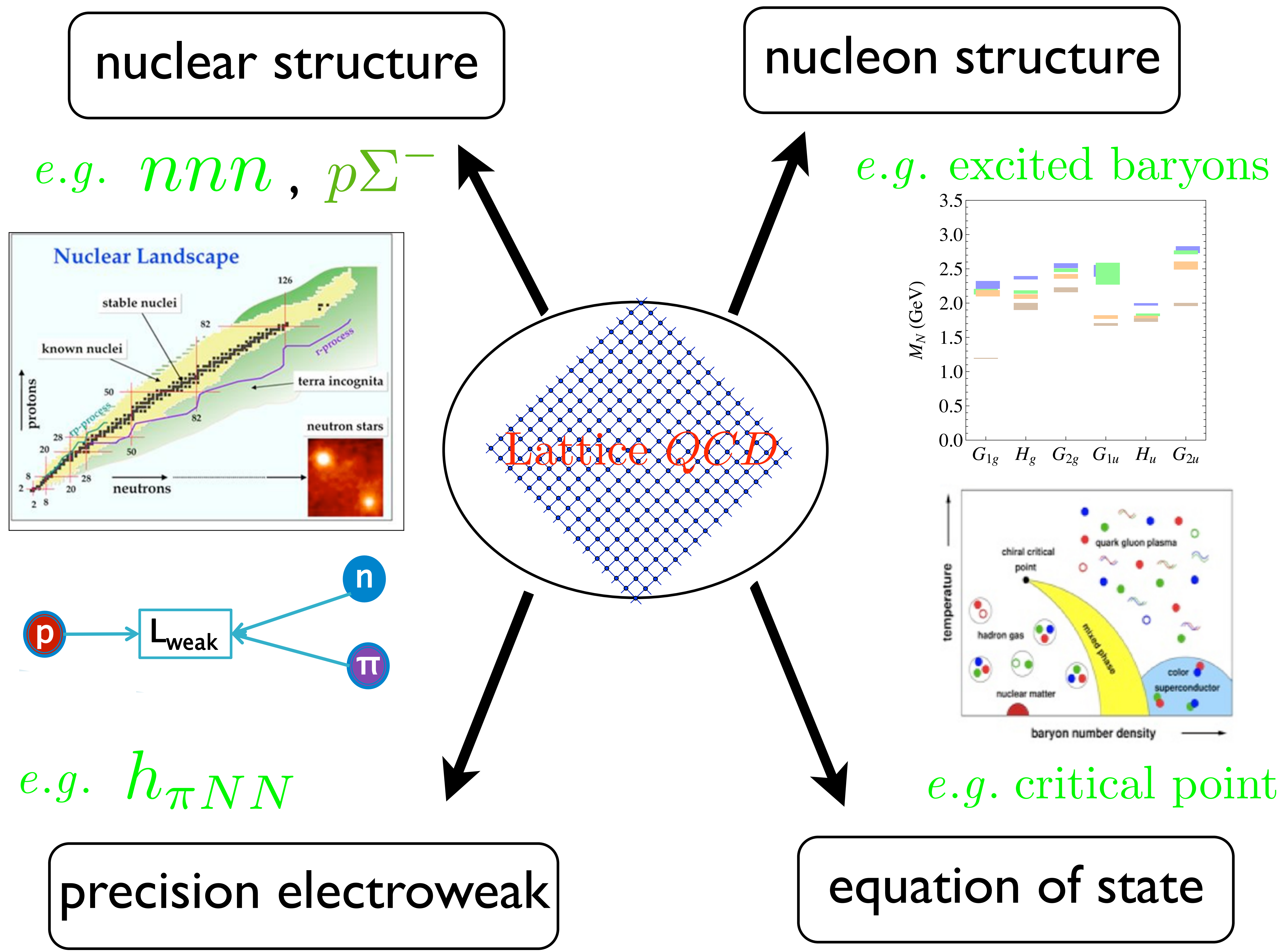
Bad Honnef 2/15/2011

Outline

- Motivation and basic technology
- Meson-Meson and Meson-Baryon
- Baryon-Baryon and nuclei
- H-dibaryon
- Conclusion



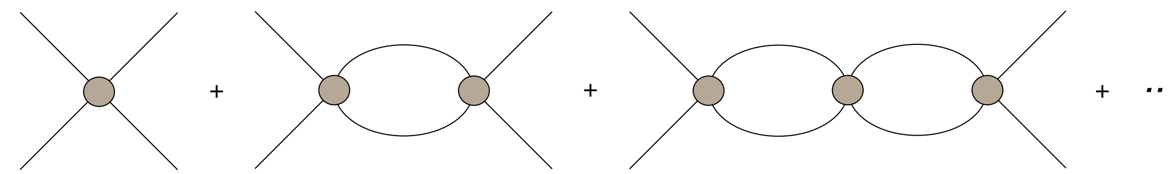
nuclear structure
e.g. nnn , pΣ⁻
Nuclear Landscape
stable nuclei
known nuclei
terra incognita
r-process
rp-process
neutron stars
protons
neutrons
126
82
50
28
20
8
2
p
n
π
L weak
e.g. h πNN
precision electroweak
Lattice QCD
nucleon structure
e.g. excited baryons
M_N (GeV)
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
G1g
Hg
G2g
G1u
Hu
G2u
chiral critical point
quark gluon plasma
hadron gas
mixed phase
nuclear matter
color superconductor
temperature
baryon number density
e.g. critical point
equation of state

How do we extract S-wave scattering information (phase shifts and binding energies) from a lattice calculation?

Recall NR scattering

$$\mathcal{A}_2(p) = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$$

=



sum of poles in a Finite Volume!

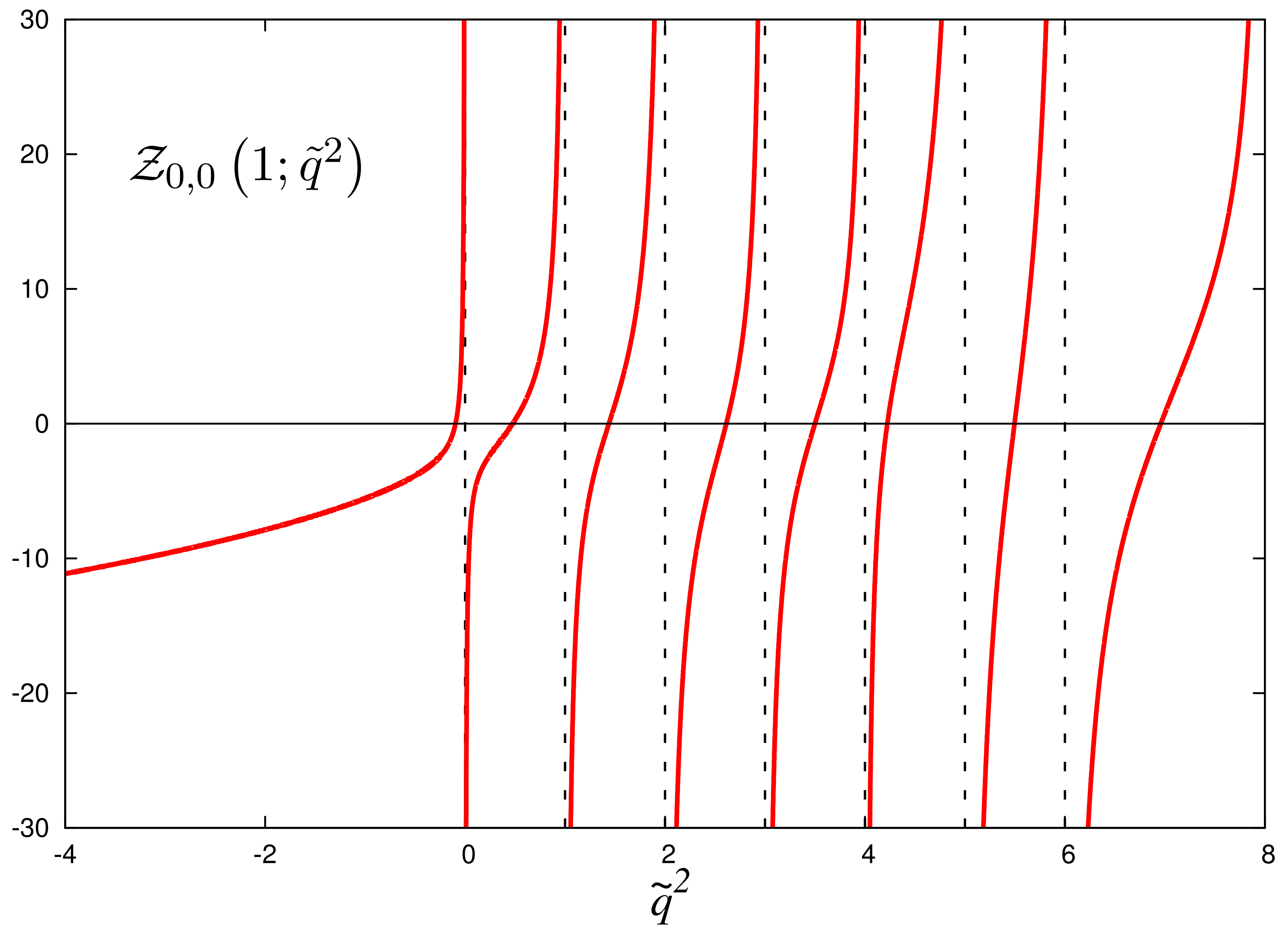
$$\boxed{\mathcal{A}_2^{-1}(p) = 0}$$

eigenvalue equation

S-wave at Finite Volume

$$q\cot\delta_0 \;=\; \frac{2}{\sqrt{\pi}L}\,\mathcal{Z}_{0,0}\left(1;\tilde{q}^2\right) \qquad \mathcal{Z}_{0,0}(1;\tilde{q}^2) \;=\; \frac{1}{\sqrt{4\pi}}\lim_{\Lambda_{\mathbf{n}}\to\infty}\left[\sum_{\mathbf{n}}^{\Lambda_{\mathbf{n}}}\frac{1}{|\mathbf{n}|^2-\tilde{q}^2} \;-\; 4\pi\Lambda_{\mathbf{n}}\right]$$

$$+\,\mathcal{O}\left(e^{-M_\pi L}\right)$$



Weak coupling expansion:

$$\Delta E_0(2,L) = \frac{4\pi\, a_{\pi\pi}}{m_\pi L^3}\left\{1 - \left(\frac{a_{\pi\pi}}{\pi L}\right)\mathcal{I} + \left(\frac{a_{\pi\pi}}{\pi L}\right)^2\left[\mathcal{I}^2 - \mathcal{J}\right] + \left(\frac{a_{\pi\pi}}{\pi L}\right)^3\left[-\mathcal{I}^3 + 3\mathcal{I}\mathcal{J} - \mathcal{K}\right]\right\} + \frac{8\pi^2 {a_{\pi\pi}}^3}{m_\pi L^6}\, r_{\pi\pi} + \mathcal{O}\left(L^{-7}\right)$$

Calculated on the lattice!

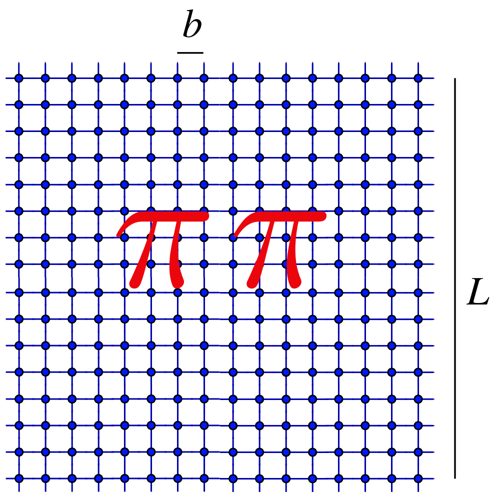
phase shift

$$\mathcal{I} = \lim_{\Lambda_j \to \infty} \sum_{\mathbf{i} \neq \mathbf{0}}^{|\mathbf{i}| \leq \Lambda_j} \frac{1}{|\mathbf{i}|^2} - 4\pi\Lambda_j = -8.91363291781$$

$$\mathcal{J} = \sum_{\mathbf{i} \neq \mathbf{0}} \frac{1}{|\mathbf{i}|^4} = 16.532315959$$

$$\mathcal{K} = \sum_{\mathbf{i} \neq \mathbf{0}} \frac{1}{|\mathbf{i}|^6} = 8.401923974433$$

$\pi\pi$ scattering in lattice QCD

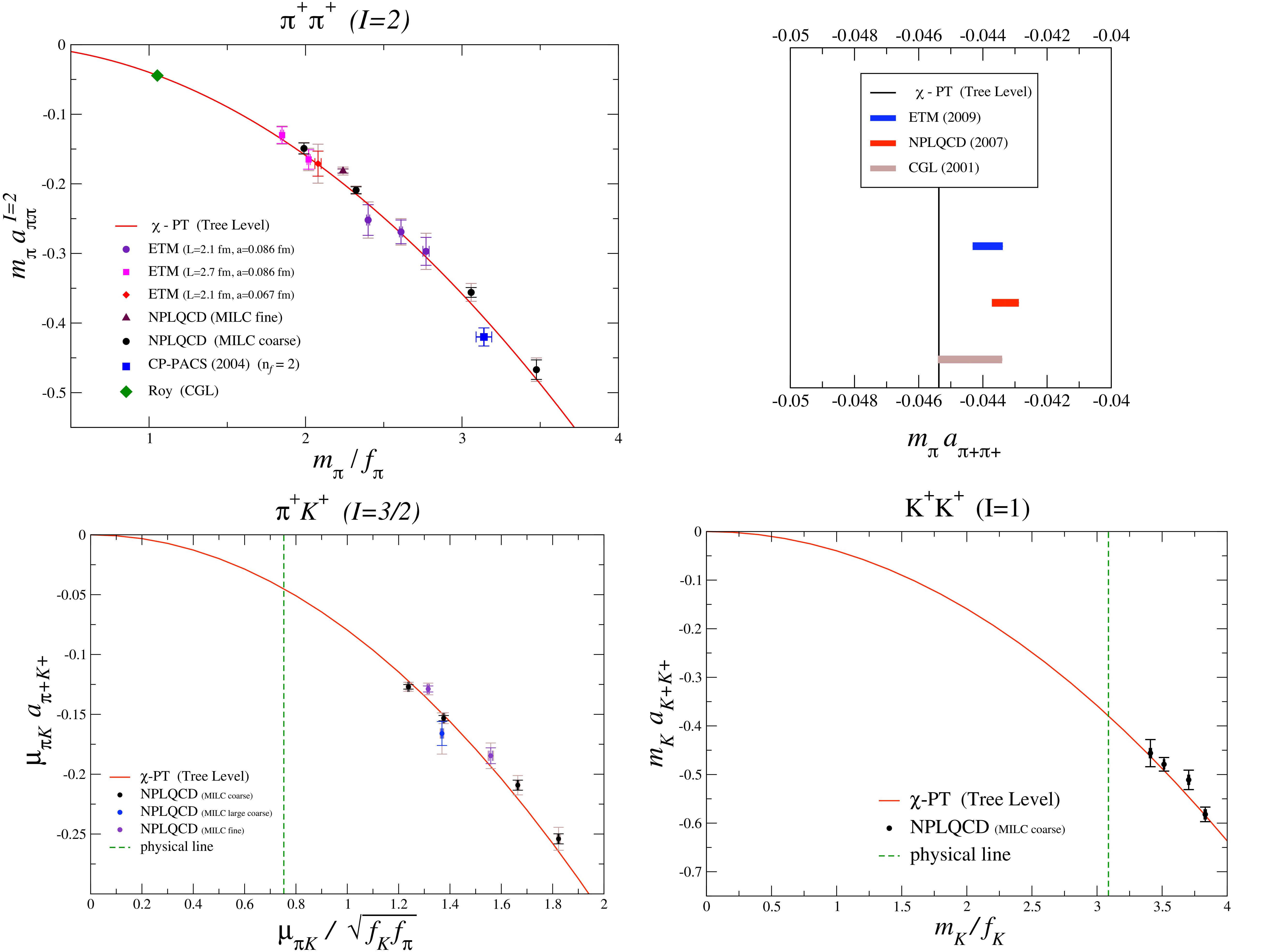


$$\mathcal{O}_{\pi^+}(t,\vec{x}) \;=\; \overline{u}(t,\vec{x})\gamma_5 d(t,\vec{x})$$

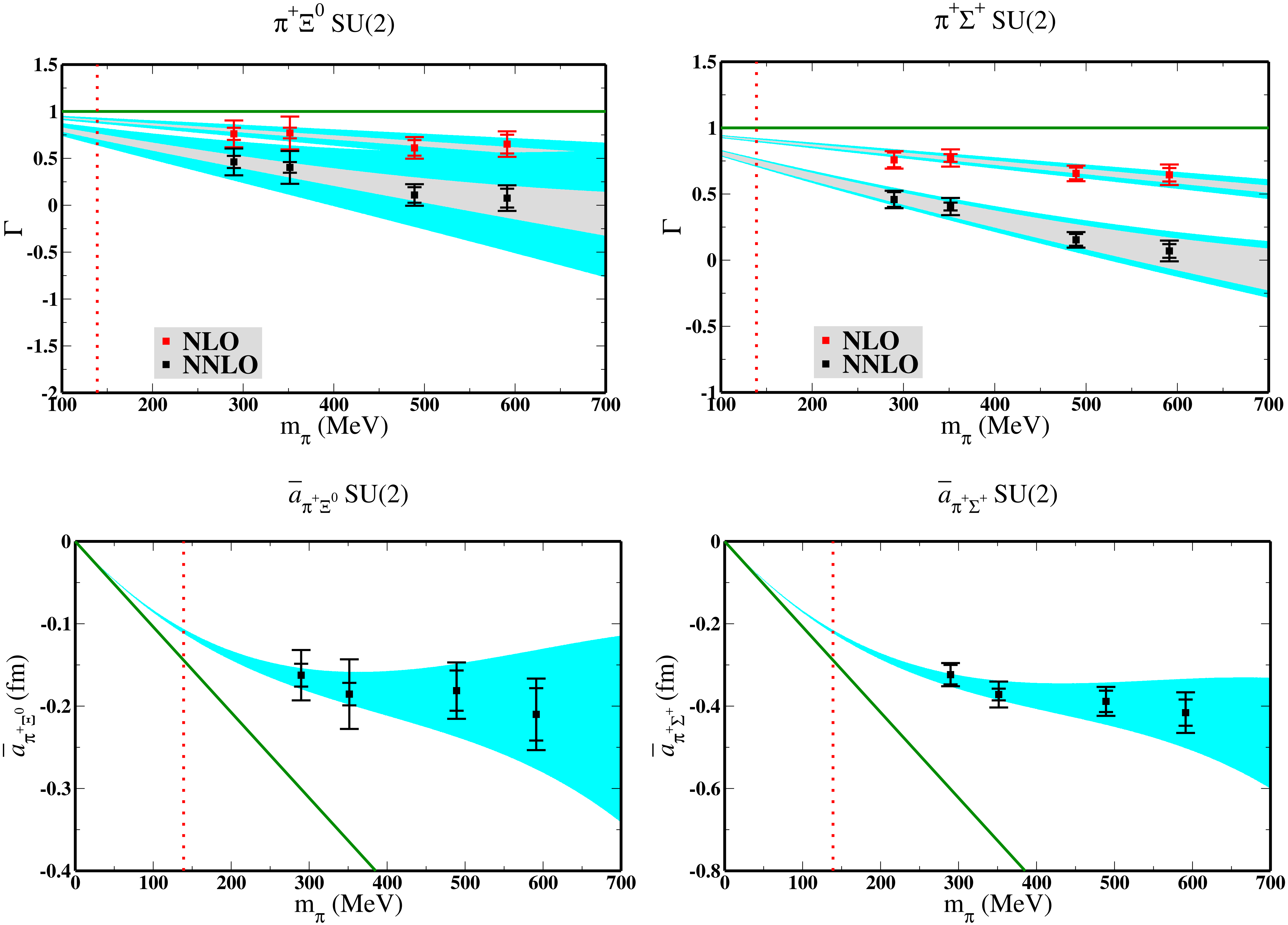
$$C_{\pi^+\pi^+}(p,t) \quad = \quad \langle 0| \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{O}_{\pi^-}(t,\mathbf{x})\ \mathcal{O}_{\pi^-}(t,\mathbf{y})\ \mathcal{O}_{\pi^+}(0,\mathbf{0})\ \mathcal{O}_{\pi^+}(0,\mathbf{0})|0\rangle$$

$$\frac{C_{\pi^+\pi^+}(p,t)}{C_{\pi^+}(t)C_{\pi^+}(t)} \quad \rightarrow \sum_{n=0}^{\infty} \mathcal{A}_n\ e^{-\Delta E_n(2,L)\ t}$$

$$\Delta E_n(2,L) \;\equiv\; 2\sqrt{\vec{p}_n^{\,2} + m_\pi^2} \;-\; 2m_\pi$$



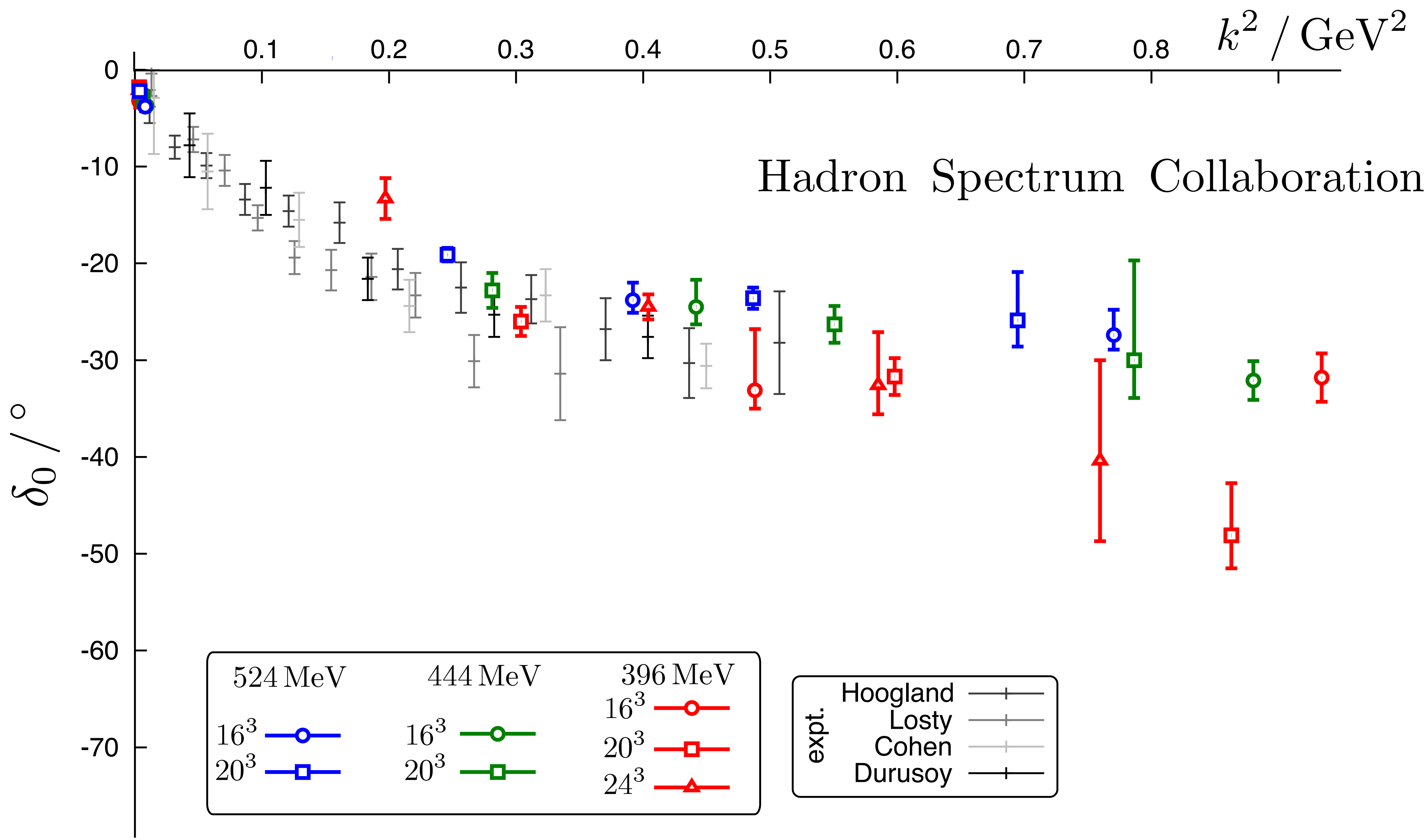
π+π+ (I=2)
m_π a_ππ^(I=2)
m_π / f_π
χ - PT (Tree Level)
ETM (L=2.1 fm, a=0.086 fm)
ETM (L=2.7 fm, a=0.086 fm)
ETM (L=2.1 fm, a=0.067 fm)
NPLQCD (MILC fine)
NPLQCD (MILC coarse)
CP-PACS (2004) (n_f = 2)
Roy (CGL)
χ - PT (Tree Level)
ETM (2009)
NPLQCD (2007)
CGL (2001)
m_π a_π+π+
π+K+ (I=3/2)
μ_πK a_π+K+
μ_πK / √(f_K f_π)
χ-PT (Tree Level)
NPLQCD (MILC coarse)
NPLQCD (MILC large coarse)
NPLQCD (MILC fine)
physical line
K+K+ (I=1)
m_K a_K+K+
m_K / f_K
χ-PT (Tree Level)
NPLQCD (MILC coarse)
physical line



$$a_{\pi\Xi}^{(3/2)} = -0.098 \pm 0.017 \text{ fm}$$ *NPLQCD*

$$a_{\pi\Sigma}^{(2)} = -0.197 \pm 0.017 \text{ fm}$$ *NPLQCD*

$\pi\pi \ I = 2$ S-wave phase shift

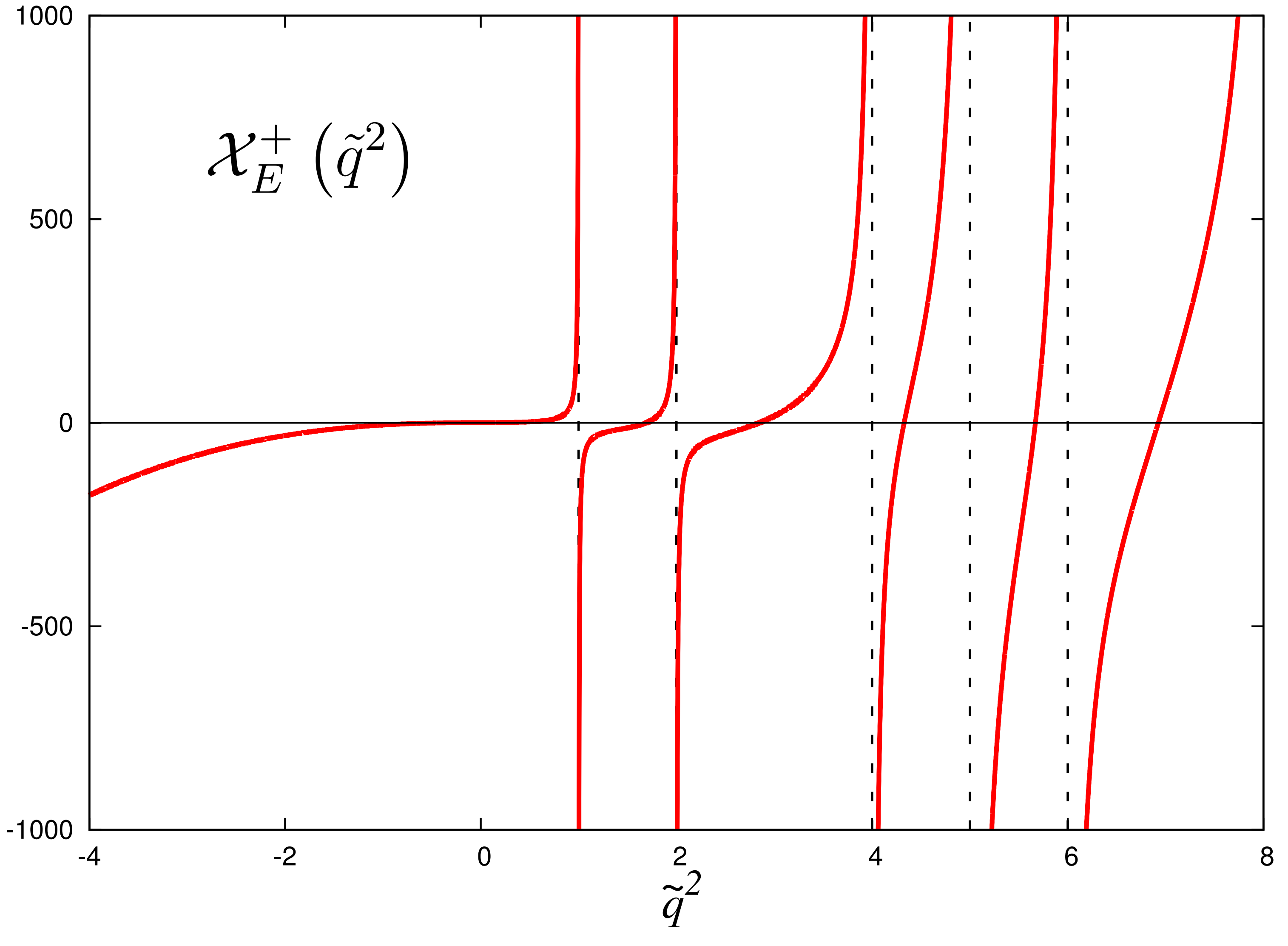


What about higher partial waves?

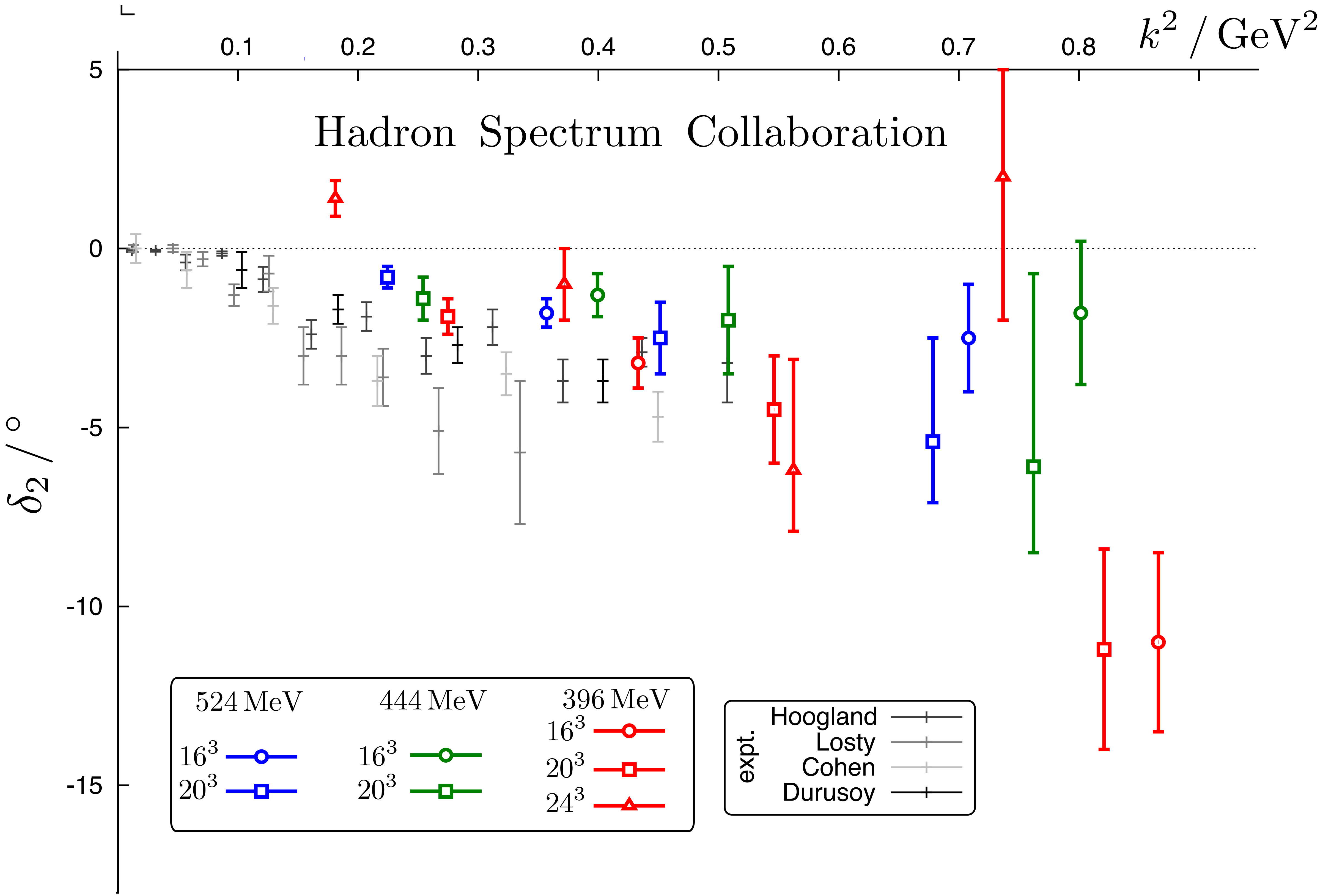
Angular Momentum, l	Irreps of the Cubic Group, $\Gamma^{(i)}$
0	A_1^+
1	T_1^-
2	$E^+ \oplus T_2^+$
3	$A_2^- \oplus T_1^- \oplus T_2^-$
4	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$
5	$E^- \oplus T_1^{-(1)} \oplus T_1^{-(2)} \oplus T_2^-$
6	$A_1^+ \oplus A_2^+ \oplus E^+ \oplus T_1^+ \oplus T_2^{+(1)} \oplus T_2^{+(2)}$
7	$A_2^- \oplus E^- \oplus T_1^{-(1)} \oplus T_1^{-(2)} \oplus T_2^{-(1)} \oplus T_2^{-(2)}$
8	$A_1^+ \oplus E^{+(1)} \oplus E^{+(2)} \oplus T_1^{+(1)} \oplus T_1^{+(2)} \oplus T_2^{+(1)} \oplus T_2^{+(2)}$
9	$A_1^- \oplus A_2^- \oplus E^- \oplus T_1^{-(1)} \oplus T_1^{-(2)} \oplus T_1^{-(3)} \oplus T_2^{-(1)} \oplus T_2^{-(2)}$

D-wave at Finite Volume

$$q^5 \cot\delta_2 = \left(\frac{2\pi}{L}\right)^5 \frac{1}{\pi^{3/2}} \mathcal{X}_E^+ \left(\tilde{q}^2\right)$$



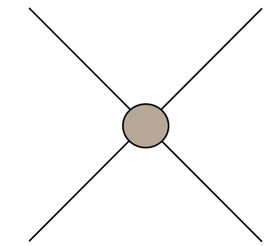
$\pi\pi \ I = 2$ D-wave phase shift



Nuclear physics

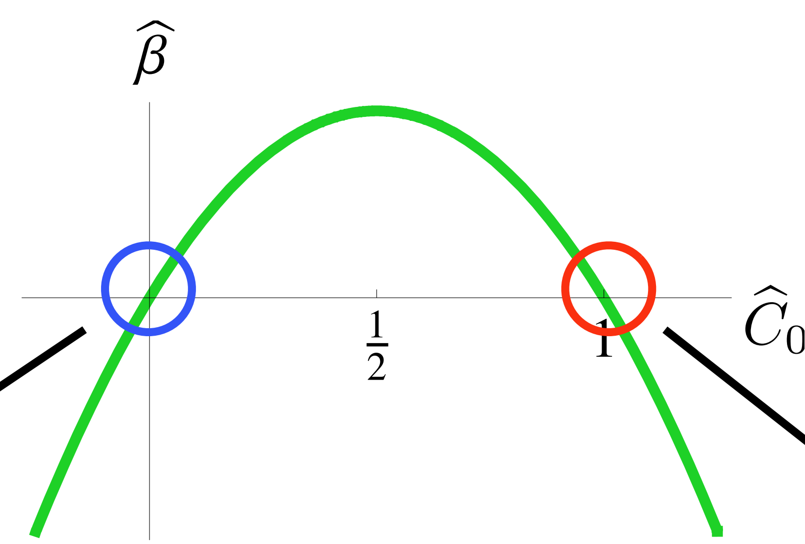
Experiment:

$$
\begin{aligned}
a_s^{{}^1S_0} &= -23.714 \text{ fm} & r_s^{{}^1S_0} &= 2.73 \text{ fm} \\
a_s^{{}^3S_1} &= 5.425 \text{ fm} & r_s^{{}^3S_1} &= 1.749 \text{ fm}
\end{aligned}
$$



$$a_s \gg \Lambda_{QCD}^{-1}$$

$$\hat{\beta}_0 = \mu \frac{d}{d\mu} \hat{C}_0 = -\hat{C}_0 (\hat{C}_0 - 1)$$

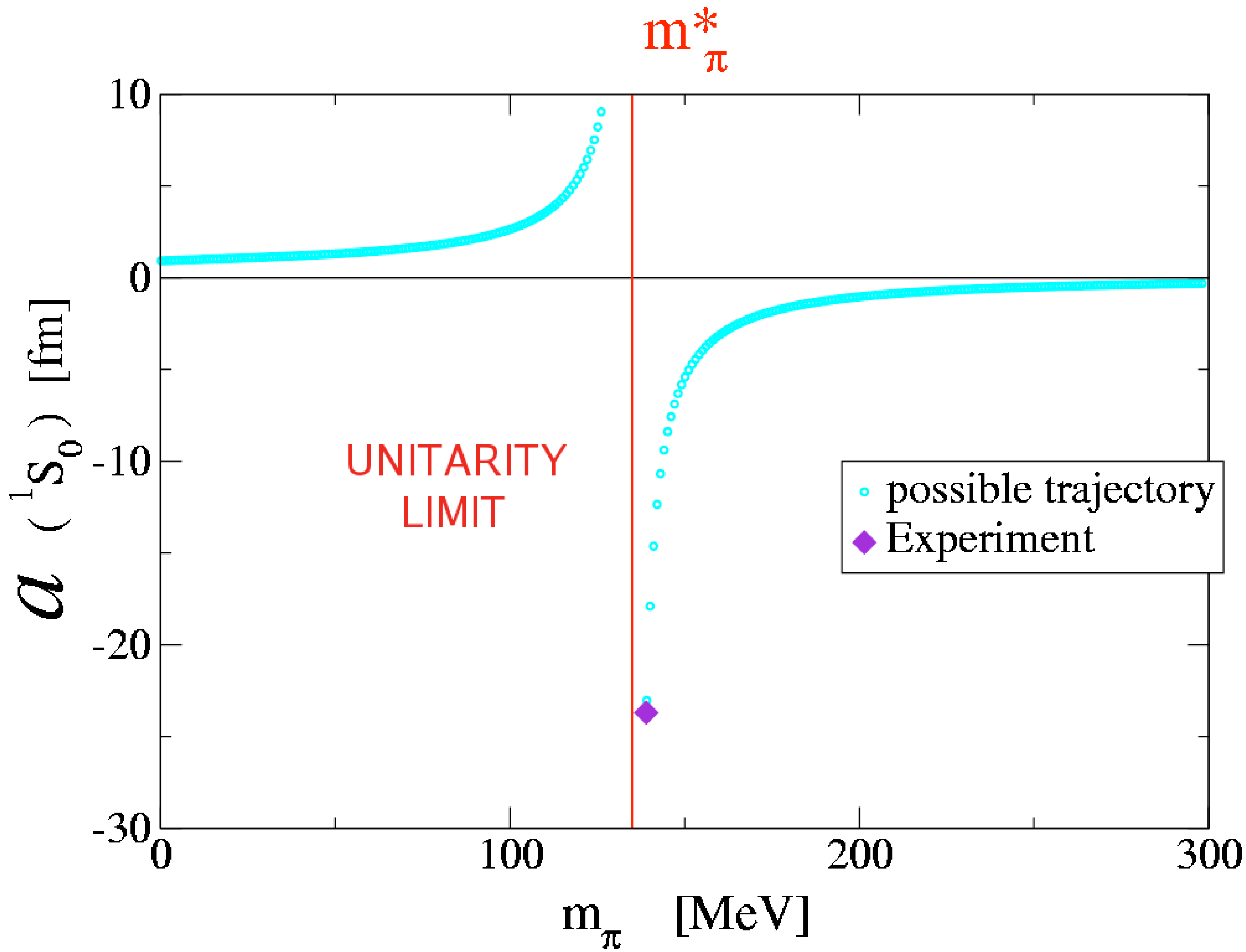


Trivial IR fixed point:
"natural case"

Nontrivial UV fixed point: "unnatural case"

"Unitarity"

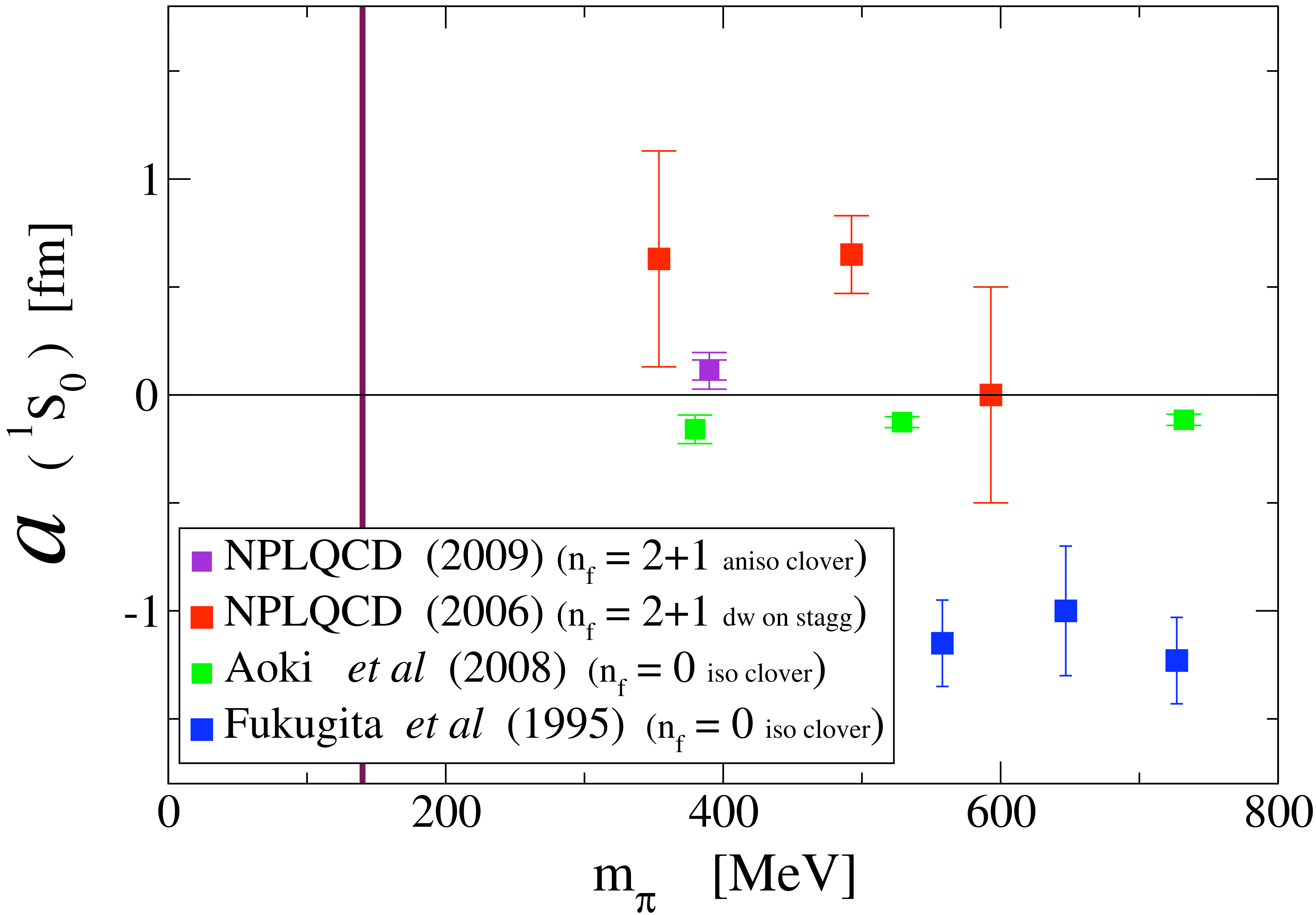
Why is nuclear physics near this UV fixed point??



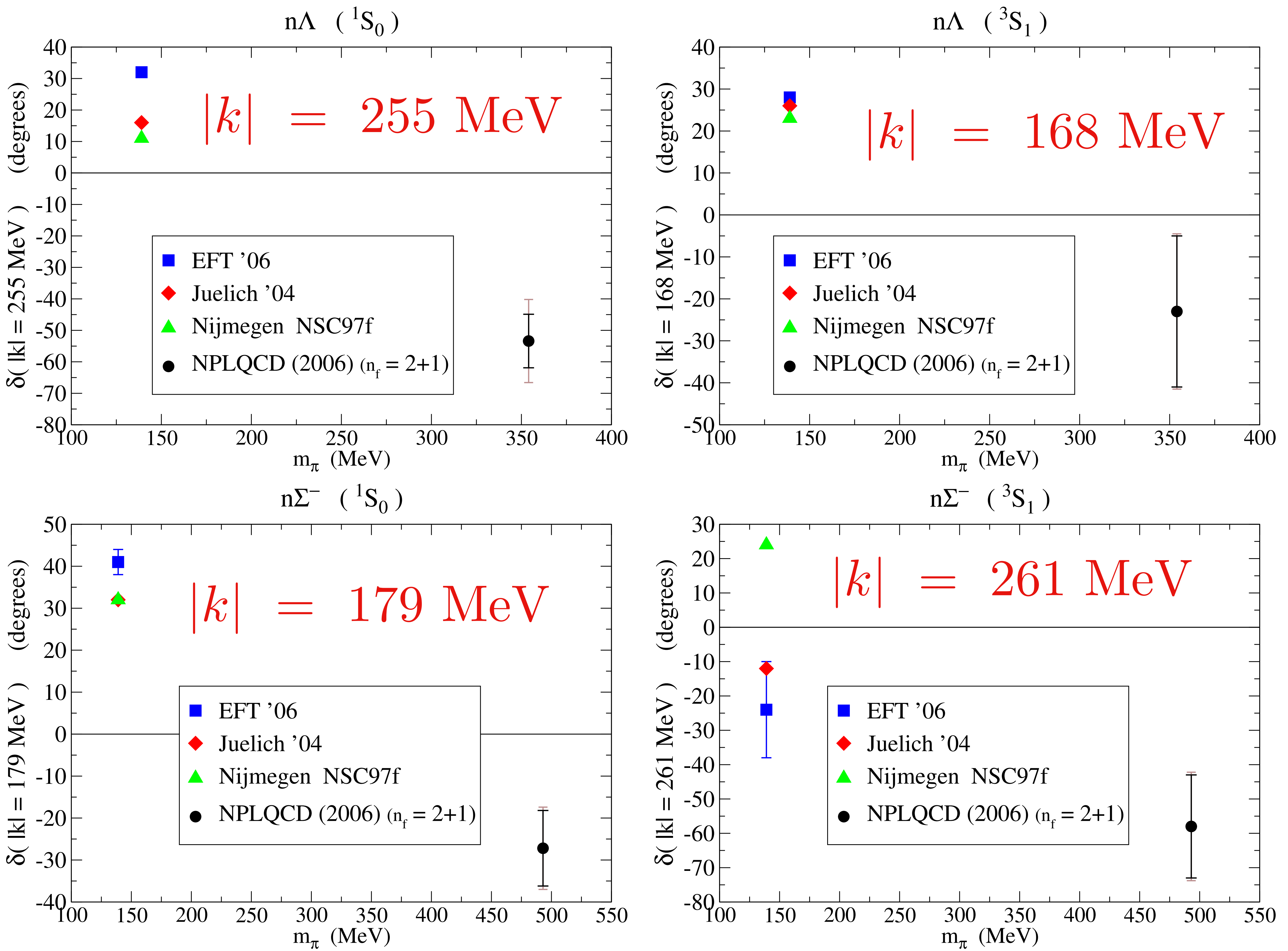
$$a_s^{-1} \sim \frac{m_\pi - m^*_\pi}{m_\pi} \Lambda_{QCD}$$

Lattice QCD will answer this question!

Lattice QCD: NN



YN interactions



Does signal/noise decay exponentially?

Does signal/noise decay exponentially?

Yes!

For a system of A *nucleons:*

$$\frac{\text{noise}}{\text{signal}} \underset{t\to\infty}{\longrightarrow} \frac{1}{\sqrt{N}}\, e^{A\left(m_p - \frac{3}{2} m_\pi\right)t}$$

Does signal/noise decay exponentially?

Yes!

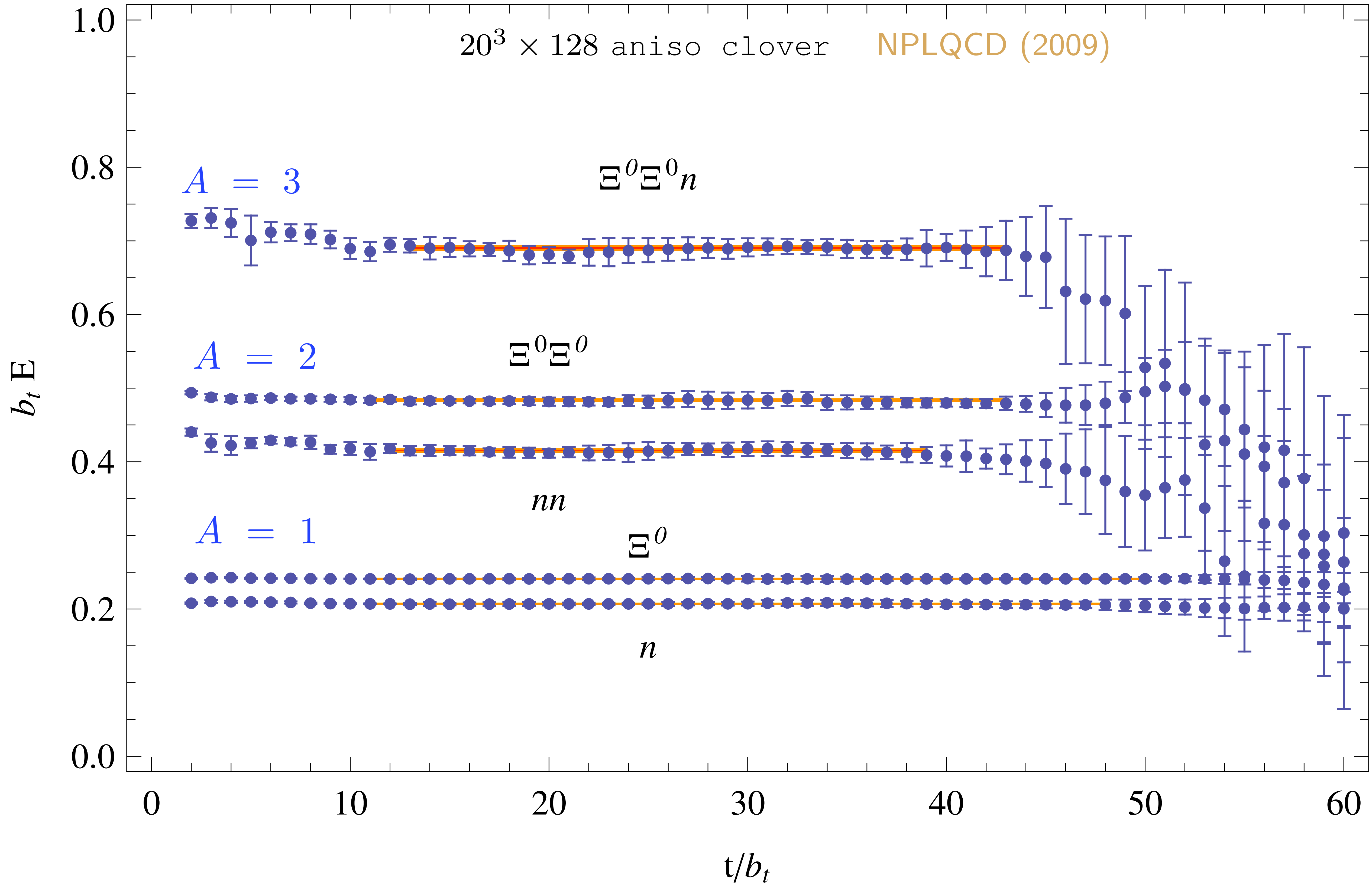
For a system of A *nucleons:*

$$\frac{\text{noise}}{\text{signal}} \xrightarrow[t \to \infty]{} \frac{1}{\sqrt{N}} \, e^{A\left(m_p - \frac{3}{2} m_\pi\right)t}$$

However, only *asymptotically*!

Is there a signal/noise problem?

related to sign problem?



Is there a signal/noise problem?

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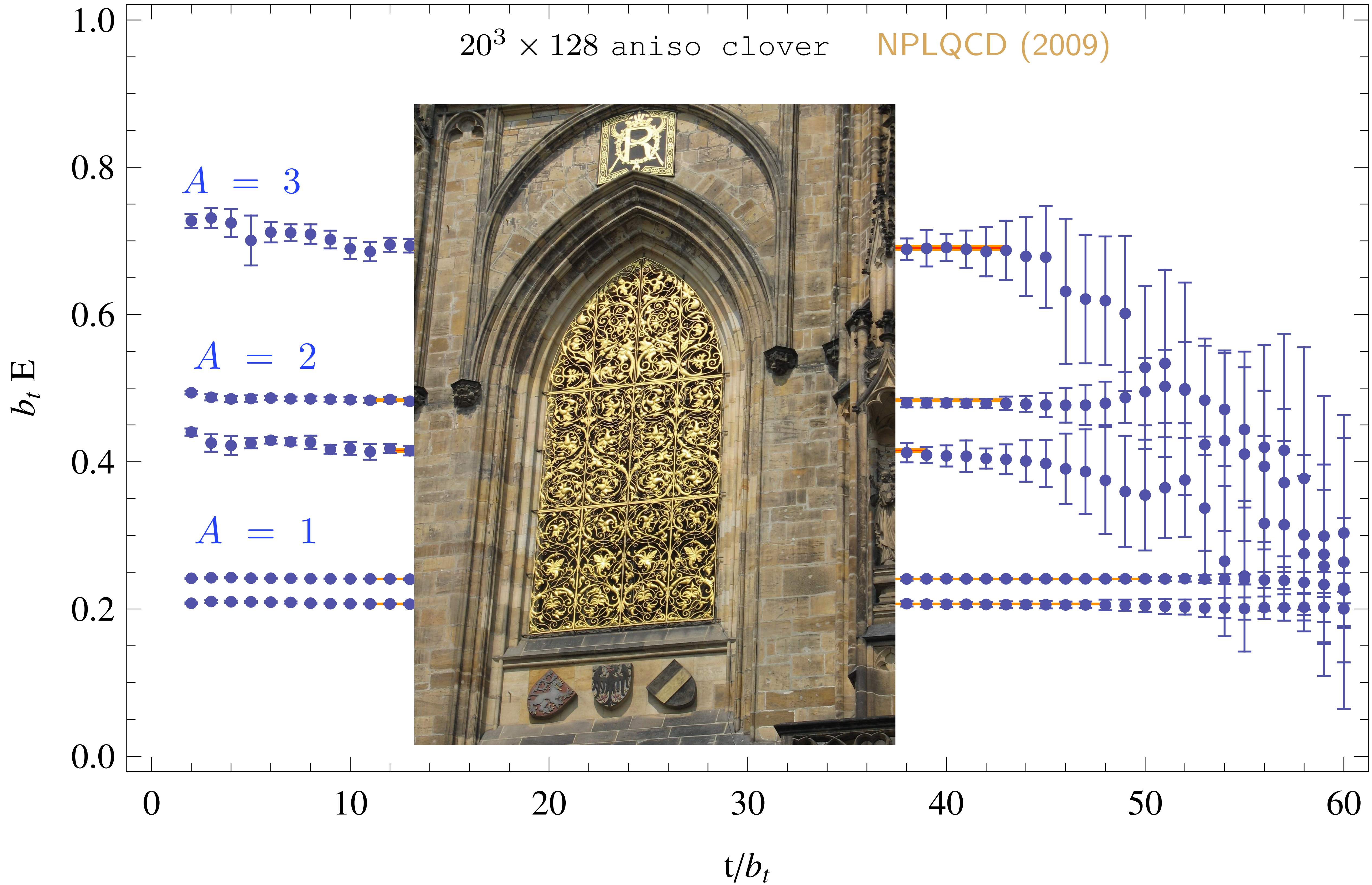


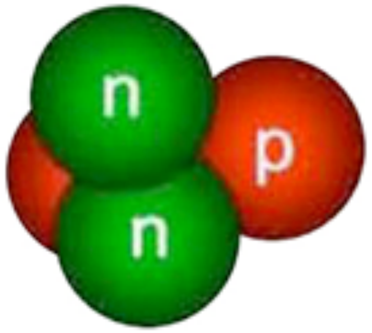
Contraction bottleneck for $A \gg 2$?

Naive factorial growth!

np: 36

nnp: 2880

npnp: 518400



$\vdots$

(A,Z): (A+Z)! (2A-Z)!

Recursion relations for mesons $\rightarrow A$ growth!

Baryon recursion relations in development!

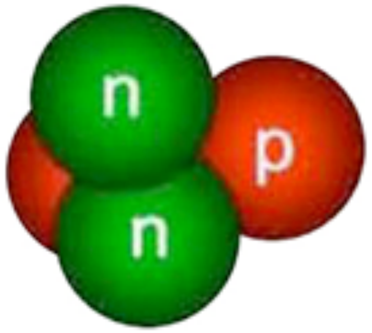
Contraction bottleneck for $A \gg 2$?

Naive factorial growth!

np: 36

Symmetries

nnp: 2880 $\longrightarrow$ 93



npnp: 518400 $\longrightarrow$ 1107

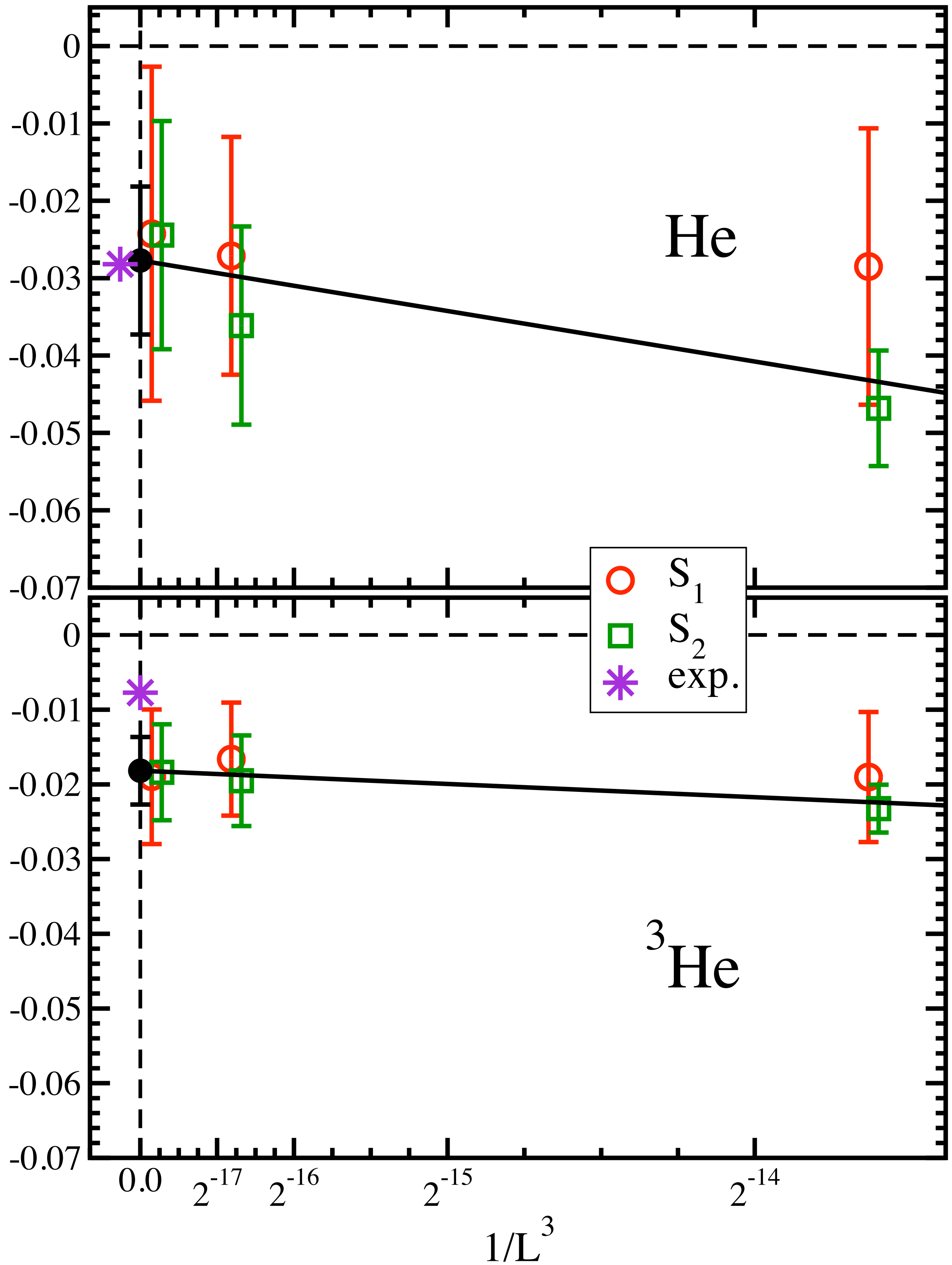
⋮

(A,Z): (A+Z)! (2A-Z)!

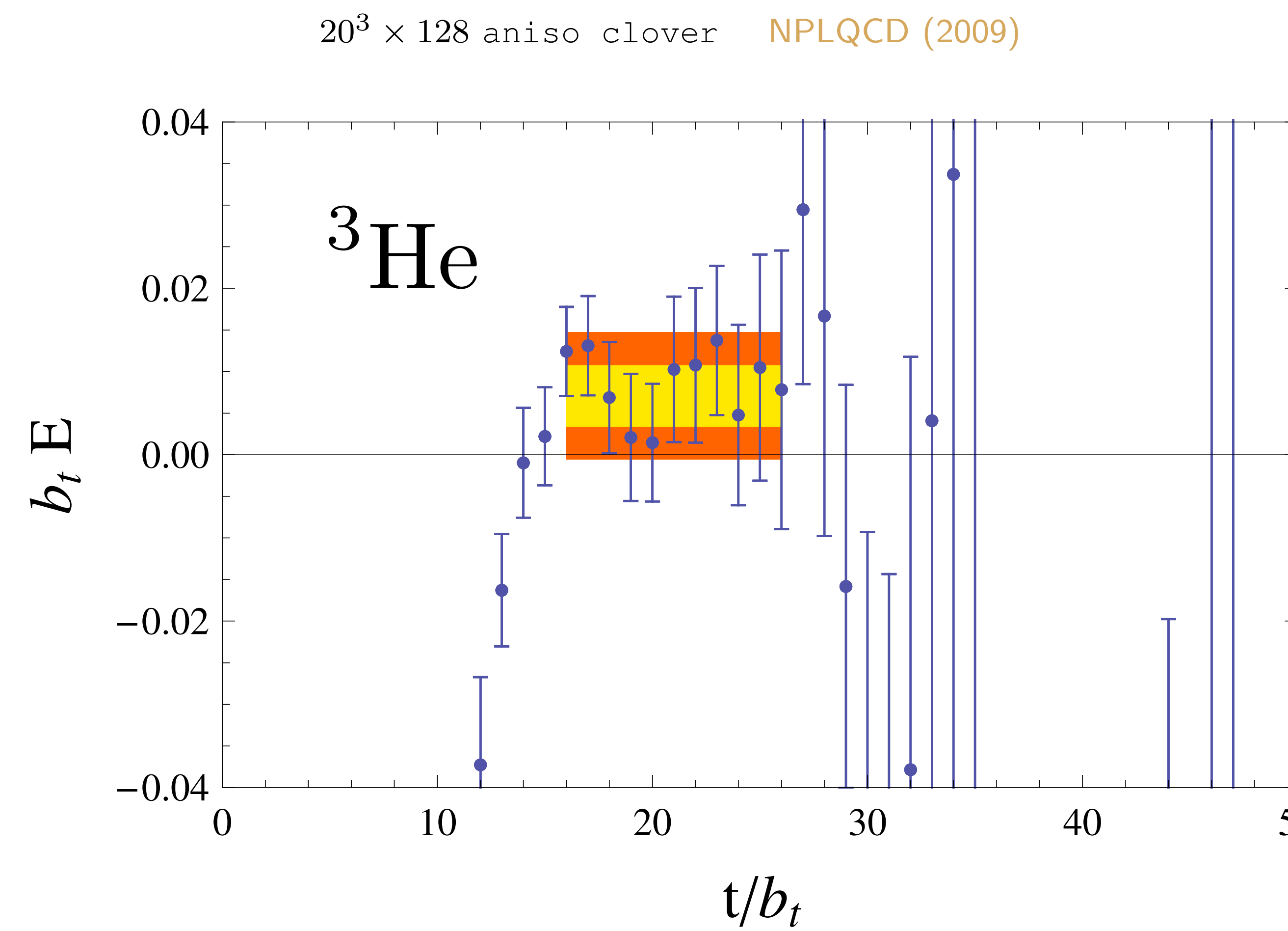
Recursion relations for mesons $\rightarrow A$ growth!

Baryon recursion relations in development!

(Yamazaki *et. al.* (2009))

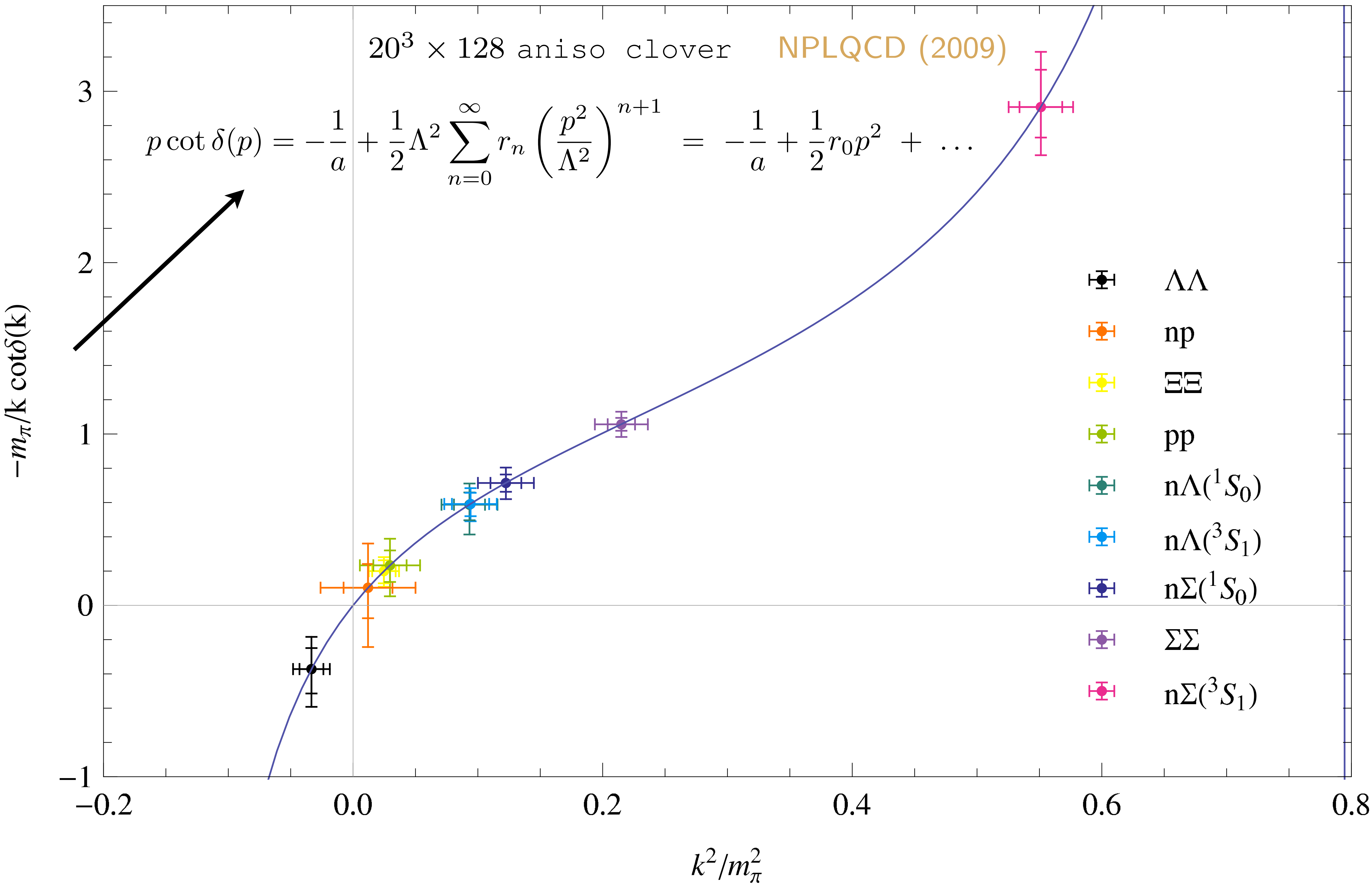


Quenched - Heavy pions



Full QCD

Lattice QCD: Baryon-Baryon



What about bound states?

$$\mathcal{A}_2(p) = \frac{8\pi}{M}\frac{1}{p\cot\delta(p) - ip} \longrightarrow \cot\delta(i\gamma) = i$$

Finite-V:

$$\cot\delta(i\kappa) = i - i\sum_{\mathbf{m}\neq 0}\frac{e^{-|\mathbf{m}|\kappa L}}{|\mathbf{m}|\kappa L}$$

$$\kappa = \gamma + \frac{6}{L}\frac{e^{-\gamma L}}{1 - \gamma r_3} + \mathcal{O}(e^{-\sqrt{2}\gamma L})$$

Need several volumes!

Is there an H-dibaryon?

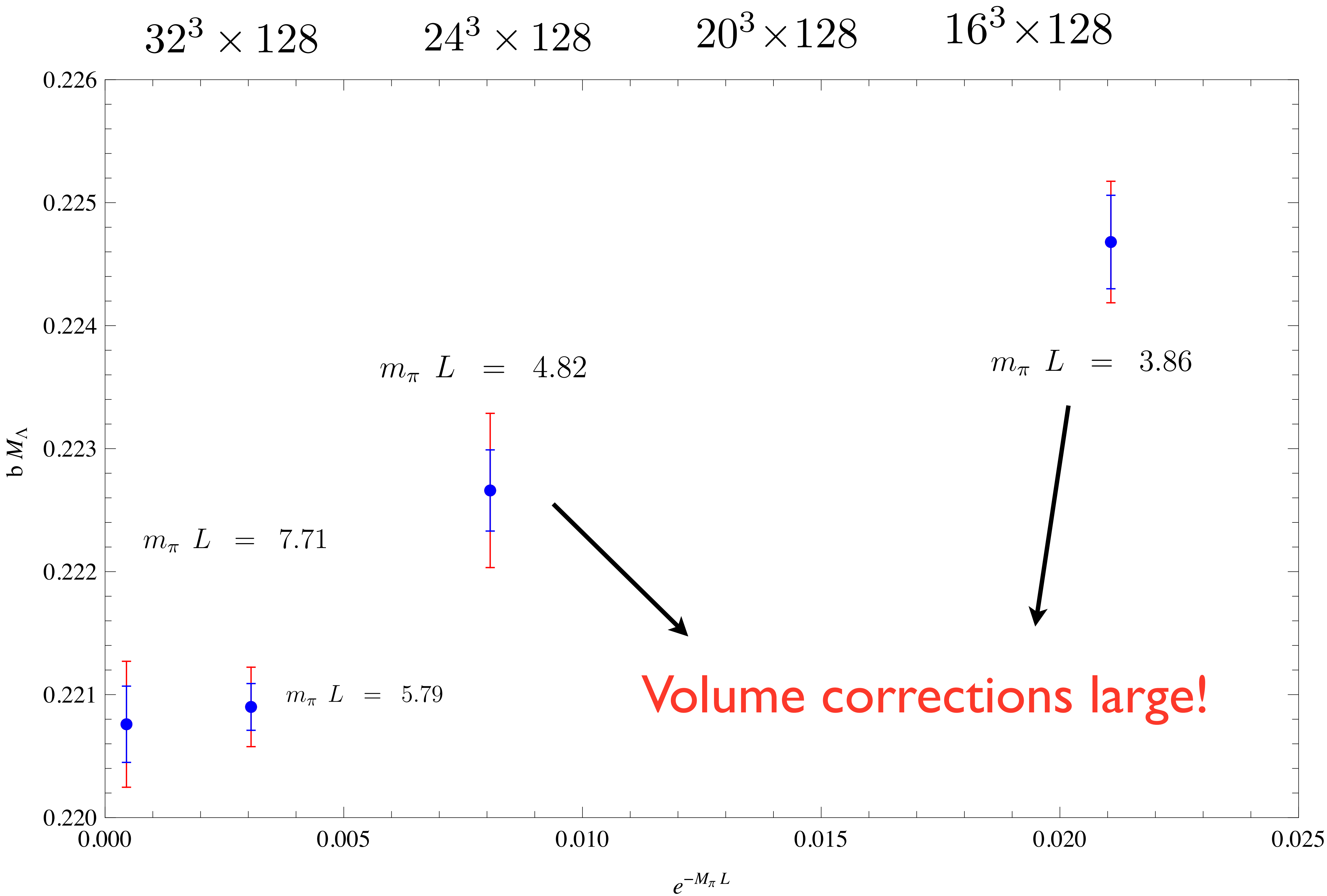
Need other volumes!

$$16^3 \times 128$$
$$20^3 \times 128$$
$$24^3 \times 128$$
$$32^3 \times 128$$

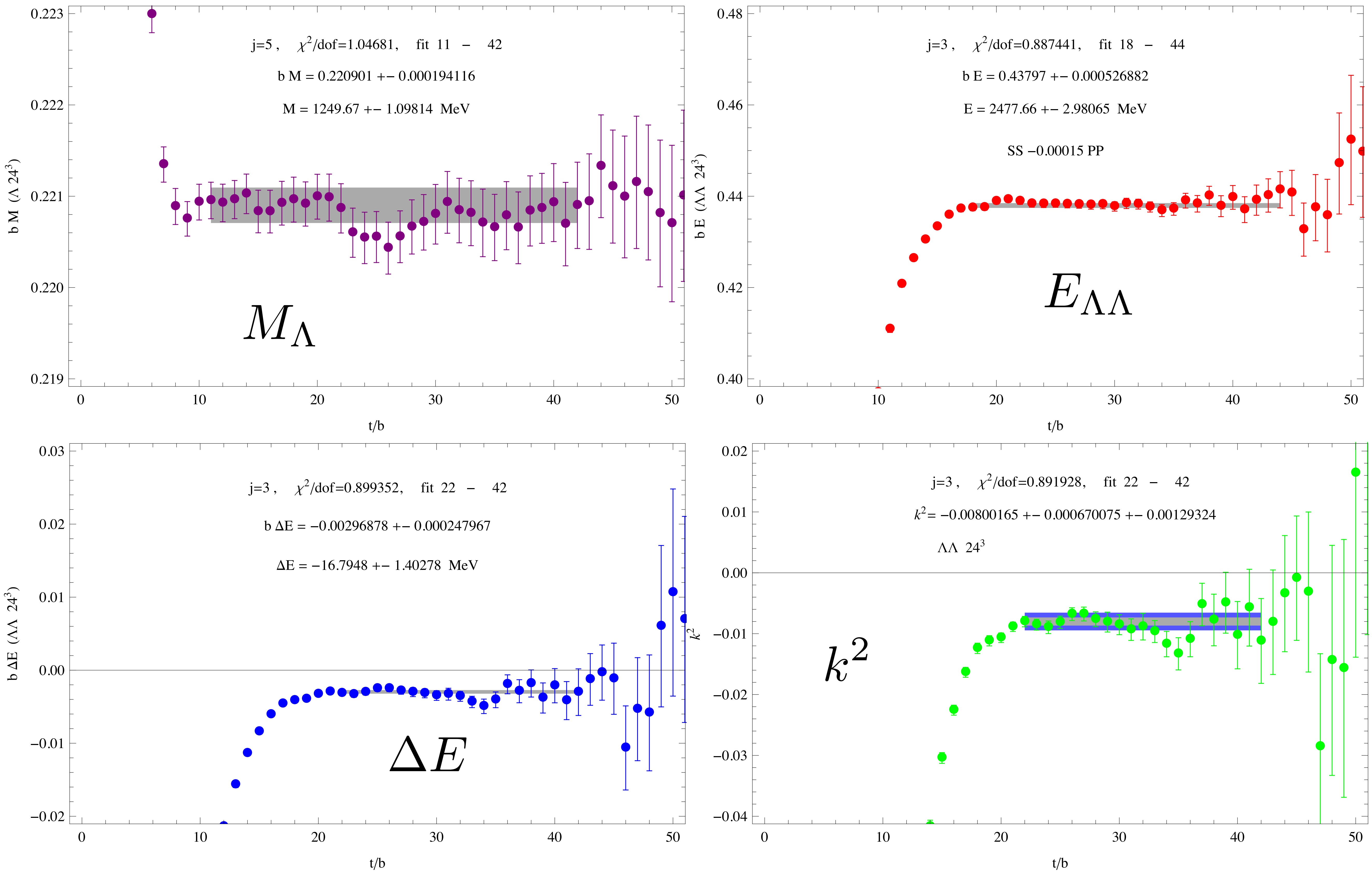
$$\kappa = \gamma + \frac{6}{L} \frac{e^{-\gamma L}}{1 - \gamma r_3} + \mathcal{O}(e^{-\sqrt{2}\gamma L})$$

$m_\pi \sim 389 \ \mathrm{MeV}$ $\quad b_s \sim 0.1227(8) \ \mathrm{fm}$ $\quad b_s/b_t = 3.500(32)$

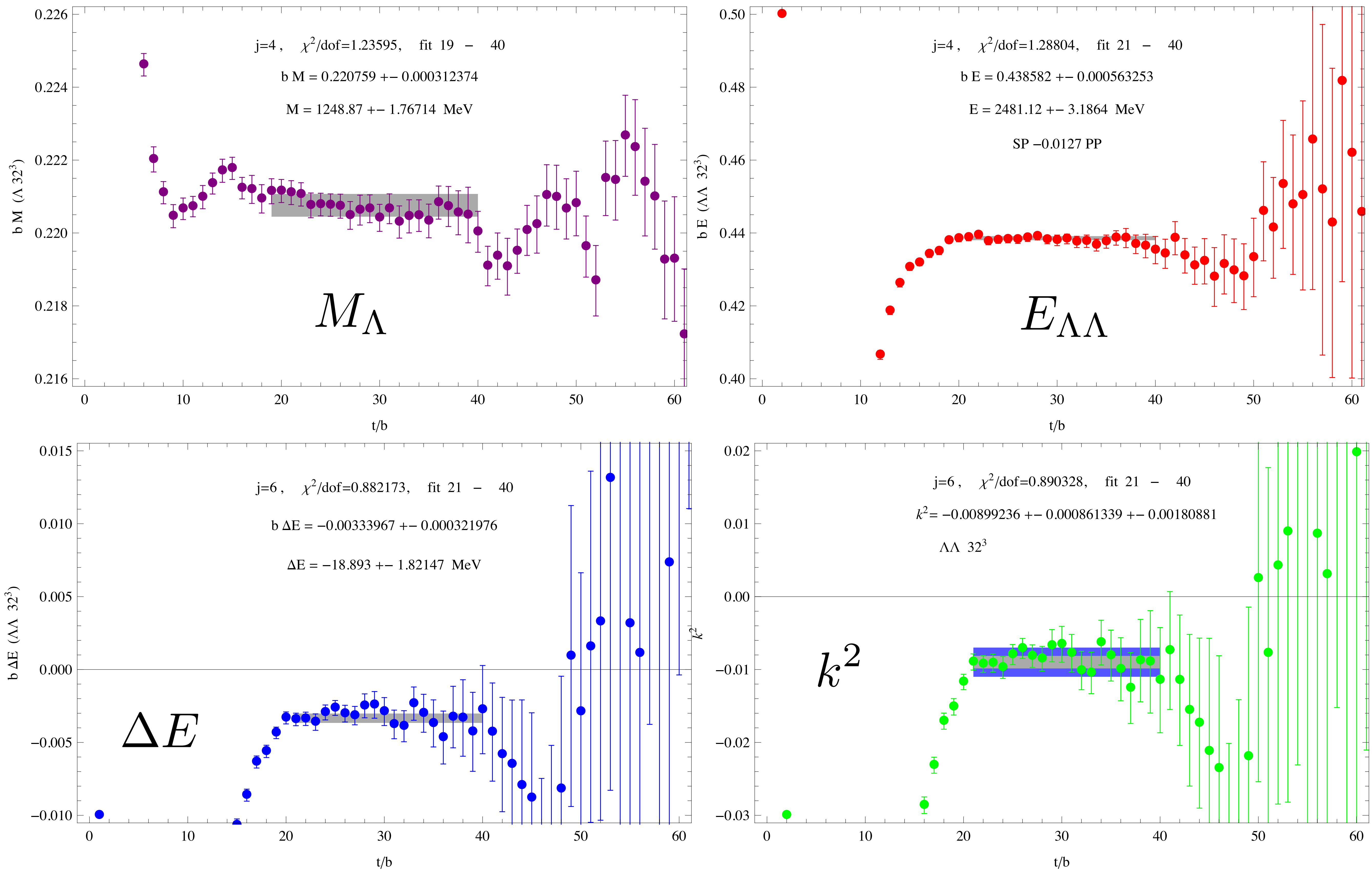
Volume dependence of the Λ mass



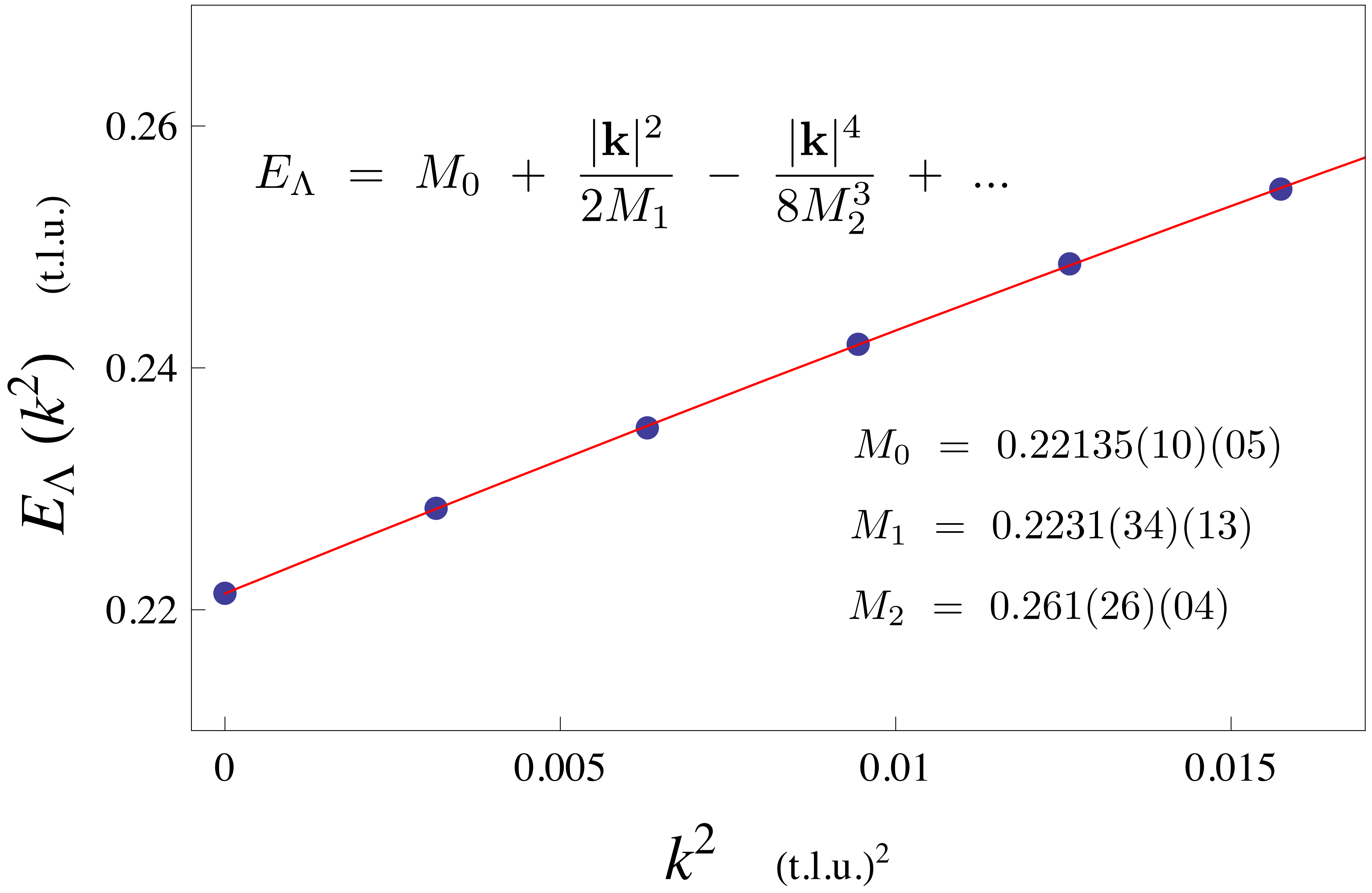
$24^3 \times 128$



$32^3 \times 256$



Energy-Momentum Relation



Special relativity satisfied!

The bottom line:

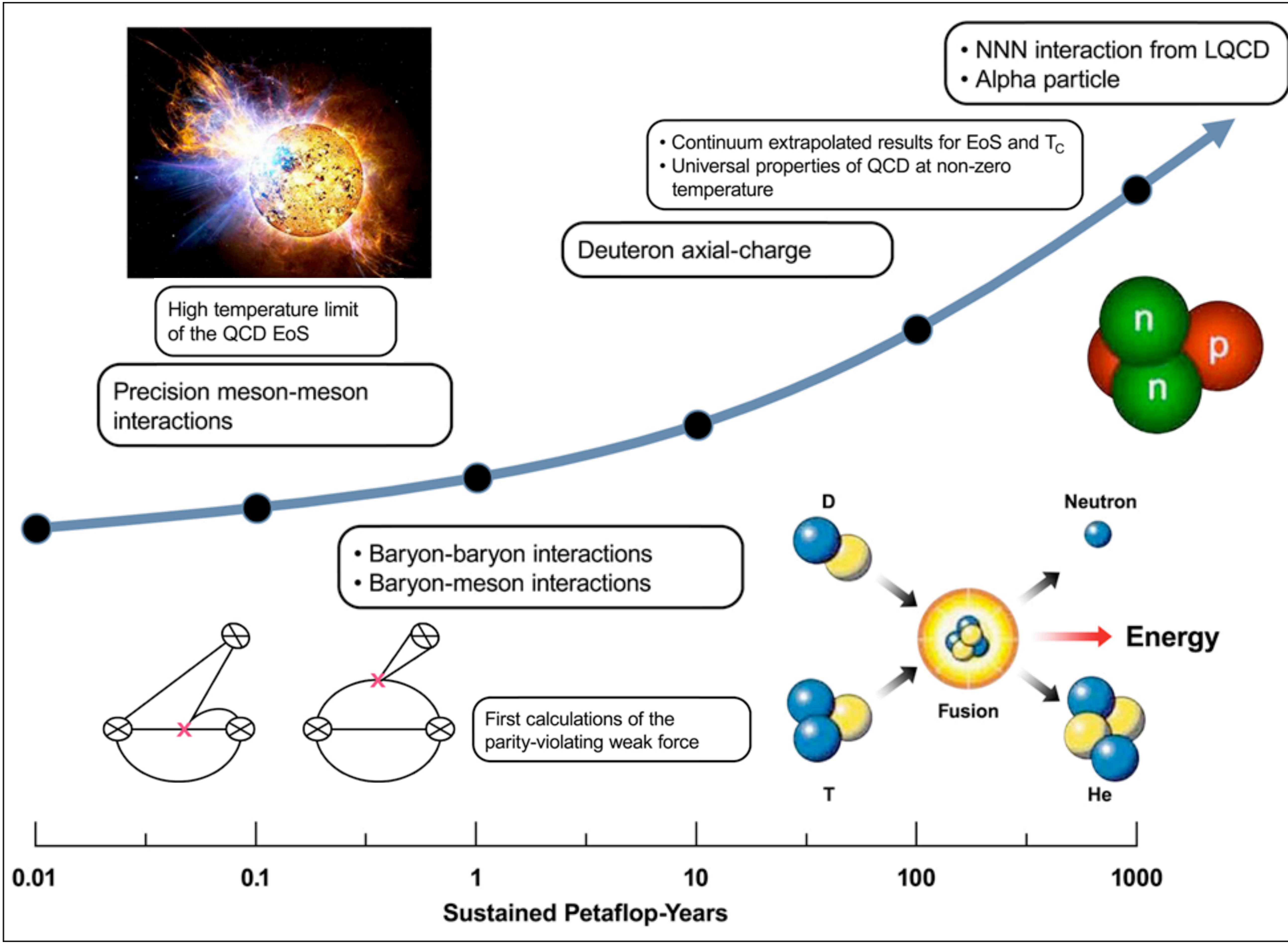
FV EM

$$B^{\mathrm{H}}_{\infty} = 16.6 \pm 2.1 \pm 4.5 \pm 1.0 \pm 0.6 \text{ MeV}$$

$$\boxed{B^{H}_{\infty} = 16.6 \pm 2.1 \pm 4.6 \text{ MeV}}$$

$m_\pi \sim 389$ MeV $\quad b_s \sim 0.1227(8)$ fm

Need smaller lattice spacing and lighter quarks!



• NNN interaction from LQCD
• Alpha particle
• Continuum extrapolated results for EoS and T_C
• Universal properties of QCD at non-zero temperature
Deuteron axial-charge
High temperature limit of the QCD EoS
Precision meson-meson interactions
n
p
n
• Baryon-baryon interactions
• Baryon-meson interactions
First calculations of the parity-violating weak force
D
Neutron
Energy
Fusion
T
He
0.01
0.1
1
10
100
1000
Sustained Petaflop-Years

Conclusion

- Stretched Meson-Meson systems are now a precision science.
- Work is beginning on obtaining hadron-hadron phase shifts.
- Two-baryon systems are currently under intense investigation. Calculation of the deuteron is a major outstanding benchmark.
- The H-dibaryon is bound at unphysical quark masses!
- Lattice QCD requires:
  - ★ the resources to move beyond the benchmarking stage.
  - ★ a strong collaborative effort among physicists, computer scientists and applied mathematicians.