How to identify hadronic molecules

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Key references:

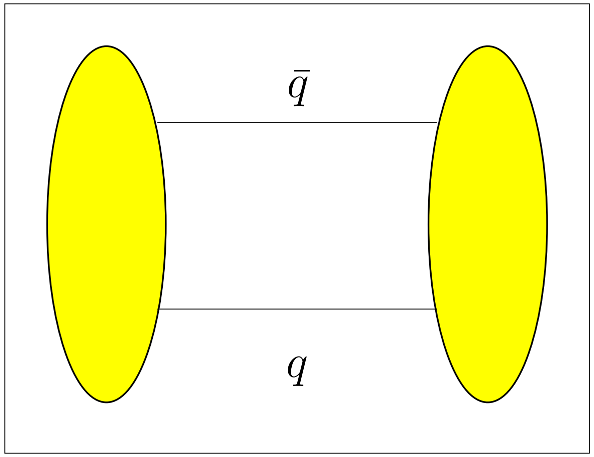
C. H., Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. D **75** (2007) 074015.

F.-K. Guo, C.H., S. Krewald, U.-G. Meißner, Phys. Lett. **B**666 (2008)251.

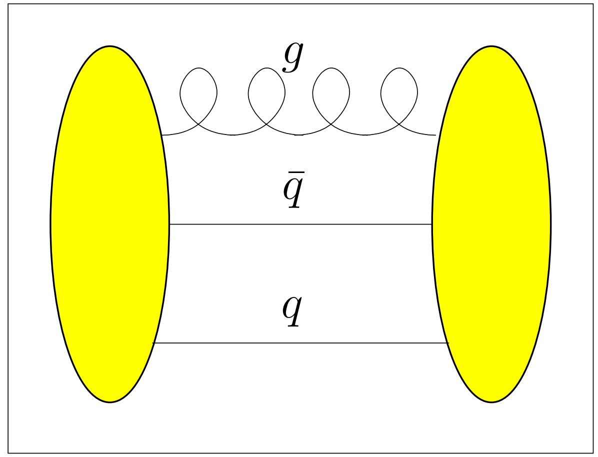
M. Cleven, F.-K. Guo, C.H., U.-G. Meißner, Eur. Phys. J **A** in print; arXiv:1009.3804 [hep-ph].

Exotics

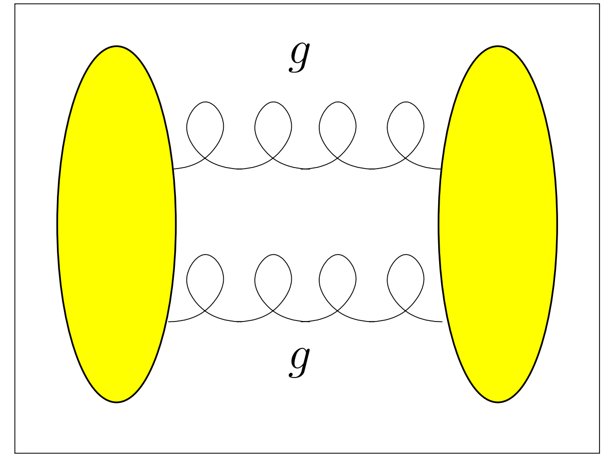
$\bar{q}q$



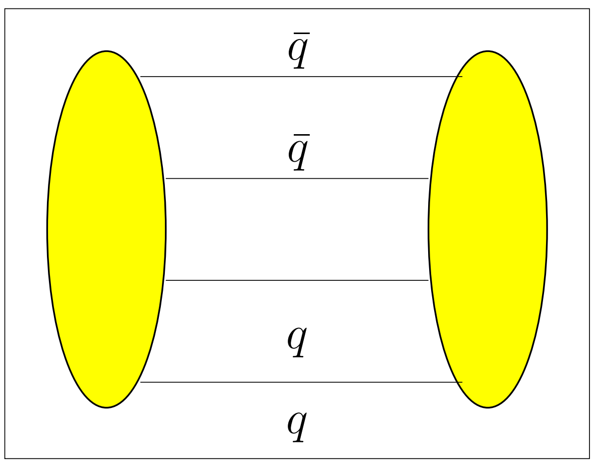
hybrid



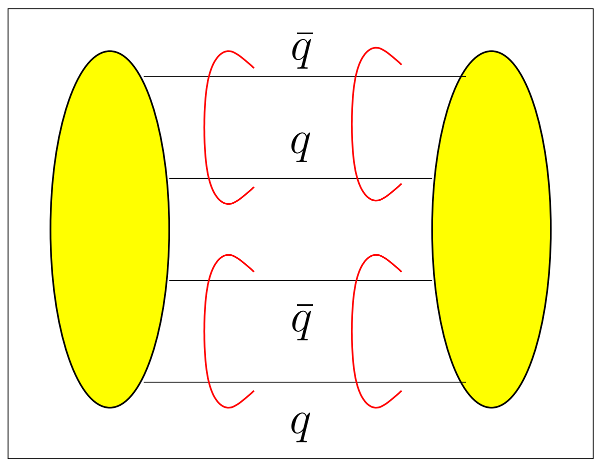
glueball



tetraquark



molecule



Only hadrons can go on-shell $\longrightarrow$ Unique analytic properties of molecular amplitude

Definition using non–rel. QM

Landau (1960), Weinberg (1963), Baru et al. (2004)

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix},$$

here $|\psi_0\rangle$ = elementary state and $|h_1 h_2\rangle$ = two–hadron cont., then λ^2 equals probability to find the bare state in the physical state

$\rightarrow$ λ^2 is the quantity of interest!

The Schrödinger equation reads

$$\hat{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle, \;\; \hat{\mathcal{H}} = \begin{pmatrix} \hat{H}_c & \hat{V} \\ \hat{V} & \hat{H}^0_{hh} \end{pmatrix} \longrightarrow \;\; \chi(p) = \lambda \, \frac{f(p^2)}{E - p^2/(2\mu)}$$

introducing the transition form factor $\langle\psi_0|\hat{V}|hh\rangle = f(p^2)$,

Note: $\hat{H}^0_{hh}$ contains only meson kinetic terms!

Effective Coupling

Therefore

$$|\Psi\rangle = \lambda \begin{pmatrix} |\psi_0\rangle \\ -\frac{f(p^2)}{\epsilon + p^2/(2\mu)} |h_1 h_2\rangle \end{pmatrix} ,$$

For the normalization of the physical state we get

$$1 = \langle \Psi | \Psi \rangle = \lambda^2 \left(1 + \int \frac{f^2(p^2)}{(\epsilon + p^2/(2\mu))^2} d^3 p \right)$$

using

$$\int \frac{f^2(p^2) d^3 p}{(p^2/(2\mu) + \epsilon)^2} = \frac{4\pi^2 \mu^2 f(0)^2}{\sqrt{2\mu\epsilon}} + \mathcal{O}\left(\frac{\sqrt{\epsilon\mu}}{\beta} \right)$$

for s–waves with β= range of forces. Using $8\pi^2 \mu f(0)^2 = g$

$$1 = \lambda^2 \left(1 + \frac{\mu g/2}{\sqrt{2\mu\epsilon}} + \mathcal{O}\left(\frac{\sqrt{\epsilon\mu}}{\beta} \right) \right)$$

Thus...

using for residue $g_{\rm eff}^2/4\pi = \lambda^2 2(m_1+m_2)^2 g$

$$\frac{g_{\rm eff}^2}{4\pi} \;=\; 4(m_1+m_2)^2(1-\lambda^2)\sqrt{2\epsilon/\mu} \le 4(m_1+m_2)^2\sqrt{2\epsilon/\mu}$$

$(1-\lambda^2)$ = Quantifies molecular component in physical state

The structure information is hidden in the

effective coupling, extracted from experiment,

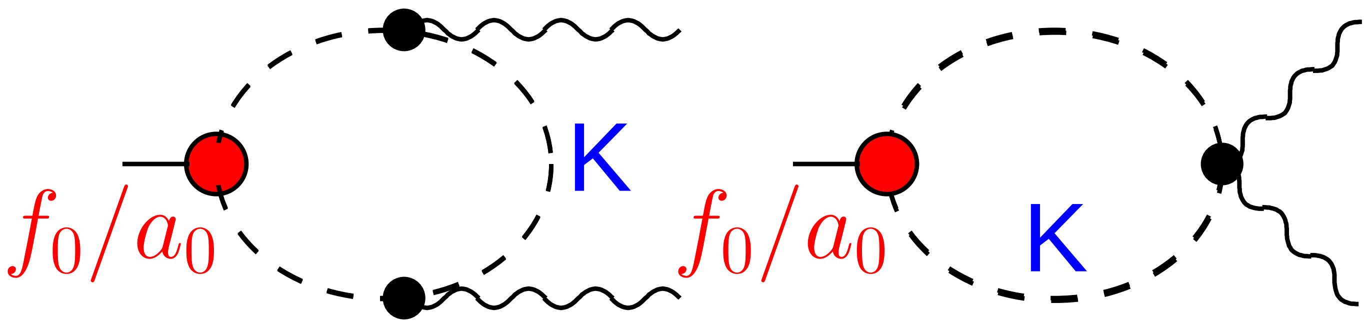
independent of the phenomenology

used to introduce the pole(s)

Formula also heavily used by Tuebingen group!

Example I: light scalar mesons

Two–photon decays of $f_0(980)/a_0(980)$ C.H. et al. (2007)



$$\Gamma_{\gamma\gamma} = g_{\rm eff}^2 \, \frac{\alpha^2}{256\pi^3}\frac{1}{x}\left(\frac{1}{x^2}\,\mathrm{arcsin}\,(x)^2 - 1\right)^2$$

where $x = 1 - \varepsilon/(2m)$. For the f_0/a_0: $\epsilon = 10$ MeV; $m = m_K$, s.t.

$\Gamma_{\gamma\gamma}^{\rm theo.} = (1{-}\lambda^2)(0.22 \pm 0.07)$ keV $+ \ldots$ vs. $\Gamma_{\gamma\gamma}^{{\rm exp.}\, f_0} = 0.22 \pm 0.02$ keV

Exp. value from R. Garcia-Martin, B. Moussallam (2010), analysing Belle data

Leading range corrections scale as $m\varepsilon/\beta^2 \simeq 1$ %

Example II: $D_s(2317)/D_s^*(2460)$

Experimental facts:

$\rightarrow$ Masses well below quark model predictions

$\rightarrow$ $M_D + M_K - M(D_s(2317)) = M_{D^*} + M_K - M(D_s^*(2460))$

$\longrightarrow$ $D_s(2317)/D_s^*(2460)$ as $KD^{(*)}$ bound states?

van Beeveren, Rupp; Oset, Gammermann; Lutz, Soyeur; ...

Here: Goldstone-Boson–$D^{(*)}$-Meson scattering

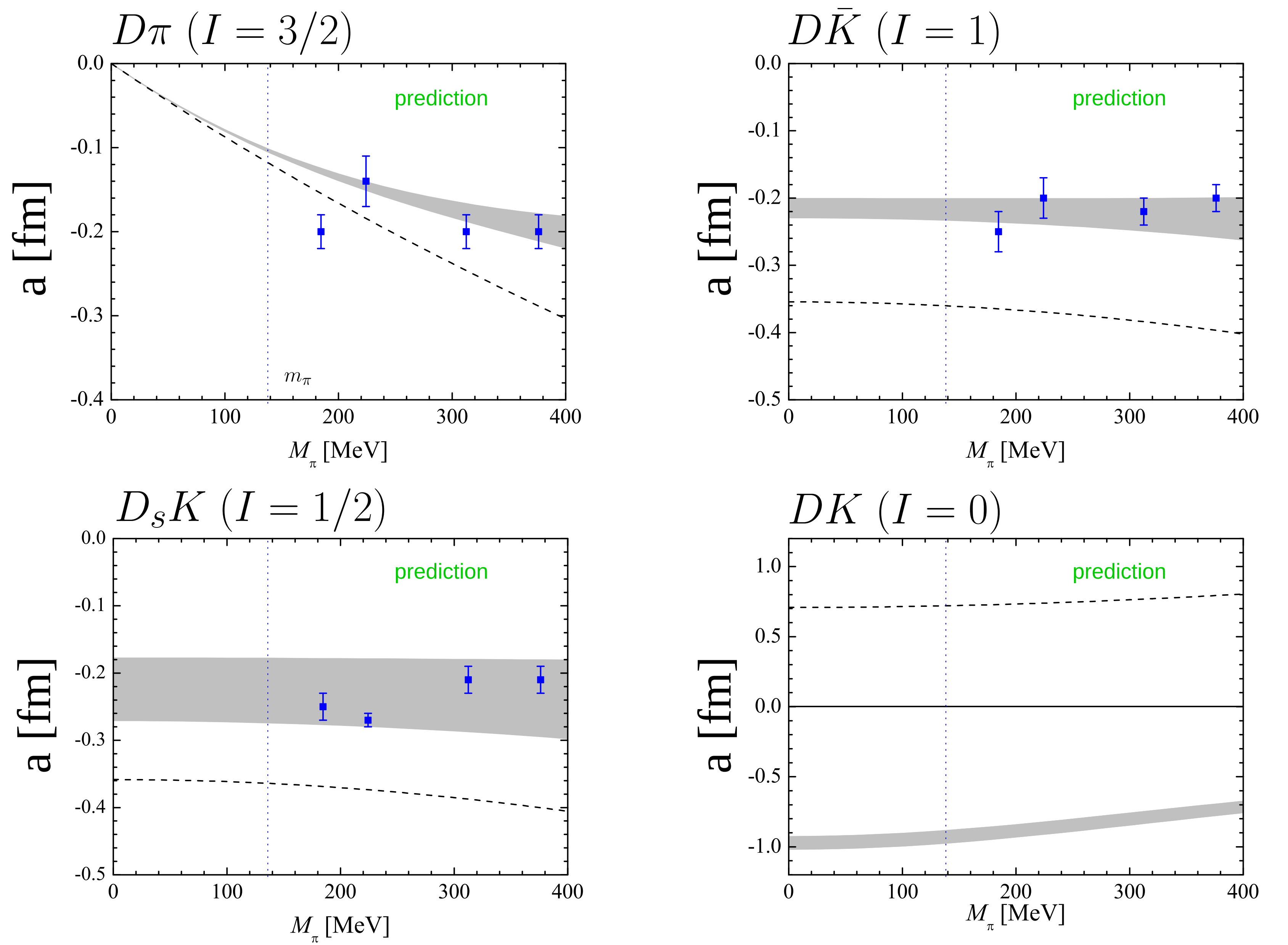
- ChPT $\rightarrow$ controlled quark mass dependence

  up to potential quark mass dep. of regulator
- with unitarization $\rightarrow$ dynamical generation of poles
- use LEC to fix pole position of $D_s(2317)$

From residues: $D_s(2317)$ $(D_s^*(2460))$ as $KD(KD^*)$ molecules

Prediction of other observables; Sensitivity to molecular nature?

Chiral extrapolation



Lattice: Liu, Lin, Orginos (2008); UChPT: Guo et al. (2009)

Chiral extrapolation

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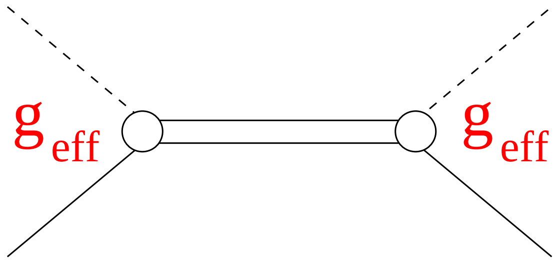
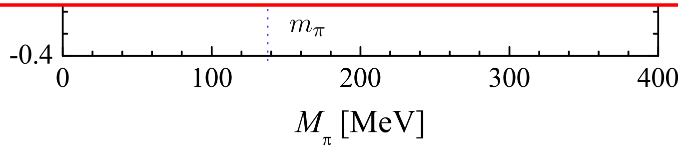
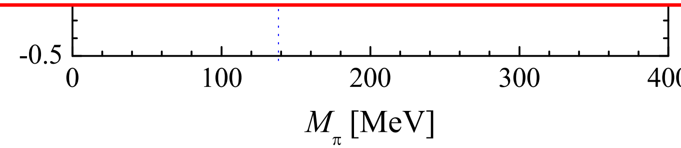
$D_S(2317)$: $a =$ $\mathbf{g}_{\rm eff}$

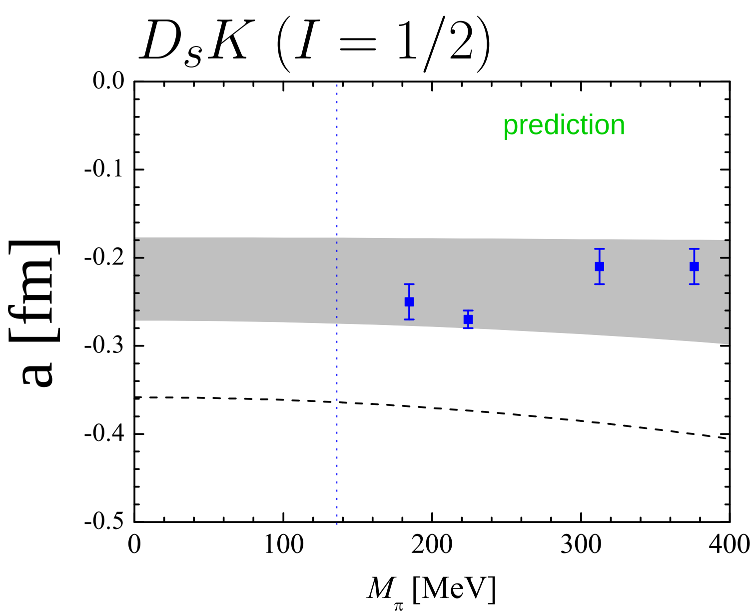


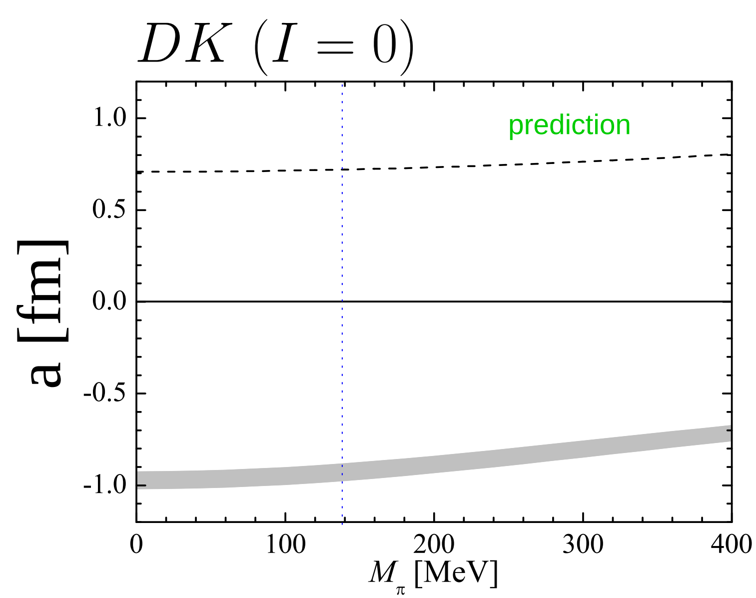
$\mathbf{g}_{\rm eff}$ $+\mathcal{O}(1/\beta) \simeq \left(\frac{2(1-\lambda^2)}{2-\lambda^2}\right)\frac{-1}{\sqrt{2m_K\epsilon}}$

$a = 1$ fm for molecule $(\lambda^2 = 0)$; smaller otherwise









Lattice: Liu, Lin, Orginos (2008); UChPT: Guo et al. (2009)

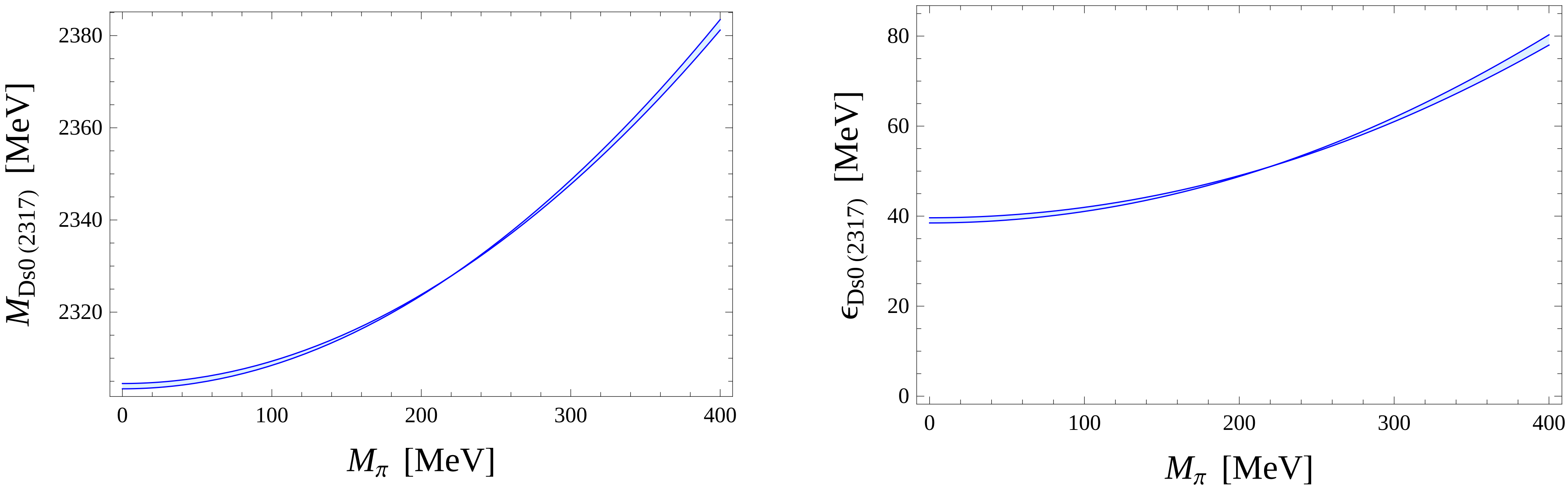
π mass dependence of poles

M. Cleven et al. (2010)

If $D_s^*(2317)$ as quark state: $c\bar{s}$

expect very weak light quark mass dependence

We find

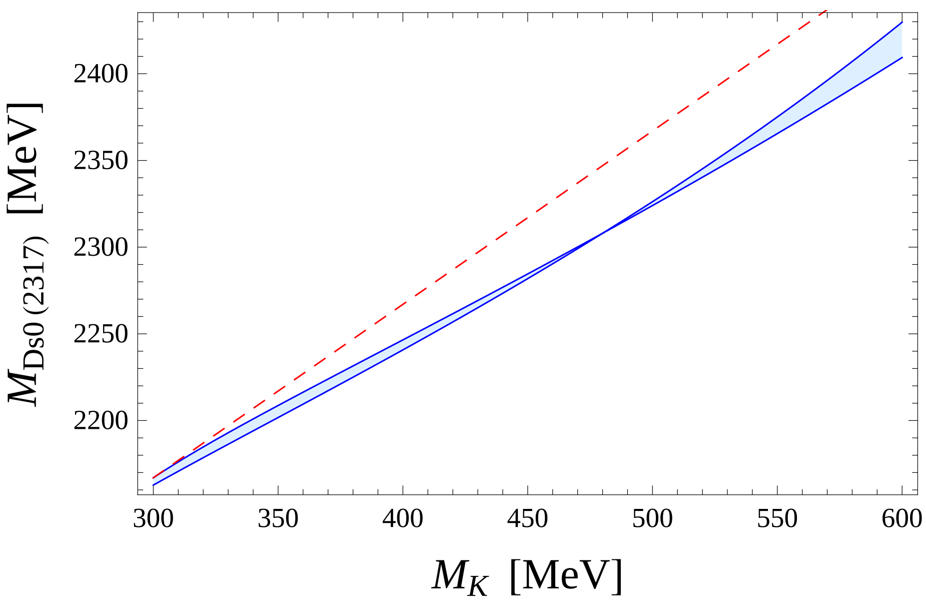


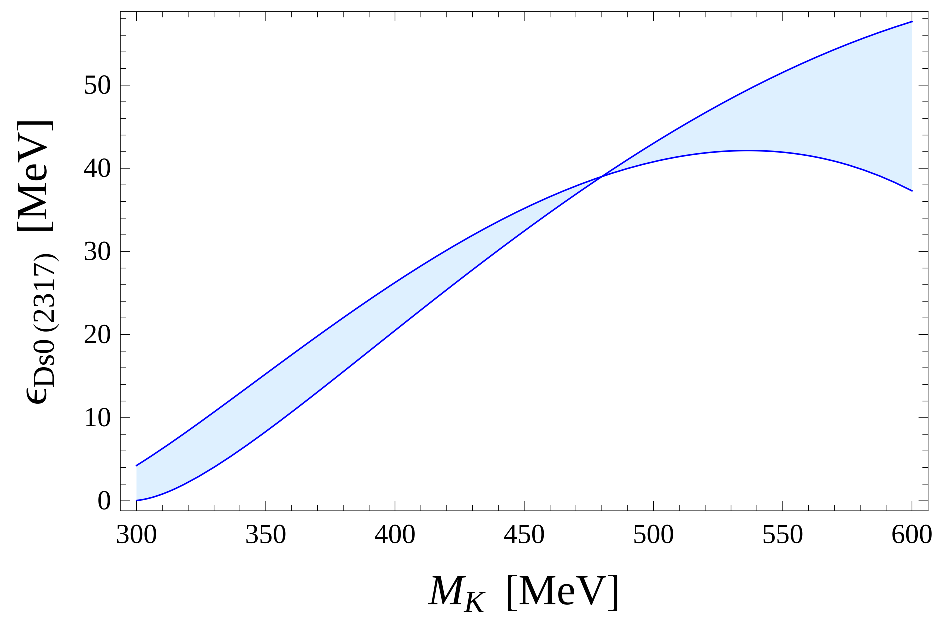
Molecule shows relatively strong light quark mass dependence

since prominent component KD contains light quarks!

K mass dependence of poles

M. Cleven et al. (2010)





→ very pronounced linear K–mass dependence

→ state follows threshold

→ for K–mass smaller than 300 MeV turns into resonance
note: interaction strength propto. M_K

$D_s(2317) \to \pi^0 D_s$

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Isospin breaking in QCD and EFT through quark mass and charge differences

The same effective operators lead to

$\rightarrow$ mass differences, e.g.

- $$m_{D^+} - m_{D^0} = \quad \Delta m^{\text{strong}} \quad + \quad \Delta m^{\text{e.m.}}$$
  $$= ((2.5 \pm 0.2) + (2.3 \pm 0.6)) \text{ MeV}$$
- $\pi^0 - \eta$ mixing $\longrightarrow$ parameters fixed

$\rightarrow$ Isospin breaking scattering amplitude

- e.g. $KD \to \pi^0 D_s$ predicted; with this

$$\Gamma(D_s(2317) \to D_s \pi^0) = (180 \pm 110)\text{keV}$$

Lutz, Soyeur (2007); complete to NLO+uncertainty estimate: Guo et al. (2008)

much smaller in quark model $\rightarrow$ direct measurement (PANDA)

Conclusion

Progress through interplay of

controlled theory,

experiments of high quality and

numerical simulations with super computers