THE MUONIC HYDROGEN LAMB SHIFT AND THE PROTON RADIUS

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Precise measurements in atomic physics → Learning about hadron structure

Hyperfine splitting (hydrogen atom):

$$E_{HF}^{exp} = E(n=1, s=1) - E(n=1, s=0) \qquad (s = \text{total spin})$$

Nature (1972)

$$\nu_{HF} = \frac{E_{HF}}{h} = 1420.4057517667(9)\ \text{MHz} \qquad (13\ \text{digits})$$

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Lamb shift (muonic hydrogen)

$$E \equiv E(2P_{3/2}(F=2)) - E(2S_{1/2}(F=1))$$

PSI: R. Pohl et al., Nature vol. 466, p. 213 (2010)

$$E_{exp} = 206.2949(32) meV$$

$$E_{th} = 209.9779(49) - 5.2262\frac{r_p^2}{fm^2} + 0.0347\frac{r_p^3}{fm^3}\ meV = 205.984\ meV$$

using CODATA value $r_p = 0.8768(69)$ fm.

$$E_{exp} - E_{th} = 0.311\ meV$$

New proposed value: $r_p = 0.84184(67)$ fm. 5 standard deviations!!

$$E_{LO} = 205.0074 = \mathcal{O}(m_r\alpha^3)$$

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Theoretical setup

We use an effective field theory, Potential Non-Relativistic QED, which describes the muonic hydrogen dynamics and profits from the hierarchy $m_\mu \gg m_\mu\alpha \gg m_\mu\alpha^2$

$$\left.\begin{array}{l}\left(i\partial_0 - \frac{\mathbf{p}^2}{2m_r} - \frac{\alpha}{r}\right)\psi(\mathbf{r}) = 0 \\ +\text{corrections to the potential} \\ +\text{interaction with ultrasoft photons}\end{array}\right\} \text{potential NRQED} \qquad E \sim mv^2$$

Scales:

$m_p \sim \Lambda_\chi$

$m_\mu \sim m_\pi \sim m_r = \frac{m_\mu m_p}{m_p+m_\mu}$

$m_r\alpha \sim m_e$

Expansion parameters, ratios between scales, mainly:

$$\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \sim \frac{1}{9}$$

$$\frac{m_r\alpha}{m_r} \sim \frac{m_r\alpha^2}{m_r\alpha} \sim \alpha \sim \frac{1}{137}$$

Needed precision $m_r\alpha^5$ (heavy quarkonium precision)

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$$\alpha_{eff}(k) = \alpha\frac{1}{1+\Pi(-\mathbf{k}^2)}\,,$$

where

$$\Pi(k^2) = \alpha\Pi^{(1)}(k^2) + \alpha^2\Pi^{(2)}(k^2) + \alpha^3\Pi^{(3)}(k^2) + \dots$$

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Vacuum polarization effects: $\mathcal{O}(m_r\alpha^3)$

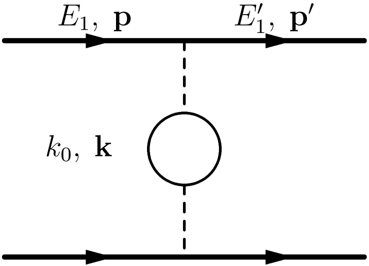
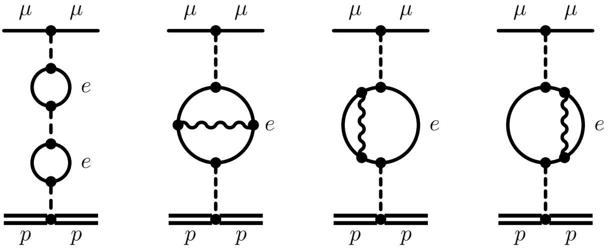


Figure: *Leading correction to the Coulomb potential due to the electron vacuum polarization.* $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ *and* $k_0 = E_1 - E_1'$.

1-loop static potential

$$E_{LO} = 205.0074 = \mathcal{O}(m_r\alpha^3)$$

Vacuum polarization effects: $\mathcal{O}(m_r\alpha^4, m_r\alpha^5)$



Pachuki/Borie
2-loop static potential is the same as two-loop vacuum polarization iterations
1.5079(*two loop vacuum polarization*)+ 0.151(*iteration one-loop*)

3-loop static potential (three loop vacuum polarization, Kinoshita-Nio, 0.0076)

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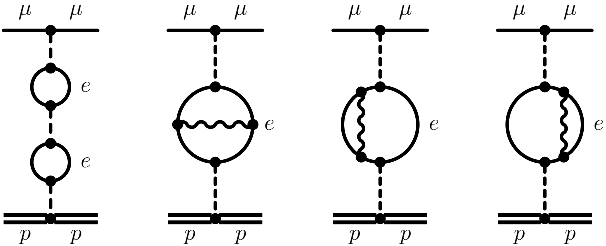

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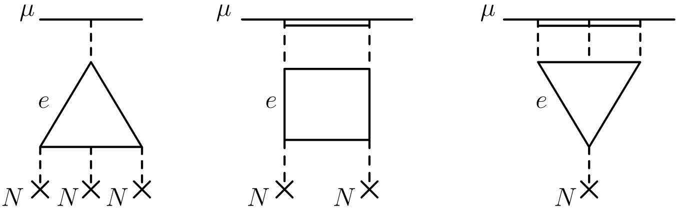
Static potential, not vacuum polarization: $\mathcal{O}(m_r\alpha^5)$



(1:3) (2:2) (3:1)

Light-by-light (Wichmann-Kroll and Delbrück) contribution very small

$$\Delta E \simeq -0.0009 \text{ mev (Karshenboim } \textit{et al}.)$$

Earlier work by Borie

Observation:
The limit $m_e \to 0$ known from QCD (Anzai et al. and Smirnov et al).
It should be possible to obtain the result with finite mass (albeit numerically) and check.

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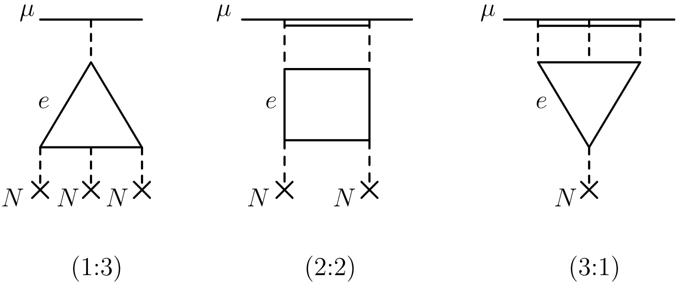

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$1/m$ potential

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\begin{aligned}
L_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} dt S^\dagger(\mathbf{x},\mathbf{X},t)\Bigg\{ i\partial_0 - \frac{\mathbf{p}^2}{2m_r} \\
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\end{aligned}
$$

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V(\mathbf{x},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m_\mu} + \frac{V^{(2)}(r)}{m_\mu^2} + \dots
$$

$$
\frac{V^{(1)}(r)}{m_\mu} \to \mathcal{O}(m_r\alpha^6)
$$

relativistic corrections+vacuum polarization

$$L_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} dt S^\dagger(\mathbf{x},\mathbf{X},t)\left\{ i\partial_0 - \frac{\mathbf{p}^2}{2m_r} - V(\mathbf{x},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) + e\mathbf{x}\cdot\mathbf{E}(\mathbf{X},t)\right\} S(\mathbf{x},\mathbf{X},t) - \int d^3\mathbf{x}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$

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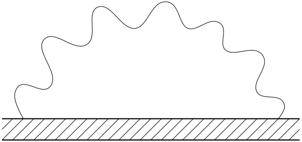
$$\frac{V^{(2)}(r)}{m_\mu^2} \rightarrow \mathcal{O}(m_r\alpha^4, \alpha^5)$$

$\mathcal{O}(m\alpha^4)$ 0.0575 (purely relativistic)
$\mathcal{O}(m\alpha^5)$ 0.0169 (Pachucki and Veitia)

Ultrasoft effects: $\mathcal{O}(m\alpha^5)$



$$\Delta E = -0.6677 \; meV$$

$$\mathcal{O}(m\alpha^5 \frac{m_\mu}{m_p}) : \qquad \Delta E = -0.045 \; meV$$

Start the overlap with hadronic effects.

Hadronic corrections

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$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.}\delta^3(\mathbf{r})$$

$$D_d^{had.} = -c_3 - 16\pi\alpha d_2 + \frac{\pi\alpha}{2}c_D$$

c_3, d_2, c_D, ... matching coefficients of NRQED.

$$HBET(m_\pi/m_\mu) \to NRQED(m_\mu\alpha) \to pNRQED$$

$$\delta\mathcal{L} = \cdots \frac{d_2}{m_p^2}F_{\mu\nu}D^2F^{\mu\nu} + \cdots - e\frac{c_D}{m_p^2}N_p^\dagger\boldsymbol{\nabla}\cdot\mathbf{E}N_p + \cdots + \frac{c_3}{m_p^2}N_p^\dagger N_p\mu^\dagger\mu$$

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$$D_d^{had.} = -c_3 - 16\pi\alpha d_2 + \frac{\pi\alpha}{2}c_D$$

c_3, d_2, c_D, ... matching coefficients of NRQED.

$$HBET(m_\pi/m_\mu) \to NRQED(m_\mu\alpha) \to pNRQED$$

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Hadronic corrections

$$L_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} dt S^\dagger(\mathbf{x},\mathbf{X},t)\Bigg\{ i\partial_0 - \frac{\mathbf{p}^2}{2m_r}$$

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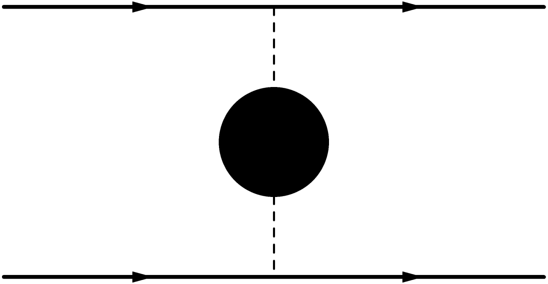

Figure: *Leading correction to the Coulomb potential due to the hadronic vacuum polarization.*

$d_2 \rightarrow$ hadronic vacuum polarization

$$\Delta E = 0.011 \text{ meV}$$

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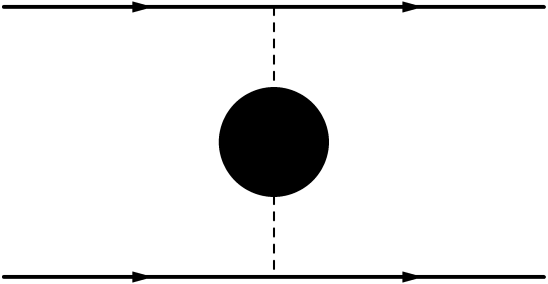

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c_3 or Zemach (r^3) effects: $\mathcal{O}(m_\mu \alpha^5 \times \frac{m_\mu^2}{\Lambda_\chi^2} \times \frac{m_\mu}{m_\pi})$

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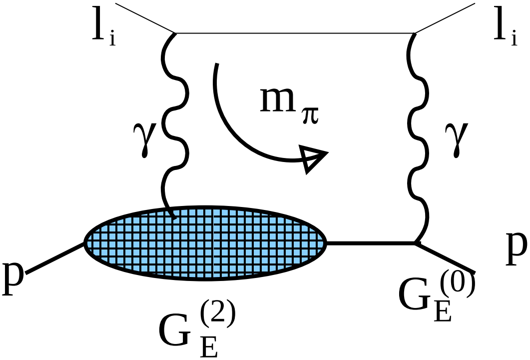

Figure: *Symbolic representation (plus permutations) of the Zemach $\langle r^3 \rangle$ correction.*

$$\Delta E = 0.047 \frac{\langle r_p^3 \rangle}{\mathrm{fm}^3}$$

$$\frac{\langle r_p^3 \rangle}{\mathrm{fm}^3} = \frac{96}{\pi} \int d^{D-1}k \, \frac{1}{\mathbf{k}^6} G_E^{(0)} G_E^{(2)}$$

$$\begin{aligned} \delta c^{pl_i}_{3,Zemach} &= \frac{\pi}{3}\alpha^2 m_p^2 m_\mu \langle r_p^3 \rangle = 2(\pi\alpha)^2 \left(\frac{m_p}{4\pi F_0}\right)^2 \frac{m_{l_i}}{m_\pi} \left\{ \frac{3}{4} g_A^2 + \frac{1}{8} \right. \\ &\left. + \frac{2}{\pi} g^2_{\pi N \Delta} \frac{m_\pi}{\Delta} \sum_{r=0}^{\infty} C_r \left(\frac{m_\pi}{\Delta}\right)^{2r} + g^2_{\pi N \Delta} \sum_{r=1}^{\infty} H_r \left(\frac{m_\pi}{\Delta}\right)^{2r} \right\} , \end{aligned}$$

where

$$C_r = \frac{(-1)^r \Gamma(-3/2)}{\Gamma(r+1)\Gamma(-3/2-r)} \left\{ B_{6+2r} - \frac{2(r+2)}{3+2r} B_{4+2r} \right\} , \qquad r \geq 0 ,$$

$$B_n \equiv \int_0^\infty dt \frac{t^{2-n}}{\sqrt{1-t^2}} \ln \left[\frac{1}{t} + \sqrt{\frac{1}{t^2} - 1} \right]$$

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Including Pions and Δ particles

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$$\frac{\langle r_p^3\rangle|_{''exp''}}{\mathrm{fm}^3} = \left\{\begin{array}{l} 2.71(13)\ \mathrm{Friar-Sick} \\ 2.50\ \mathrm{Arrington} \\ 2.85(8)\ \mathrm{Bernauer-Arrington} \end{array}\right\} \rightarrow \Delta E = 0.025 - 0.029$$

Not the reason for the discrepancy.

$\langle r_p^3\rangle \sim 35$ De Rujula, not consistent neither with experiment nor chiral symmetry.

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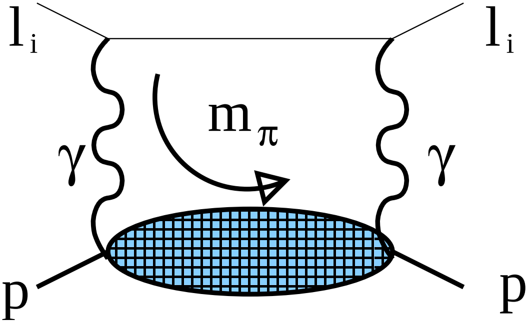

$$\Delta E|_{Fitexp} = 0.012(\mathrm{Pachucki})/0.015(\mathrm{Borie})\ mev$$

$$\Delta E|_{\chi PT}(pions) = 0.018(\mathrm{Nevado-Pineda})\ mev$$

$$c_{3,NR}^{pl_i} = -e^4 m_p m_{l_i} \int \frac{d^4 k_E}{(2\pi)^4} \frac{1}{k_E^4} \frac{1}{k_E^4 + 4m_{l_i}^2 k_{0,E}^2}$$
$$\times \left\{ (3k_{0,E}^2 + \mathbf{k}^2) S_1(ik_{0,E}, -k_E^2) - \mathbf{k}^2 S_2(ik_{0,E}, -k_E^2) \right\}$$

$$T^{\mu\nu} = i \int d^4x \, e^{iq\cdot x} \langle p, s | T J^\mu(x) J^\nu(0) | p, s \rangle \, ,$$

which has the following structure ($\rho = q \cdot p/m$):

$$\begin{aligned} T^{\mu\nu} &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) S_1(\rho, q^2) \\ &+ \frac{1}{m_p^2} \left(p^\mu - \frac{m_p \rho}{q^2} q^\mu \right) \left(p^\nu - \frac{m_p \rho}{q^2} q^\nu \right) S_2(\rho, q^2) \\ &- \frac{i}{m_p} \epsilon^{\mu\nu\rho\sigma} q_\rho s_\sigma A_1(\rho, q^2) \\ &- \frac{i}{m_p^3} \epsilon^{\mu\nu\rho\sigma} q_\rho \left((m_p \rho) s_\sigma - (q \cdot s) p_\sigma \right) A_2(\rho, q^2) \end{aligned}$$

$$
\begin{aligned}
c_{3,NR}^{pl_i} = & -e^4 m_p^2 \frac{m_{l_i}}{m_\pi}\left(\frac{g_A}{f_\pi}\right)^2 \int \frac{d^{D-1}k_E}{(2\pi)^{D-1}} \frac{1}{(1+\mathbf{k}^2)^4} \\
& \times \int_0^\infty \frac{dw}{\pi} w^{D-5} \frac{1}{w^2 + 4\frac{m_{l_i}^2}{m_\pi^2}\frac{1}{(1+\mathbf{k}^2)^2}} \\
& \times \left\{(2+(1+\mathbf{k}^2)^2)A_E(w^2,\mathbf{k}^2) + (1+\mathbf{k}^2)^2\mathbf{k}^2 w^2 B_E(w^2,\mathbf{k}^2)\right\}
\end{aligned}
$$

where (for $D = 4$)

$$
A_E = -\frac{1}{4\pi}\left[-\frac{3}{2} + \sqrt{1+w^2} + \int_0^1 dx \frac{1-x}{\sqrt{1+x^2w^2+x(1-x)w^2\mathbf{k}^2}}\right],
$$

$$
\begin{aligned}
B_E = \frac{1}{8\pi}\Bigg[& \int_0^1 dx \frac{1-2x}{\sqrt{1+x^2w^2+x(1-x)w^2\mathbf{k}^2}} \\
& -\frac{1}{2}\int_0^1 dx \frac{(1-x)(1-2x)^2}{(1+x^2w^2+x(1-x)w^2\mathbf{k}^2)^{\frac{3}{2}}}\Bigg].
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Definition of the proton radius

$$\langle p', s | J^\mu | p, s \rangle = \bar{u}(p') \left[F_1(q^2)\gamma^\mu + iF_2(q^2)\frac{\sigma^{\mu\nu} q_\nu}{2m_p} \right] u(p) \,,$$

$$F_i(q^2) = F_i + \frac{q^2}{m_p^2} F_i' + \ldots$$

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Infrared divergent! → Wilson coefficient

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Precise determination of hadronic parameters from alternative sources (experiment).
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The proton radius is a matching coefficient of the effective theory. In general an scheme/scale dependent object.

Precise determination of hadronic parameters from alternative sources (experiment).
Non-trivial double checks by chiral perturbation theory.

Previous claims about r^3 unfounded.

Theory appears to be solid, not to say extremely reliable. Only few a places where one should look "again" (out of desperation). Two/three-loop vacuum polarization potential? "Scheme" dependence? ...

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Order $1/m^2$

. :

$$\tilde{V}^{(b)} = \frac{\pi\alpha_{eff}(k)}{2}\left[Z_p\frac{c_D^{(\mu)}}{m_\mu^2} + Z_\mu\frac{c_D^{(p)}}{m_p^2}\right],$$

$$\tilde{V}^{(c)} = -i2\pi\alpha_{eff}(k)\frac{(\mathbf{p}\times\mathbf{k})}{\mathbf{k}^2}\cdot\left\{Z_p\frac{c_S^{(\mu)}\mathbf{s}_1}{m_\mu^2} + Z_\mu\frac{c_S^{(p)}\mathbf{s}_2}{m_p^2}\right\},$$

$$\tilde{V}^{(d)} = -Z_\mu Z_p 16\pi\alpha\left(\frac{d_2^{(\mu)}}{m_\mu^2} + \frac{d_2^{(\tau)}}{m_\tau^2} + \frac{d_{2,NR}}{m_p^2}\right),$$

$$\tilde{V}^{(e)} = -Z_\mu Z_p\frac{4\pi\alpha_{eff}(k)}{m_\mu m_p}\left(\frac{\mathbf{p}^2}{\mathbf{k}^2} - \frac{(\mathbf{p}\cdot\mathbf{k})^2}{\mathbf{k}^4}\right),$$

$$\tilde{V}^{(f)} = -\frac{i4\pi\alpha_{eff}(k)}{m_\mu m_p}\frac{(\mathbf{p}\times\mathbf{k})}{\mathbf{k}^2}\cdot\left(Z_p c_F^{(\mu)}\mathbf{s}_1 + Z_\mu c_F^{(p)}\mathbf{s}_2\right),$$

$$\tilde{V}^{(g)} = \frac{4\pi\alpha_{eff}(k)c_F^{(1)}c_F^{(2)}}{m_\mu m_p}\left(\mathbf{s}_1\cdot\mathbf{s}_2 - \frac{\mathbf{s}_1\cdot\mathbf{k}\mathbf{s}_2\cdot\mathbf{k}}{\mathbf{k}^2}\right),$$

$$\tilde{V}^{(h)} = -\frac{1}{m_p^2}\left\{(c_{3,NR}^{pl_i} + 3c_{4,NR}^{pl_i}) - 2c_{4,NR}^{pl_i}\mathbf{S}^2\right\}.$$

$$\tilde{V}^{(a)}_{1\,loop} = \frac{Z_\mu^2 Z_p^2 \alpha^2}{m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} - \frac{8}{3}\log 2 + \frac{5}{3}\right),$$

$$\tilde{V}^{(b,c)}_{1\,loop} = \frac{4 Z_\mu^2 Z_p^2 \alpha^2}{3 m_\mu m_p}\left(\log\frac{\mathbf{k}^2}{\mu^2} + 2\log 2 - 1\right).$$

Hadronic corrections: Spin-dependent

$$L_{pNRQED} = \int d^3\mathbf{x} d^3\mathbf{X} dt S^\dagger(\mathbf{x},\mathbf{X},t)\Biggl\{ i\partial_0 - \frac{\mathbf{p}^2}{2m_r}$$

$$-V(\mathbf{x},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) + e\mathbf{x}\cdot\mathbf{E}(\mathbf{X},t)\Biggr\} S(\mathbf{x},\mathbf{X},t) - \int d^3\mathbf{x}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$

$$V(\mathbf{x},\mathbf{p},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m_\mu} + \frac{V^{(2)}(r)}{m_\mu^2} + \ldots$$

$$\frac{\delta V^{(2)}(r)}{m_\mu^2} \to \frac{1}{m_p^2} D_d^{had.}(\mathbf{S}_1+\mathbf{S}_2)^2\delta^3(\mathbf{r})$$

$$D_s^{had.} = 2c_4$$

c_4, matching coefficient of NRQED.

$$HBET(m_\pi/m_\mu) \to NRQED(m_\mu\alpha) \to pNRQED$$

$$\delta\mathcal{L} = \cdots - \frac{c_4}{m_p^2} N_p^\dagger \boldsymbol{\sigma} N_p \mu^\dagger \boldsymbol{\sigma}\mu$$

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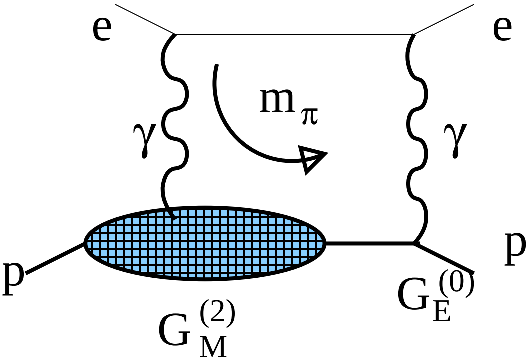

Figure: *Symbolic representation (plus permutations) of the Zemach correction.*

$$\delta c^{pl}_{4,Zemach} = (4\pi\alpha)^2 m_p \frac{2}{3} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \frac{1}{\mathbf{k}^4} G_E^{(0)} G_M^{(2)}\,.$$

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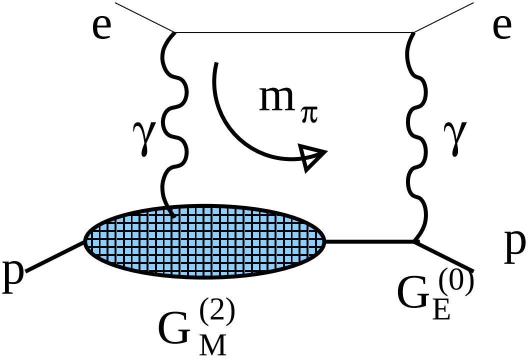

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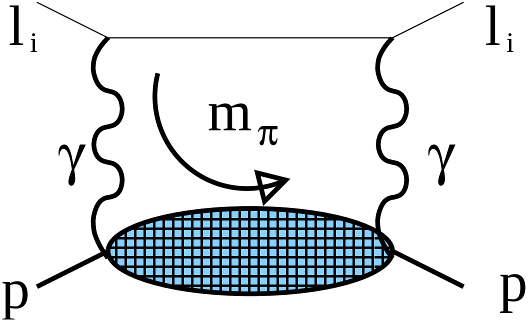

Figure: *Symbolic representation (plus permutations) of the spin-dependent polarizability correction.*

$$\delta c^{pl}_{4,pol} = -\frac{ig^4}{3}\int \frac{d^Dk}{(2\pi)^D}\frac{1}{k^2}\frac{1}{k^4-4m_l^2k_0^2}\left\{A_1(k_0,k^2)(k_0^2+2k^2)+3k^2\frac{k_0}{m_p}A_2(k_0,k^2)\right\}$$

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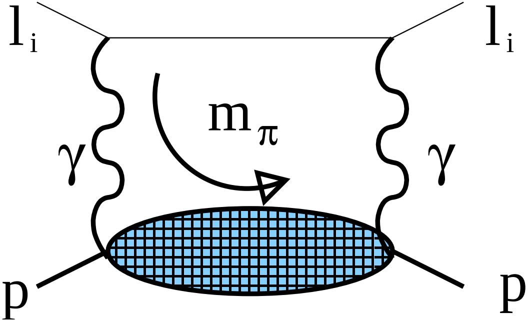

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$$T^{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle p,s|TJ^{\mu}(x)J^{\nu}(0)|p,s\rangle\,,$$

which has the following structure ($\rho = q\cdot p/m$):

$$\begin{aligned}
T^{\mu\nu} \quad = \quad & \left(-g^{\mu\nu}+\frac{q^{\mu}q^{\nu}}{q^2}\right)S_1(\rho,q^2)\\
& +\frac{1}{m_p^2}\left(p^{\mu}-\frac{m_p\rho}{q^2}q^{\mu}\right)\left(p^{\nu}-\frac{m_p\rho}{q^2}q^{\nu}\right)S_2(\rho,q^2)\\
& -\frac{i}{m_p}\,\epsilon^{\mu\nu\rho\sigma}q_{\rho}s_{\sigma}A_1(\rho,q^2)\\
& -\frac{i}{m_p^3}\,\epsilon^{\mu\nu\rho\sigma}q_{\rho}\left((m_p\rho)s_{\sigma}-(q\cdot s)p_{\sigma}\right)A_2(\rho,q^2)
\end{aligned}$$

Only logarithmically chiral enhanced but they can be determined from hydrogen hyperfine splitting.

$$\begin{aligned}\delta c_{4,NR}^{pl} \simeq & \left(1-\frac{\mu_p^2}{4}\right)\alpha^2 \ln\frac{m_l^2}{\nu^2} \\ & +\frac{b_{1,F}^2}{18}\alpha^2\ln\frac{\Delta^2}{\nu^2}+\frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{2}{3}\left(\frac{2}{3}+\frac{7}{2\pi^2}\right)\pi^2 g_A^2\ln\frac{m_\pi^2}{\nu^2} \\ & +\frac{m_p^2}{(4\pi F_0)^2}\alpha^2\frac{8}{27}\left(\frac{5}{3}-\frac{7}{\pi^2}\right)\pi^2 g_{\pi N\Delta}^2\ln\frac{\Delta^2}{\nu^2}\,,\end{aligned}$$

$$E_{\rm HF}=4\frac{c_{4,NR}^{pl}}{m_p^2}\frac{1}{\pi}\left(\frac{\mu_{lp}\alpha}{n}\right)^3.$$

Hydrogen. By fixing the scale $\nu=m_\rho$ we obtain the following number for the total sum in the SU(2) case:

$$E_{\rm HF,logarithms}(m_\rho)=-0.031\ {\rm MHz}\,,$$

which accounts for approximately 2/3 of the difference between theory (pure QED) and experiment. What is left gives the expected size of the counterterm. Experimentally what we have is $c_{4,NR}^{pl}=-48\alpha^2$ and $c_{4,R}^{pl}(m_\rho)\simeq c_{4,R}^{p}(m_\rho)\simeq -16\alpha^2$.