EFT approach to quarkonium at finite temperature

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- Motivation : quarkonium as signal for deconfinement
- EFT approach in the small distance region
- EFT approach in the intermediate and large distance region
- Possible phenomenological implications
- Summary

Strong Interactions: from Method to Structures, Bad Honnef, February 12-16, 2011

Quarkonium as signal for deconfinement and color screening

Matsui, Satz, PLB 178 (86) 416

Color screening

⇓

potential becomes short range

⇓

melting of quarkonium bound states

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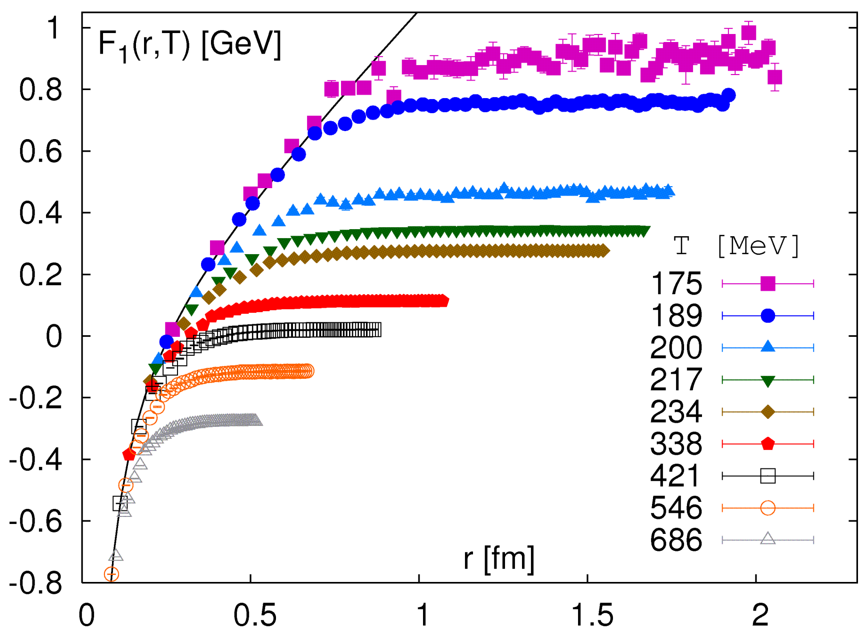
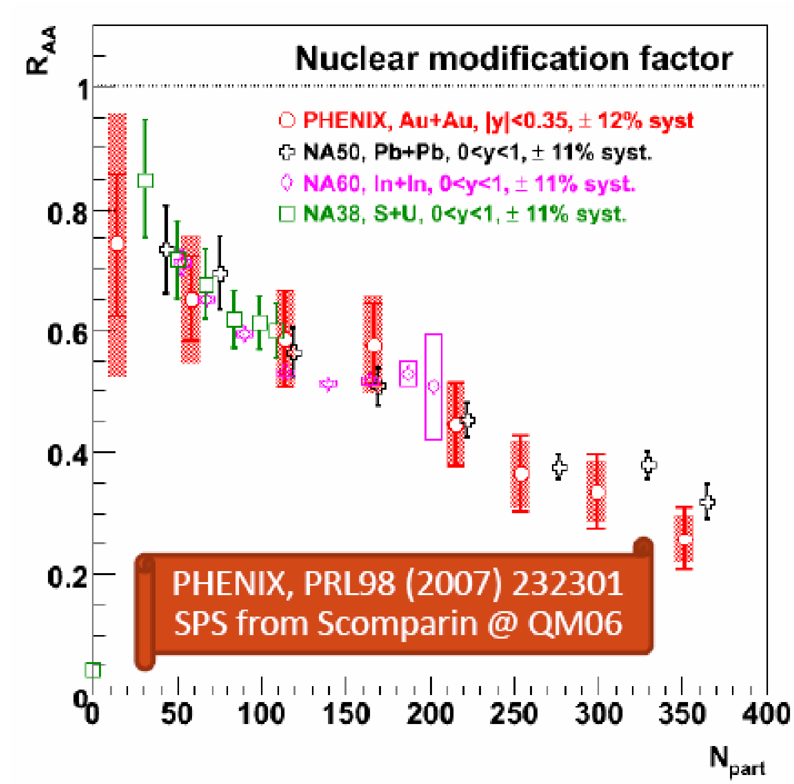
Suppression of quarkonium yield in heavy ion collisions relative to scaled pp

R. Granier de Cassagnac,
Joint CATHIE-INT mini program
Quarkonium in Hot Media, June 2009





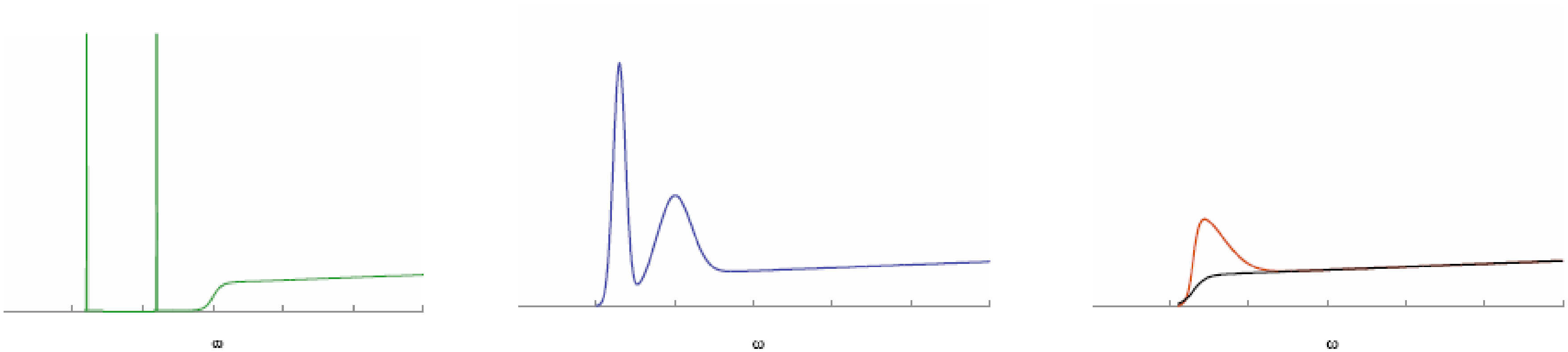
How to define the potential at $T>0$?

How to study the melting of the bound states from 1st principles ?

Quarkonium spectral functions

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

$$\sigma(\omega,p,T)=\frac{1}{2\pi}\,\mathrm{Im}\int_{-\infty}^{\infty}dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(x,0)]\rangle_T$$



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice => need to reconstruct the spectral functions

$$G(\tau,p,T)=\int d^3x e^{ipx}\langle J(x,-i\tau),J(x,0)\rangle_T$$

$$G(\tau,p,T)=\int_0^{\infty}d\omega\sigma(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

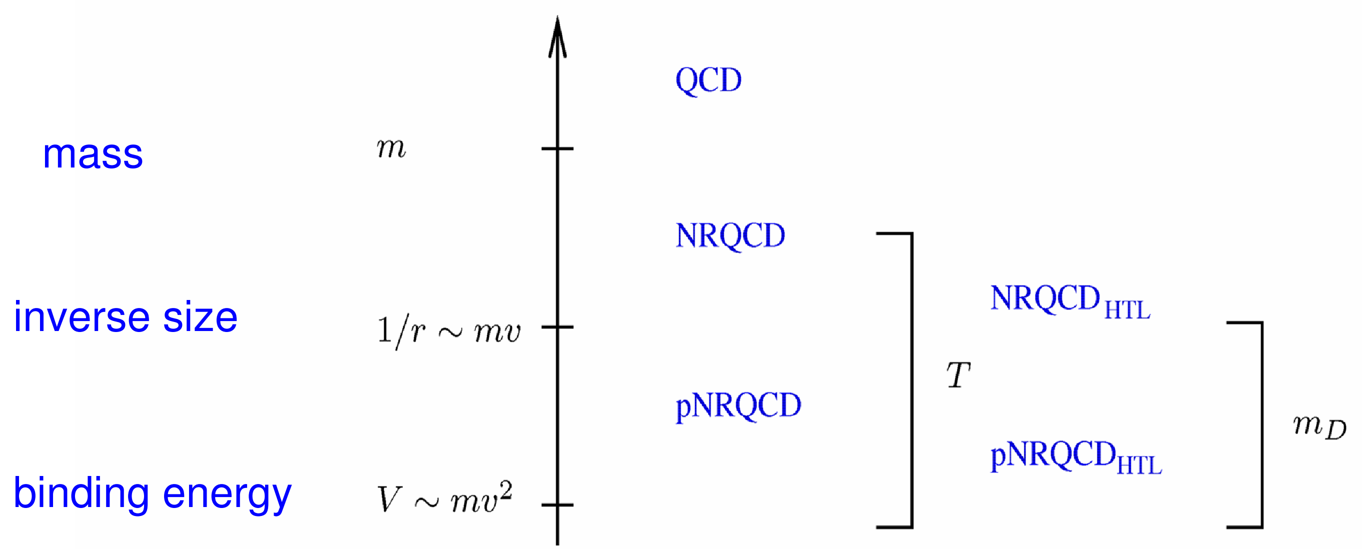
MEM $\Rightarrow$ *σ(ω,p,T)*

1S charmonium survives to $1.6T_c$??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Effective field theory approach for heavy quark bound states and potential models

The heavy quark mass provides a hierarchy of different energy scales



The scale separation allows to construct sequence of effective field theories:
NRQCD, pNRQCD

Potential model appears as the tree level approximation of the EFT
and can be systematically improved

Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

pNRQCD at finite temperature for static quarks

EFT for energy scale : $E_{bind} \sim \Delta V = (V_o - V_s) \sim mv^2$

Ultrasoft quark and gluons

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i$$

Singlet $Q\bar{Q}$ field Octet $Q\bar{Q}$ field

$$+ \int d^3r \, \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - \frac{-\nabla^2}{m} - V_o(r,T) \right] \mathrm{O} \right\}$$

$$+ V_A \, \mathrm{Tr}\left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{S} + \mathrm{S}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \vec{r} \cdot g\vec{E} \, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \vec{r} \cdot g\vec{E} \right\} + \ldots$$

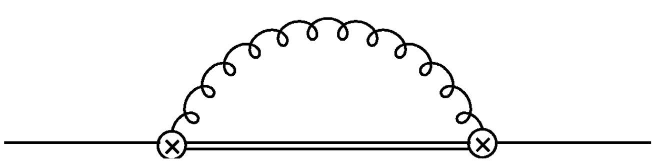
potential is the matching parameter of EFT !

Free field limit => Schrödinger equation

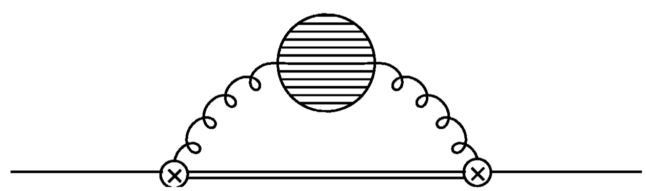
$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T) \right] S(r,t) = 0$$

$E_{bind} \sim \Delta V \sim \alpha_s / r \ll T, \; m_D$ there are thermal contribution to the potentials

Singlet-octet transition :



Landau damping :



Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (08) 014017

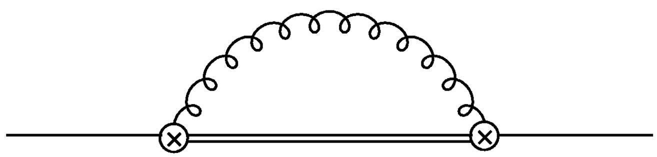
Thermal pNRQCD in the small distance regime

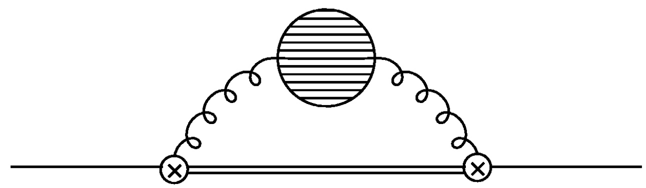
$$r \ll 1/T \ll 1/m_D$$

The heavy quarks do not feel the medium and the quark anti-quark pair interacts with the medium as a dipole

$$NRQCD \;\; {}^{1/r} \longrightarrow pNRQCD \;\; {}^{T,\;\; m_D} \longrightarrow pNRQCD_{therm}$$

Contribution from scale *T:*





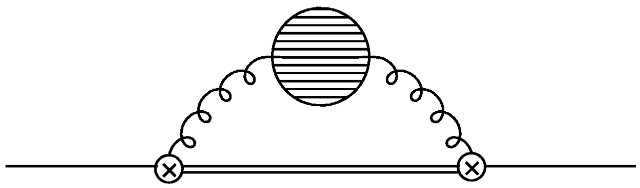
$$\delta V_s(r,T) = \frac{\pi}{9} N_c C_F \alpha_s^2 r T^2 - \frac{i}{6} N_c^2 C_F \alpha_s^3 T$$

$$-\frac{3}{2}\zeta(3) C_F \frac{\alpha_s}{\pi} r^2 T m_D^2 + \frac{2}{3}\zeta(3) N_c C_F \alpha_s^2 r^2 T^3$$

$$+i\left[\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(\frac{1}{\epsilon} - \ln\frac{T^2}{\mu_1^2}\right) + \frac{4\pi}{9}\ln 2 N_c C_F \alpha_s^2 r^2 T^3\right]$$

The *1/ε* pole is of IR origin and will cancel against UV poles from lower scales

Contribution from scale m_D :



$$\delta V_s(r,T) = -\frac{C_F}{6}\alpha_s r^2 m_D^3 + i\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(-\frac{1}{\epsilon} - \ln\frac{\mu_2^2}{m_D^2}\right)$$

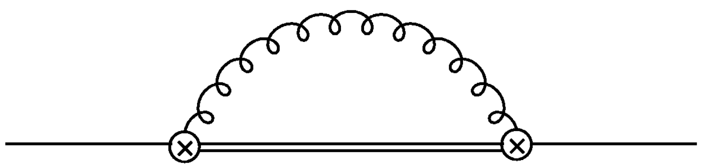
The $1/\varepsilon$ pole is of UV origin and will cancel against IR poles from scale T giving a finite imaginary part that contains a term :

$$-i\frac{C_F}{6}\alpha_s r^2 T m_D^2 \left(\ln\frac{T^2}{m_D^2} + const.\right)$$

The logarithm ensures that the imaginary part is always negative in the weak coupling regime ($m_D << T$)

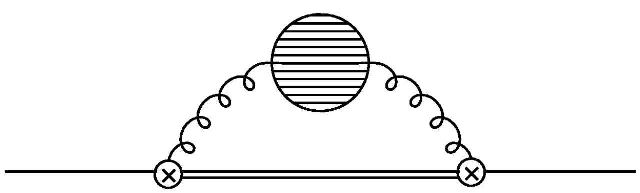
The potential for $r \ll 1/T \ll 1/m_D$:

	$\mathrm{Re}V_s(r,T)$	$\mathrm{Im}V_s(r,T)$
	$-C_F\frac{\alpha_s}{r}$	0



$\Delta V = V_o - V_s$

T:	$g^2T^3r^2 \times \frac{\Delta V}{T} \sim \alpha_s^2 T^2 r$	$g^2T^3r^2 \times \left(\frac{\Delta V}{T}\right)^2 \sim \alpha_s^3 T$



T:	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$
m_D:	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^3$	$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

Thermal pNRQCD in the large distance regime

$$1/T \ll r$$

Heavy quarks interact with the medium which generates thermal mass and thermal width

$$NRQCD \;\; {}^{T} \longrightarrow NRQCD_{HTL} \;\; {}^{1/r, m_D} \longrightarrow pNQRCD_{HTL}$$

$$\mathcal{L} \to -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i \, i\not{D} q_i + \delta\mathcal{L}_{HTL}$$

no modification of the heavy quark sector at LO

1) *1/r >> m_D* => scales *1/r* and m_D are integrated out subsequently

1/ε poles of IR and UV origin appear in ImV_s when scales *1/r* and m_D are integrated out, but these poles cancel in the sum as this happened in the short distance regime

2) *1/r ~ m_D* => scales *1/r* and m_D are integrated out simultaneously

Singlet part of the lagrangian becomes :

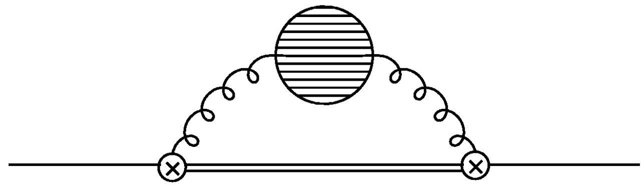
$$\int d^3 r \; \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 + V_s(r,T) + 2\delta m \right] \mathrm{S} \right.$$

$$\delta m = -\frac{C_F}{2} \alpha_s (m_D + iT)$$

thermal mass and width of the heavy quark

The potential for $r \ll 1/m_D$:

$\mathrm{Re}V_s(r,T)$



$\mathrm{Im}V_s(r,T)$

$1/r$:

$-C_F\frac{\alpha_s}{r} - C_F\alpha_s T^2 r\left(\frac{m_D}{T}\right)^2$

$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

m_D:

$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^3$

$g^2T^3r^2 \times \left(\frac{m_D}{T}\right)^2$

The potential for $r \sim 1/m_D$:

$$V_s(r,T) = -C_F\frac{\alpha_s}{r}\exp(-m_D r) \; + iC_F\alpha_s T\frac{2}{rm_D}\int_0^\infty dx \frac{\sin(rm_D x)}{(x^2+1)^2}$$

Laine, Philipsen, Romatschke, Tassler, JHEP 073 (2007) 054

$\mathrm{Re}(V_s(r,T) + 2\delta m)$ is identical to the LO singlet free energy $F_1(r,T)$

$$r \sim 1/m_D \leftrightarrow T \sim gm \Rightarrow \mathrm{Re}V_s \sim g^2/r \sim g^4 m \ll \mathrm{Im}V_s \sim g^2 T \sim g^3 m$$

The imaginary part of the potential is larger than the real part => quarkonium melting is determined by Landau damping and not by screening as originally suggested by Matusi and Satz

The role of the imaginary part for bottomonium

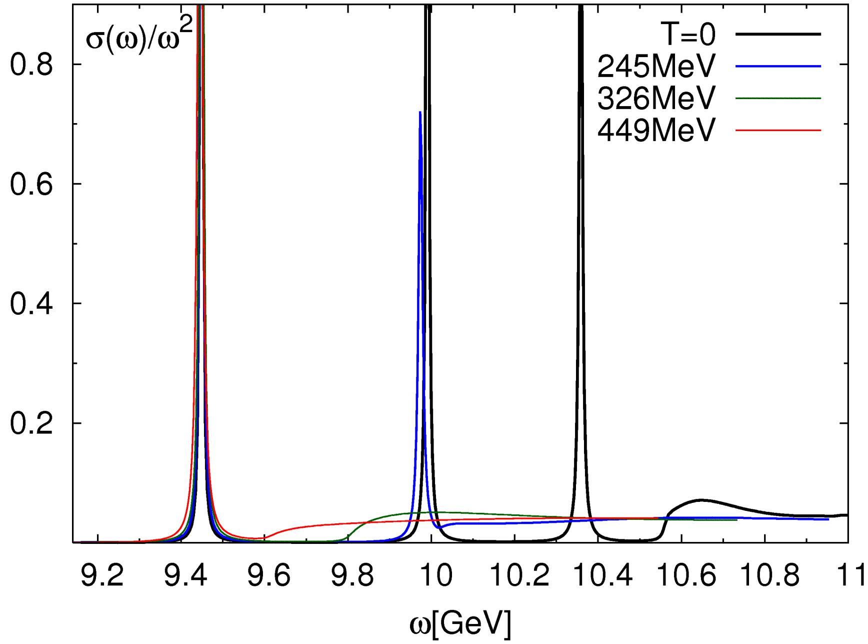
Take the singlet free energy from lQCD as the real part of the potential
Take the perturbative imaginary part the potential and the code from
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

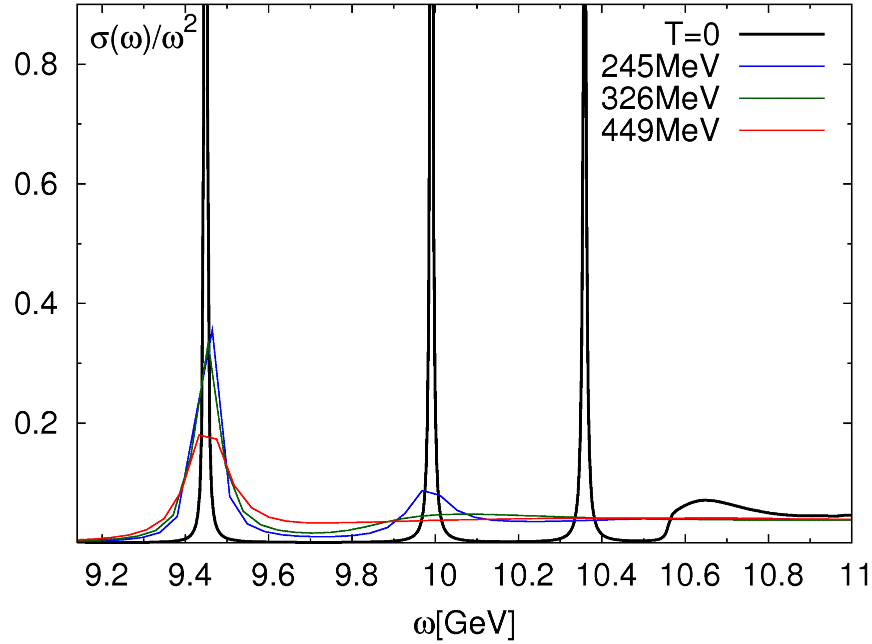
$$\left[-\frac{1}{m}\nabla^2 + V(\vec{r}) + E\right] G^{NR}(\vec{r},\vec{r}\,',E) = \delta^3(\vec{r}-\vec{r}\,')$$

$$\sigma(E) = \frac{2N_c}{\pi} \operatorname{Im} G^{NR}(\vec{r},\vec{r}\,',E)_{\vec{r}=\vec{r}\,'=0}$$

Im $V_s(r) = 0$:
2S state survives for $T > 245$ MeV
1S state could survive for $T > 450$ MeV

with imaginary part:
$2S$ state dissolves for $T > 240$ MeV
$1S$ states dissolves for $T > 450$ MeV





Miao, Mocsy, P.P. arXiv:1012.4433

no bottomonium state could survive for $T > 450$ MeV

for more systematic calculations see talk J. Ghiglieri on Wednesday

Summary

- EFT approach provides a tool to systematically study quarkonium properties at $T>0$ (can be applied for bottomonium, see talk by Jacopo Ghiglieri)

- The potentials can be rigorously defined at finite temperatures as parameters of the EFT lagrangian

- The potentials at $T>0$ have both real and imaginary part and different from the free energy of static quarks calculated in lattice QCD

- The potential receives power law corrections in the small distance, $r<1/T$ and intermediate distance regimes $r<1/m_D$

- The real part of the potential potential is exponentially screened in the large distance regime $r>1/m_D$, and $ReV << ImV$ but even for intermediate distances the imaginary part could be very important for quarkonium spectral functions

- EFT approach could be used to calculate Euclidean time static quark correlators (see talk by Antonio Vairo) ;
Need to extend the calculations to the octet sector of pNRQCD at $T>0$