Heavy quarkonium in a weaky-coupled plasma in an EFT approach

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in collaboration with
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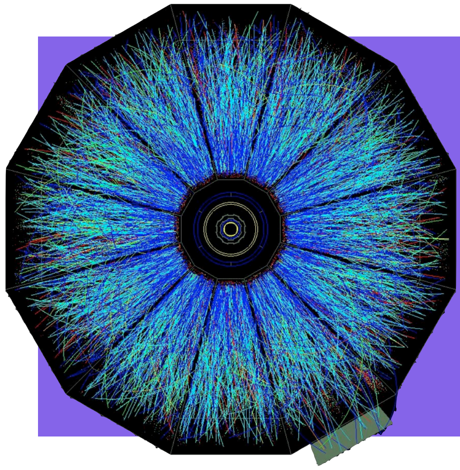
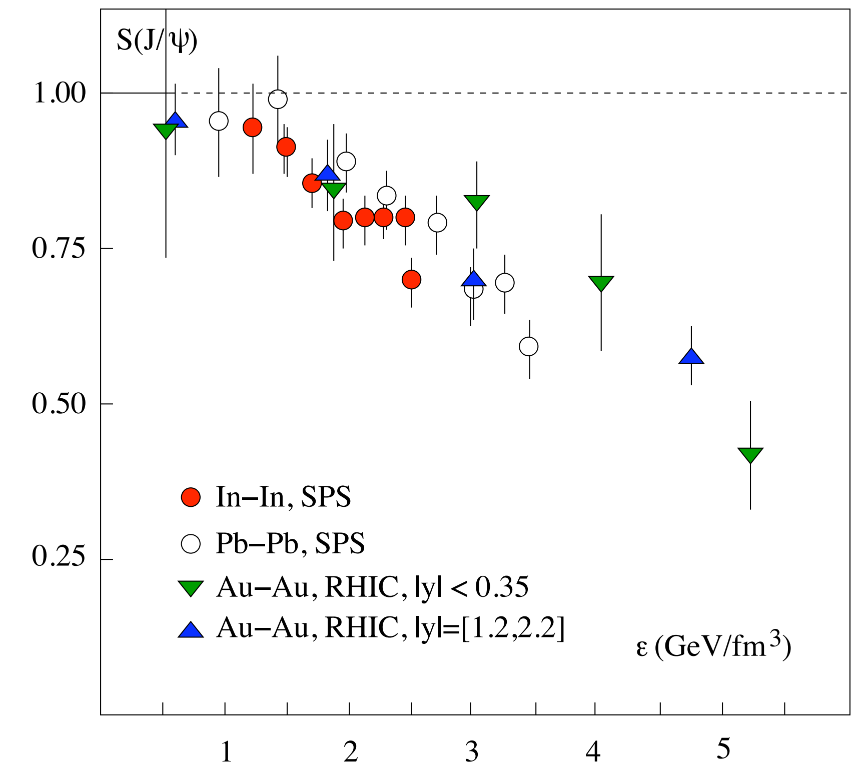
Outline

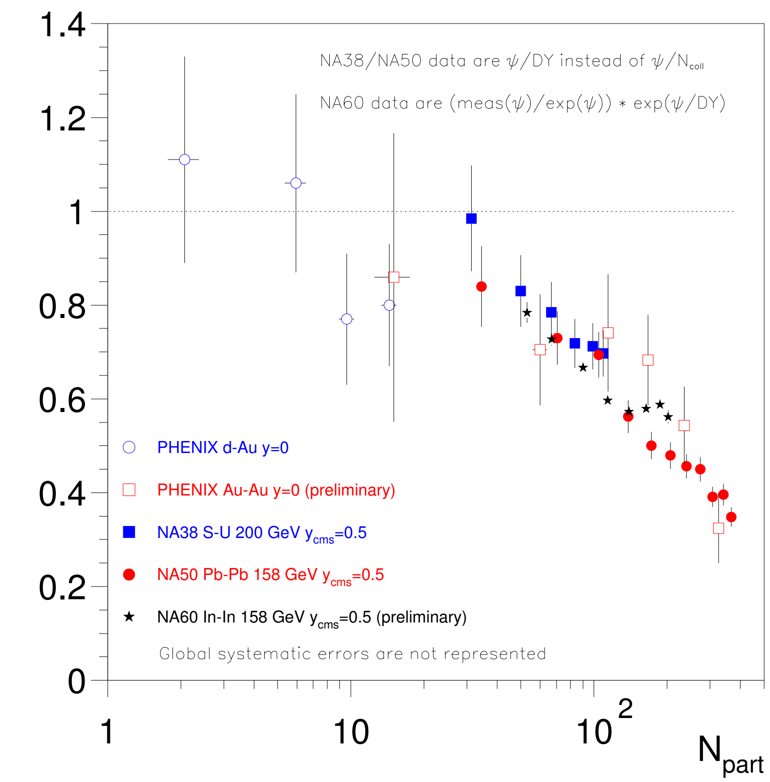
- Quarkonium in media: the EFT approach
- Building blocks: non-relativistic EFTs at zero temperature and the real-time formalism
- Integrating out in succession the relevant scales and calculating the spectrum and width of quarkonium
- Conclusions and outlook

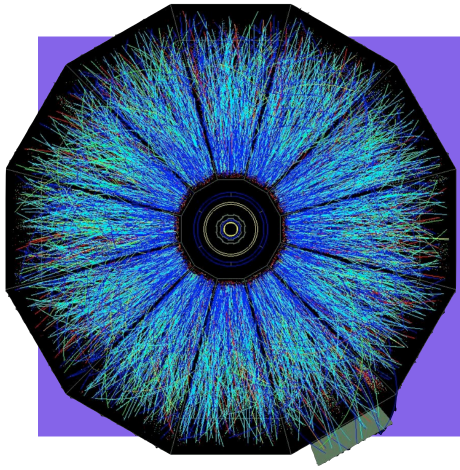


The structure: Quarkonium suppression

- Experimental data show a suppression pattern







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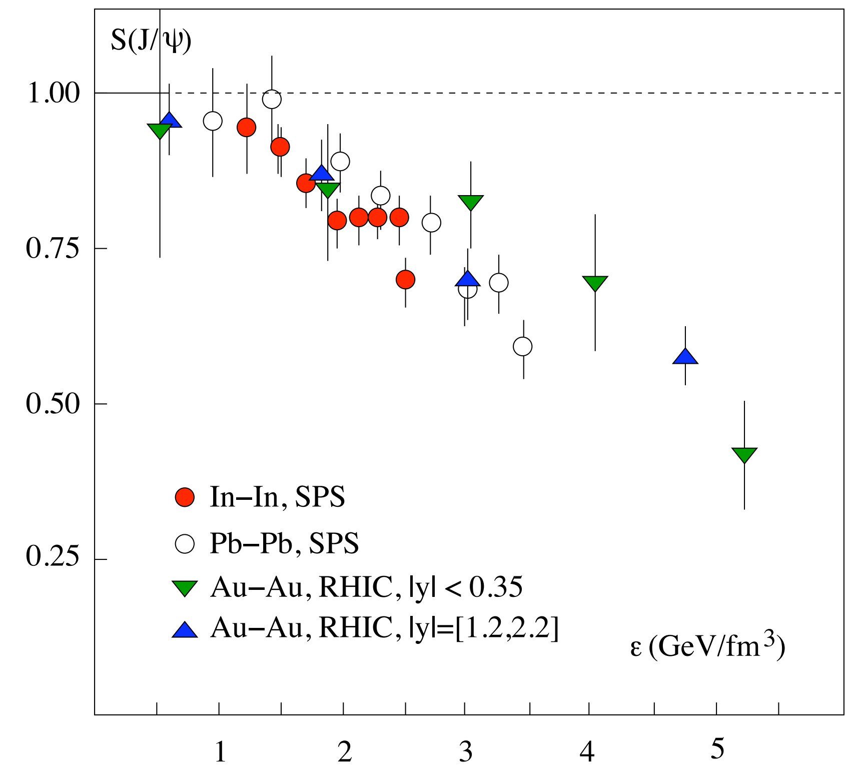

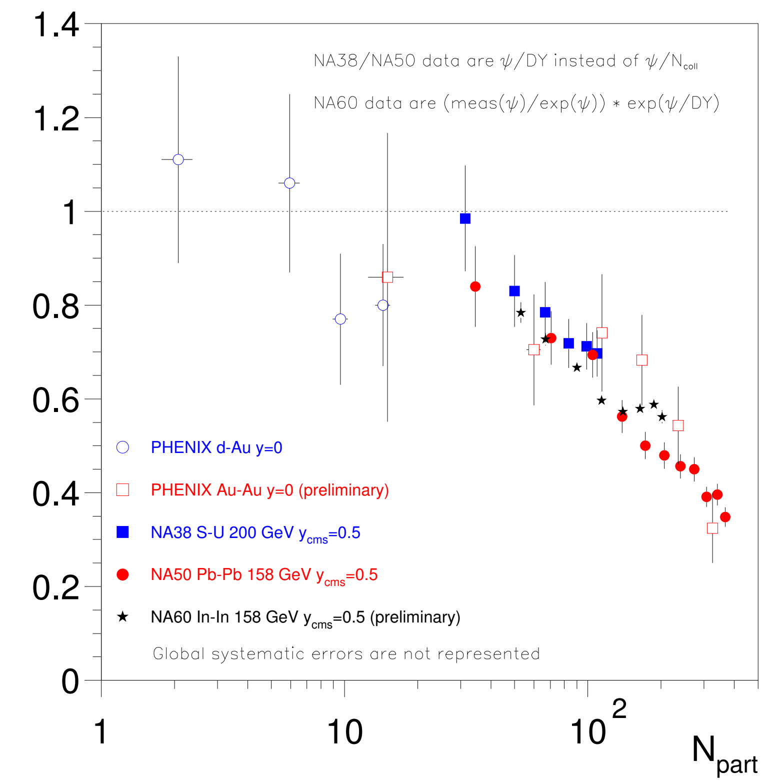

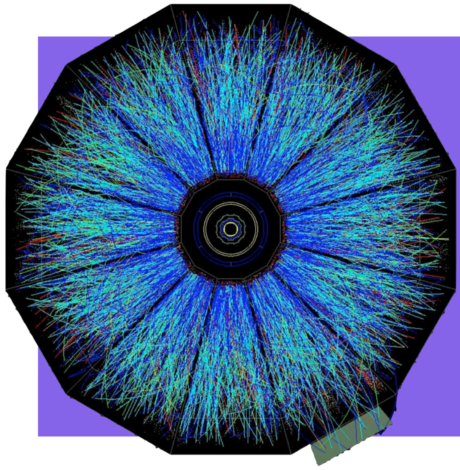

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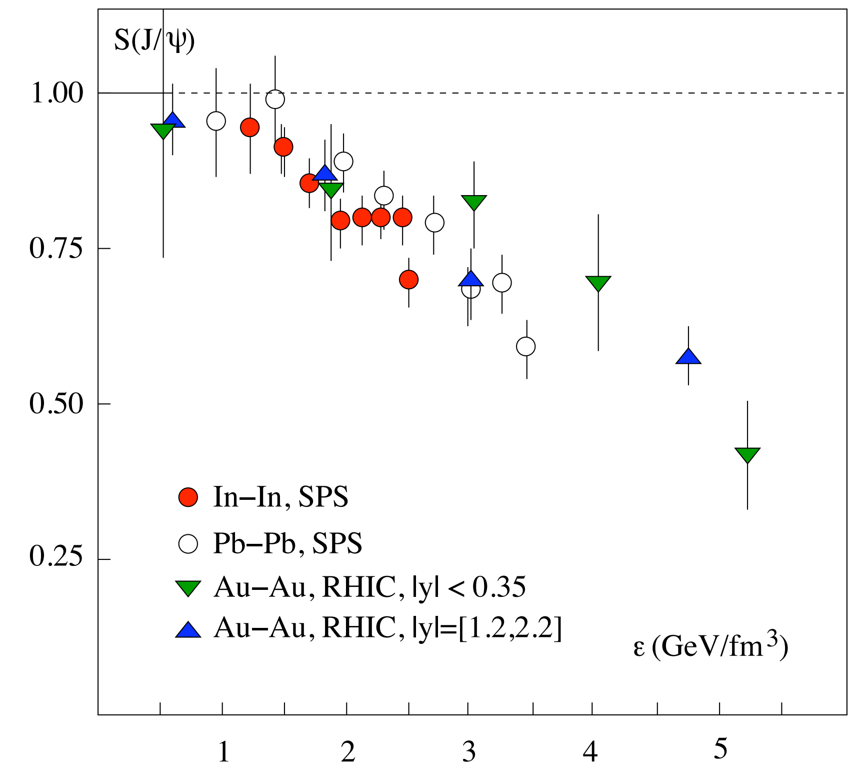

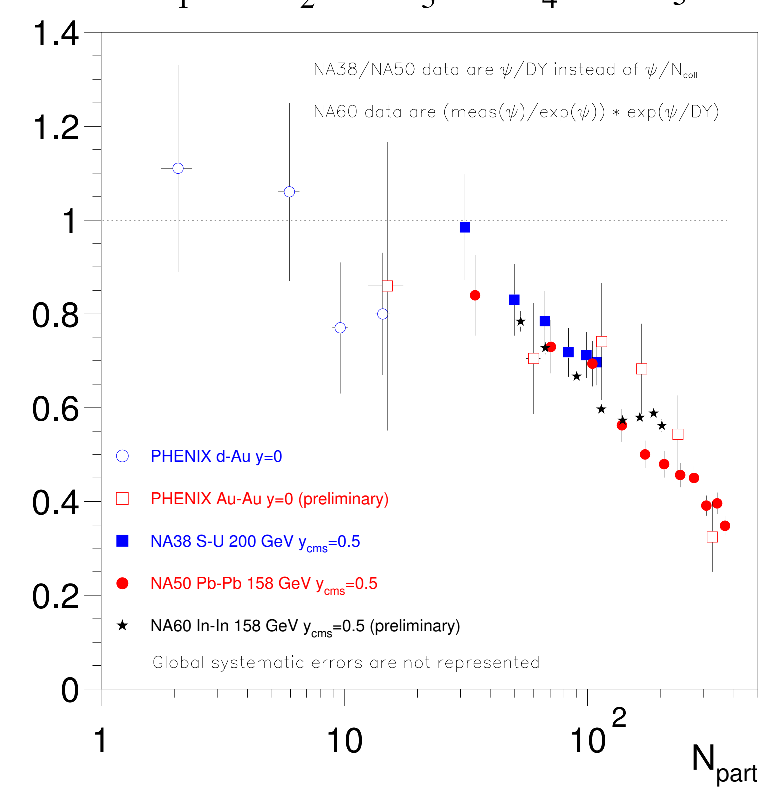

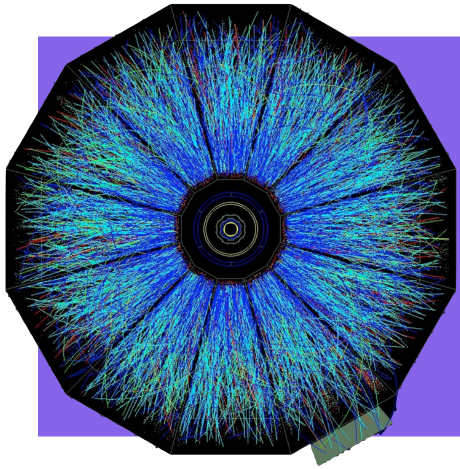

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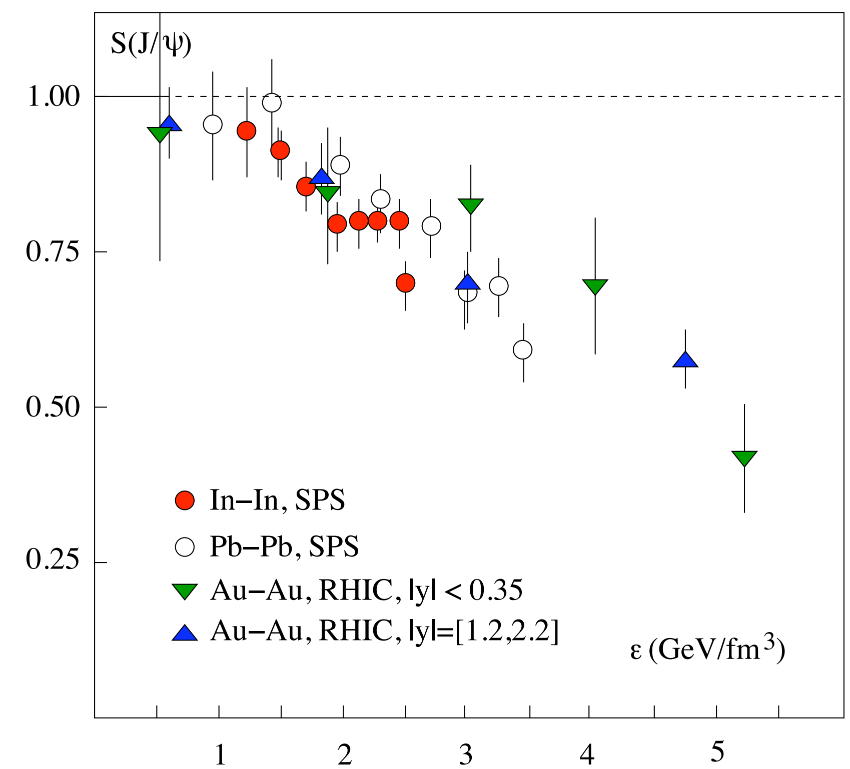

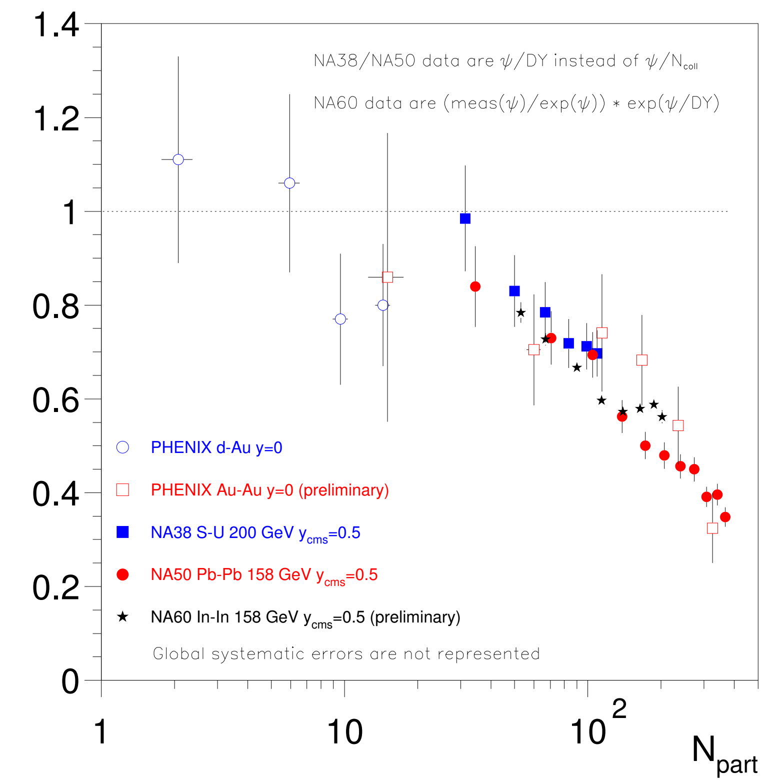

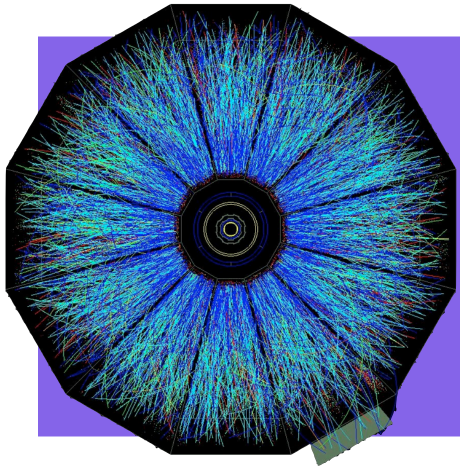

The structure: Quarkonium suppression

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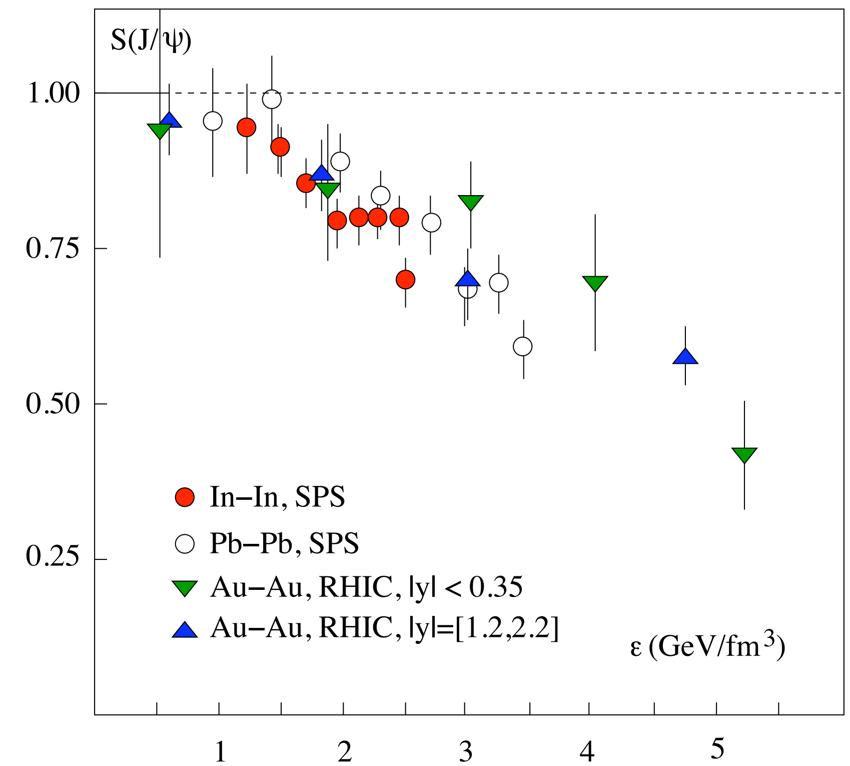

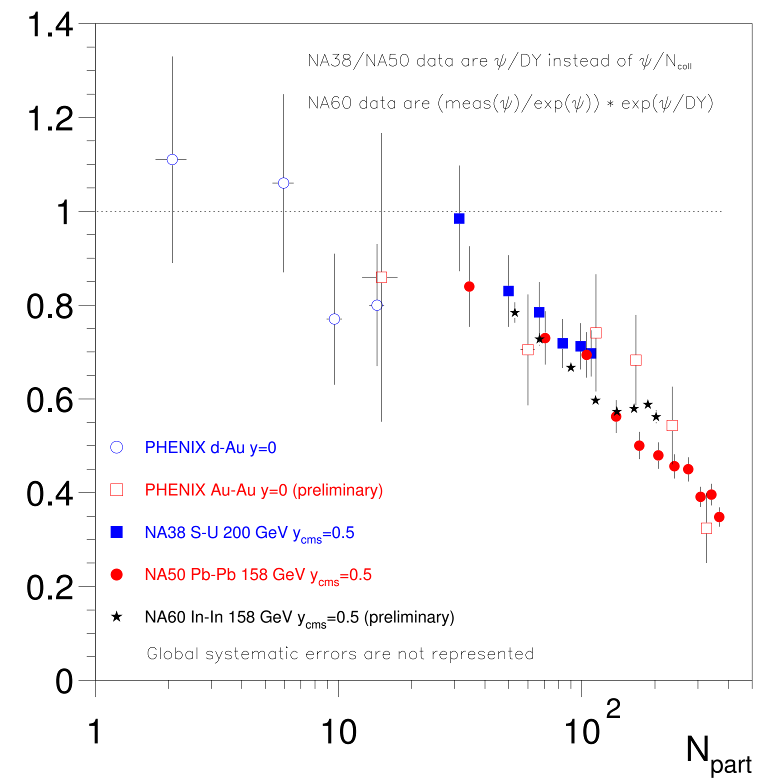

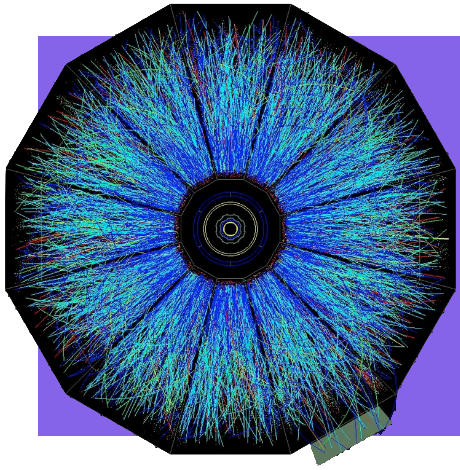

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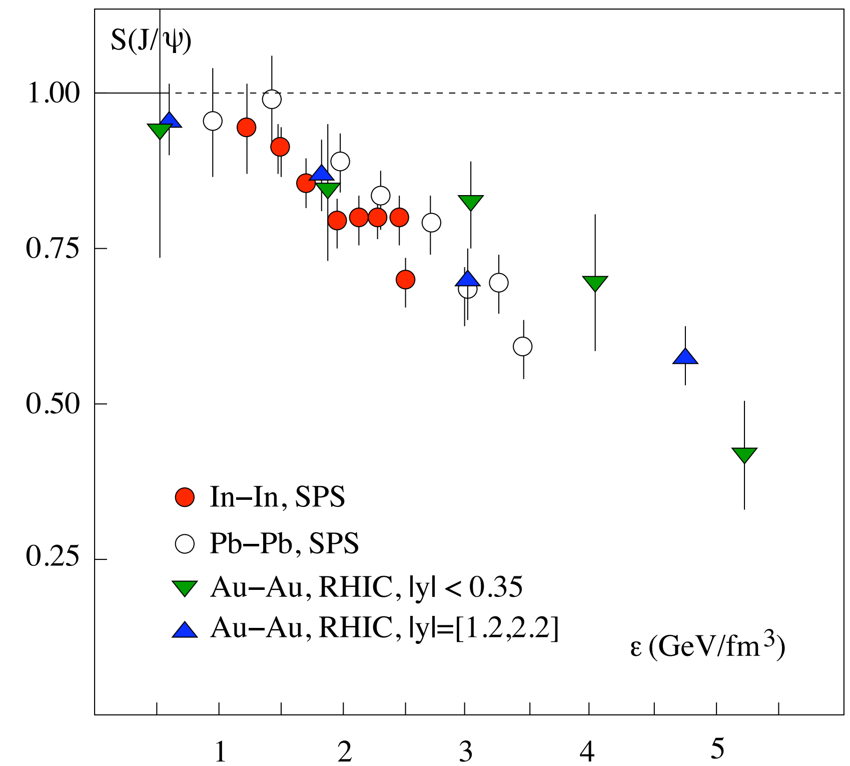

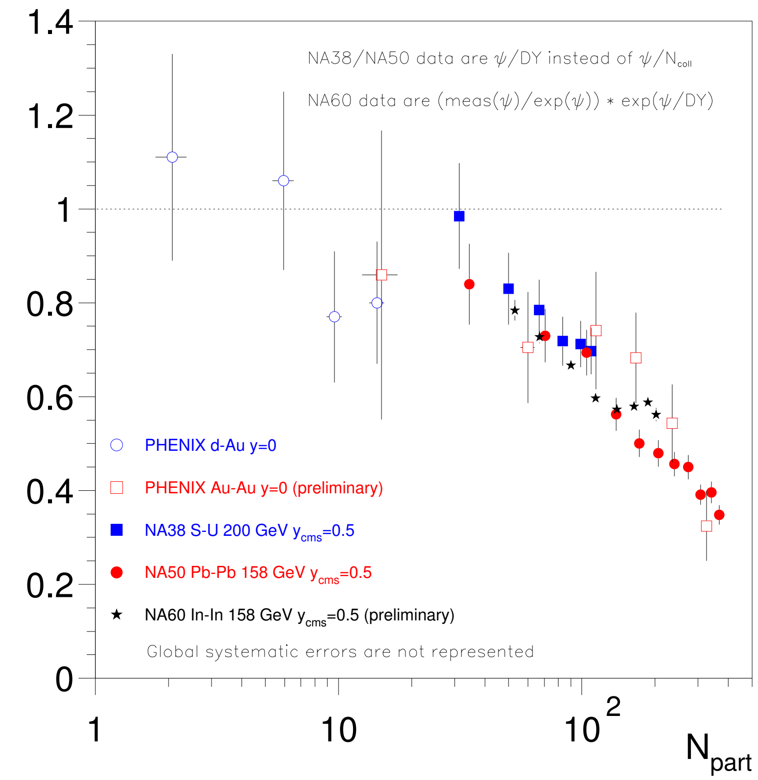

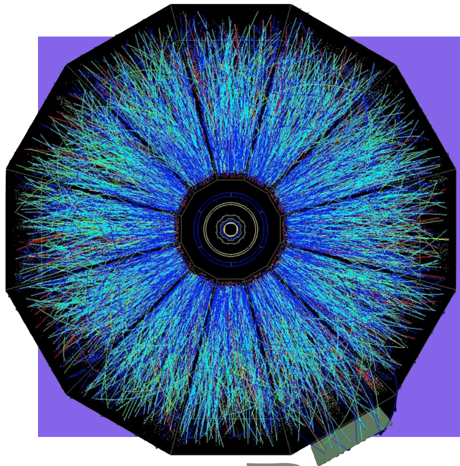

Quarkonium suppression

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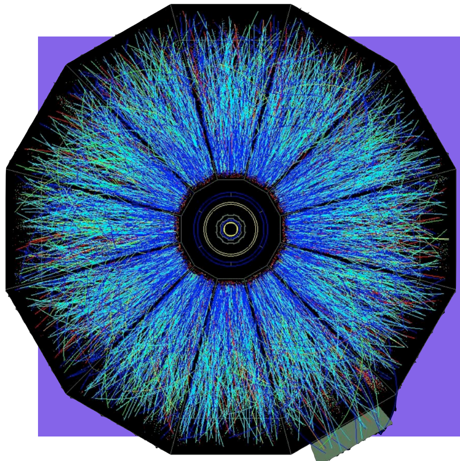
  Matsui Satz **PLB178** (1986)



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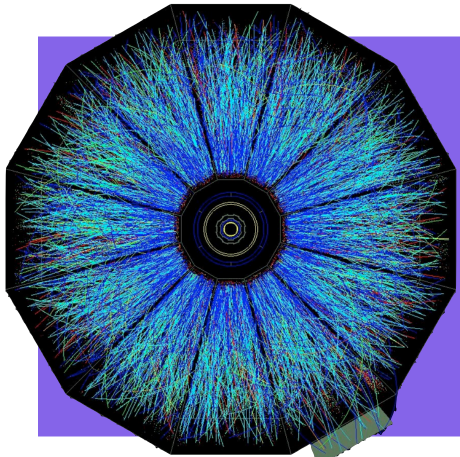

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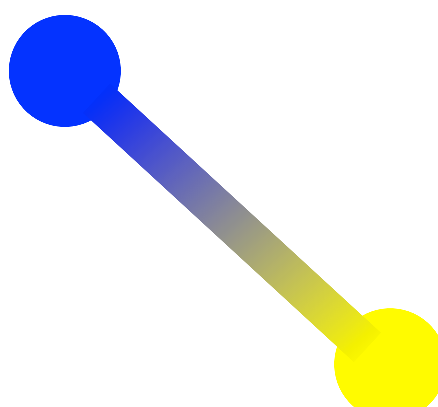

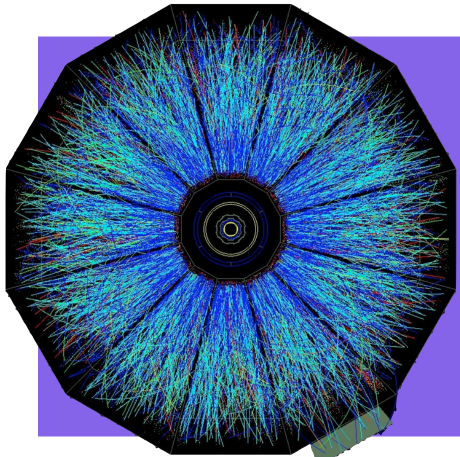

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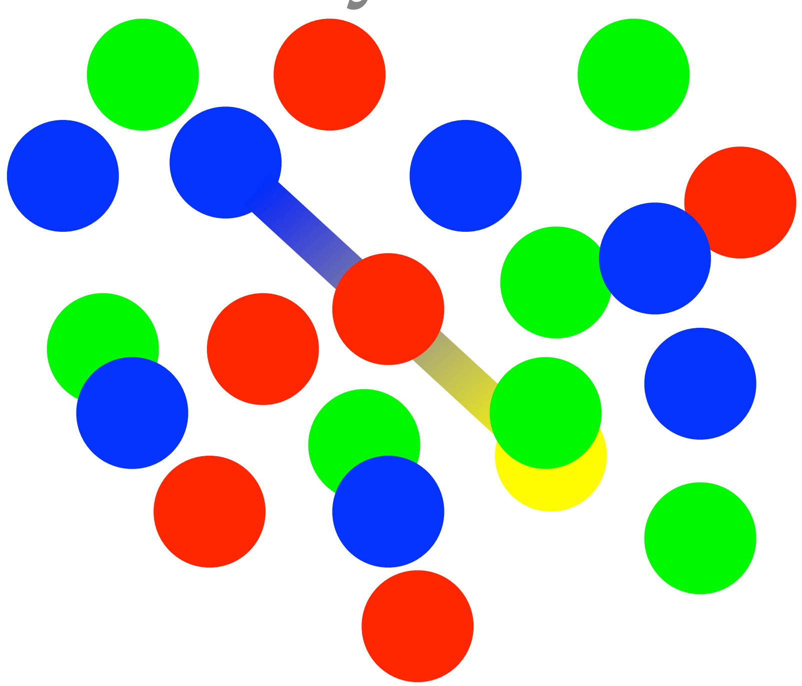

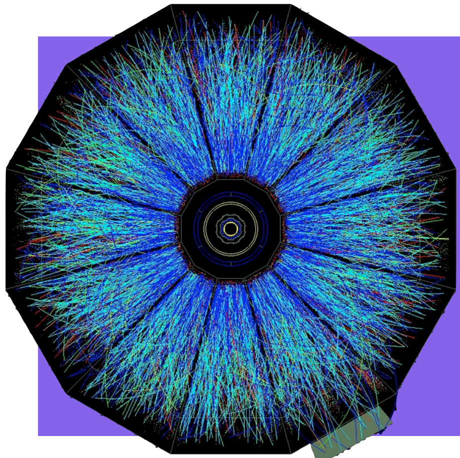

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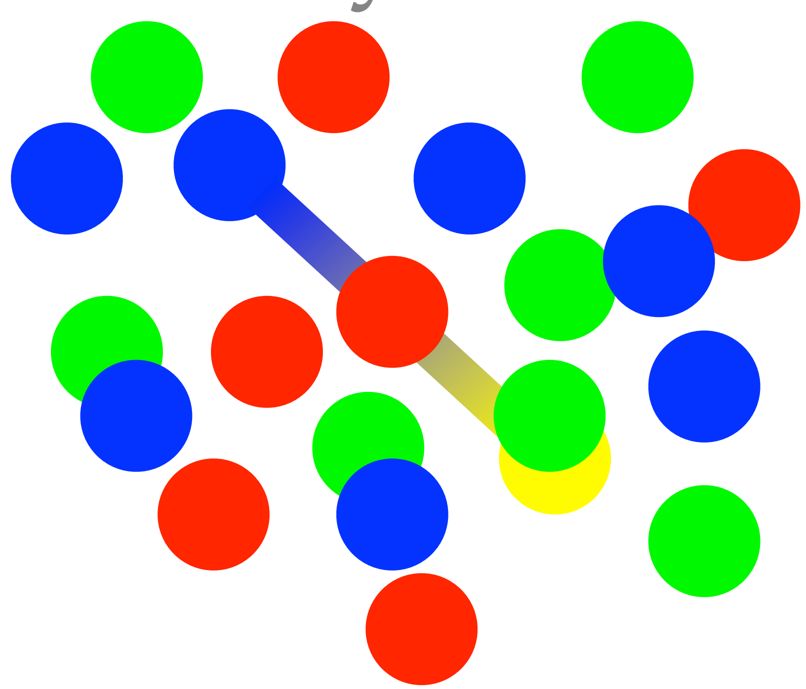
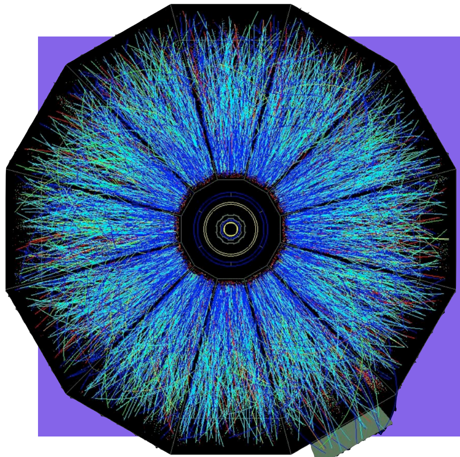


$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$



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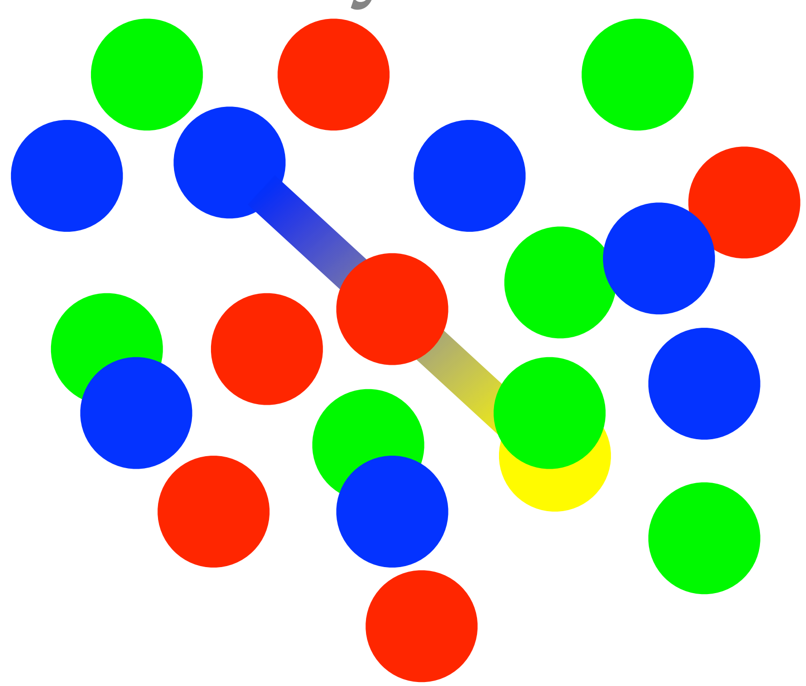

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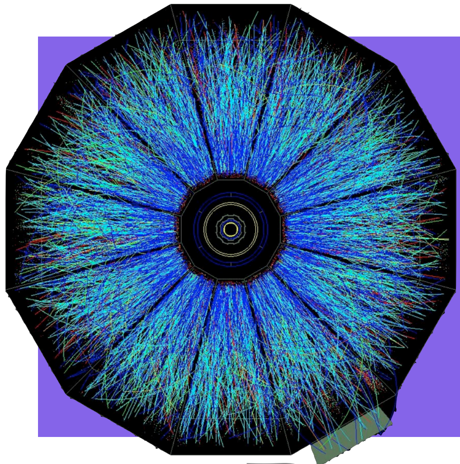
$$r \sim \frac{1}{m_D} \longrightarrow$$ Bound state dissolves



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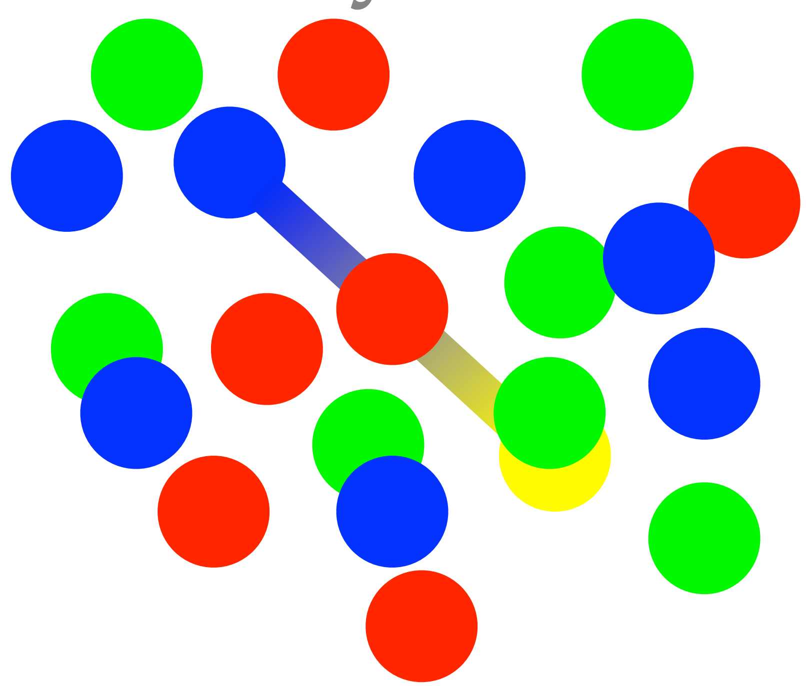

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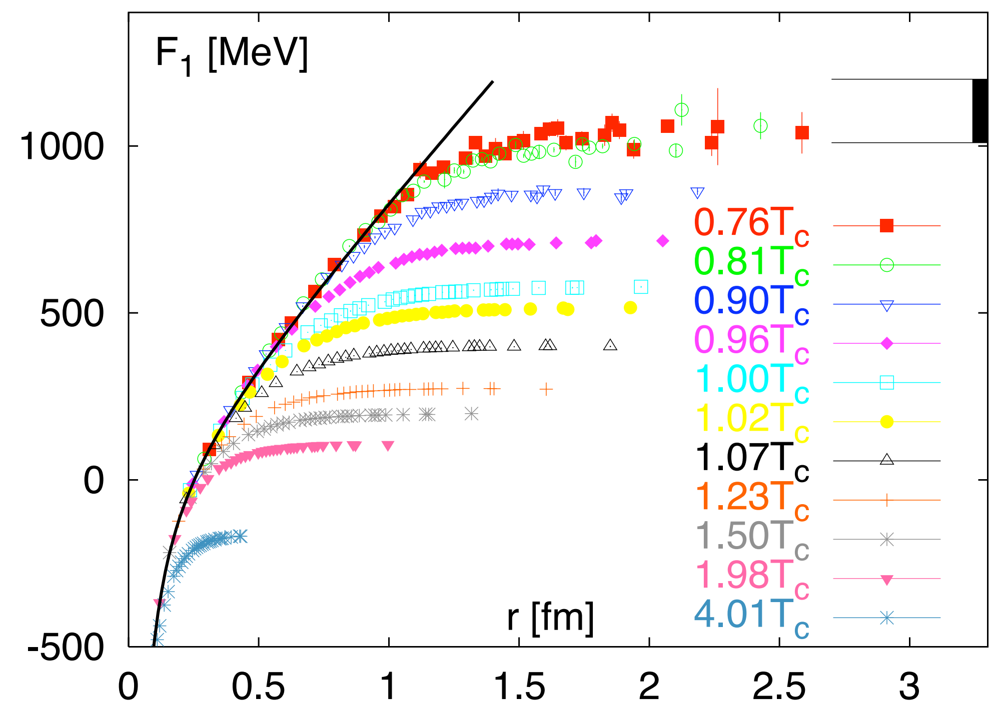
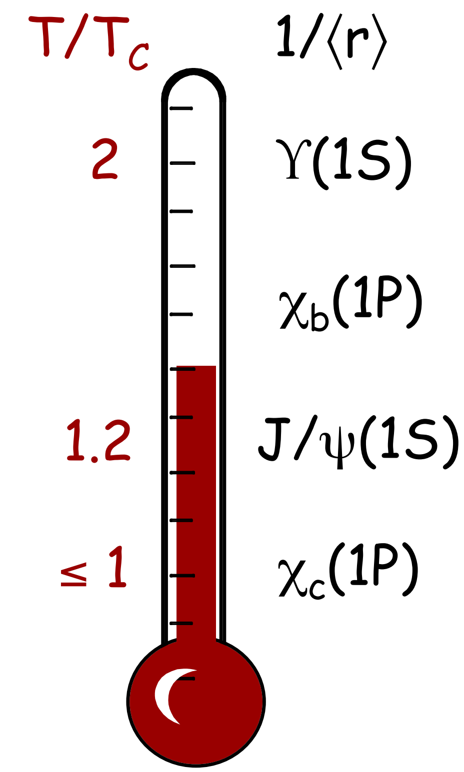
- Studied with potential models, lattice spectral functions, ...

Potential models

Digal, Petreczky, Satz 01
Wong 05-07
Mannarelli, Rapp 05
Mocsy, Petreczky 05-08
Alberico, Beraudo, Molinari, de Pace 05-08
Cabrera, Rapp 2007
Wong, Crater 07
Dumitru, Guo, Mocsy, Strickland 09
Rapp, Riek 10



Kaczmarek Zantow



- There is qualitative agreement on this picture of sequential melting
- However there are still issues with the definition of an in-medium potential

The method: an EFT approach

- Perturbative computation of the real-time potential between a static quark and antiquark for *T>>1/r*:

$$V_{\mathrm{HTL}}(r) = -\alpha_s C_F \left(\frac{e^{-m_D r}}{r} - i \frac{2T}{m_D r} f(m_D r) \right)$$

When $r \sim \frac{1}{m_D}$ $\mathrm{Im}V \gg \mathrm{Re}V$

Laine Philipsen Romatschke Tassler **JHEP0703** (2007)

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Escobedo Soto **PRA78** (2008), **PRA82** (2010)

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- EFT approach for *static* quark-antiquark pairs in finite *T* pNRQCD (Non-Abelian, static)

Brambilla JG Petreczky Vairo **PRD78** (2008)

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When $r \sim \frac{1}{m_D}$ $\mathrm{Im}V \gg \mathrm{Re}V$

Laine Philipsen Romatschke Tassler **JHEP0703** (2007)

- EFT approach for *heavy* quark-antiquark pairs in finite T

pNRQCD (Non-Abelian, non-static)

Brambilla Escobedo JG Soto Vairo **JHEP1009** (2010)

The EFT approach

- Extend the well-established zero temperature EFT framework for heavy quark-antiquark bound states (NRQCD, pNRQCD) to finite temperature
- Systematic approach with modern, rigorous definition of the potential (real-time matching coefficient)
- Transparent connection with quantum mechanics and the Schrödinger equation picture, which appears as the zeroth-order approximation in the EFT framework

Building blocks: NR EFTs and the real-time formalism

Effective Field Theories

- EFTs prove to be a valuable tool for physical problems characterized by various sufficiently separated energy/ momentum scales

- An EFT is constructed by integrating out modes of energy and momentum larger than the cut-off $\mu \ll \Lambda$

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(\mu/\Lambda) \frac{O_n}{\Lambda^{d_n - 4}}$$

Wilson coefficient

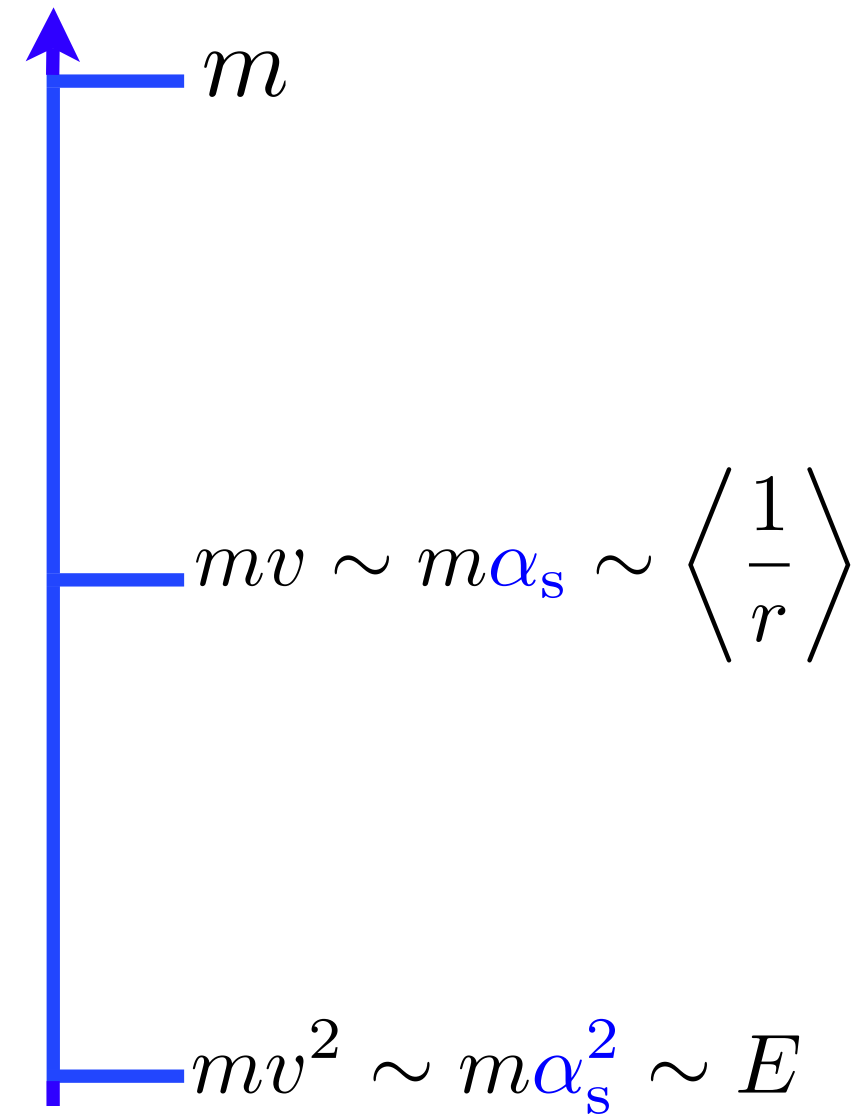
Low-energy operator/ large scale

- The Wilson coefficient are obtained by matching appropriate Green functions in the two theories

- The procedure can be iterated $\ldots \ll \mu_2 \ll \Lambda_2 \ll \mu_1 \ll \Lambda_1$

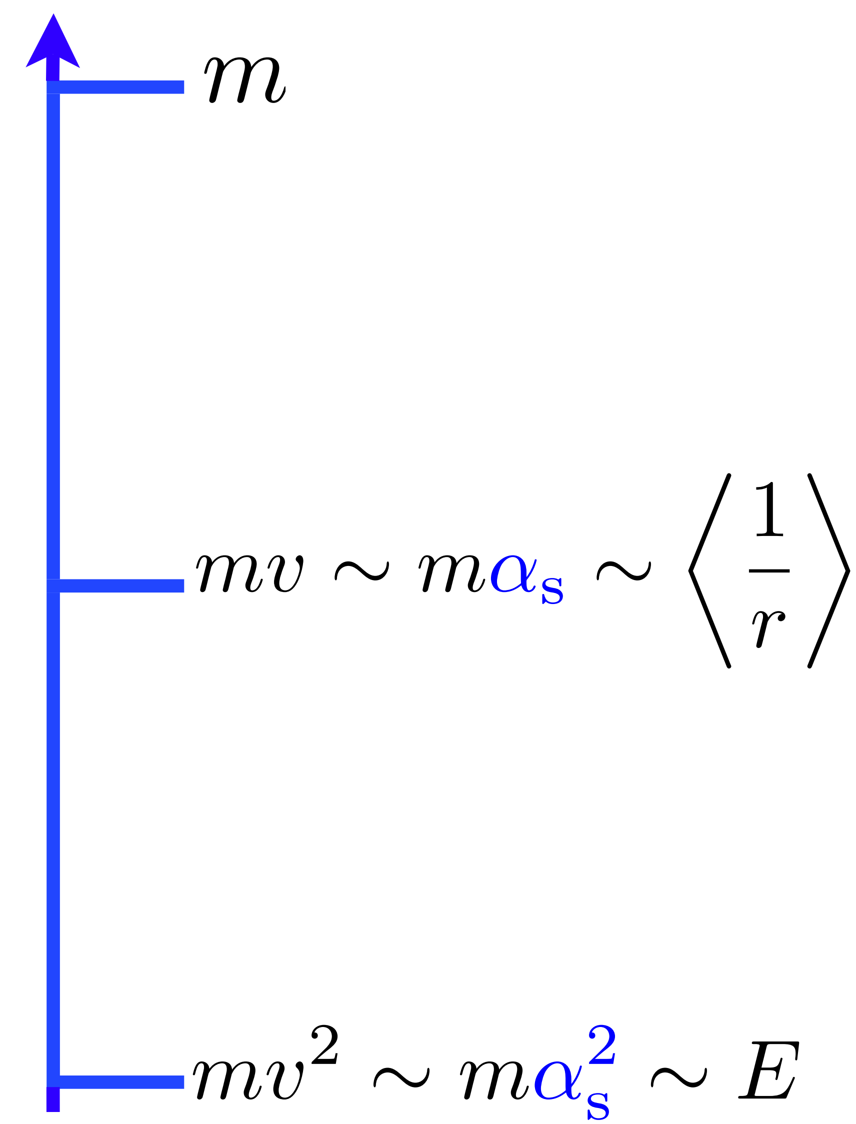
T=0 NR bound states

- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, momentum and energy scales

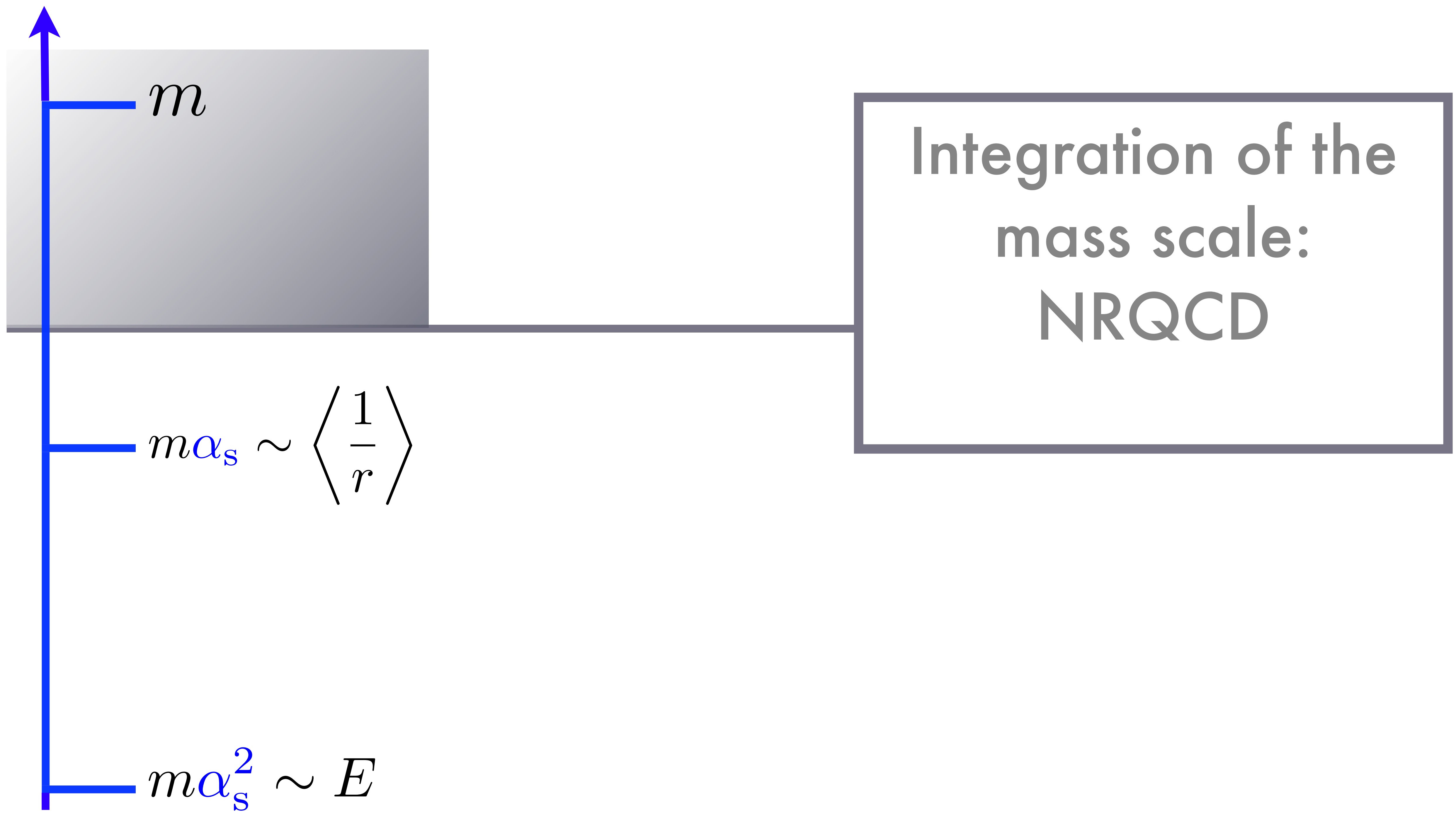


T=0 NR bound states

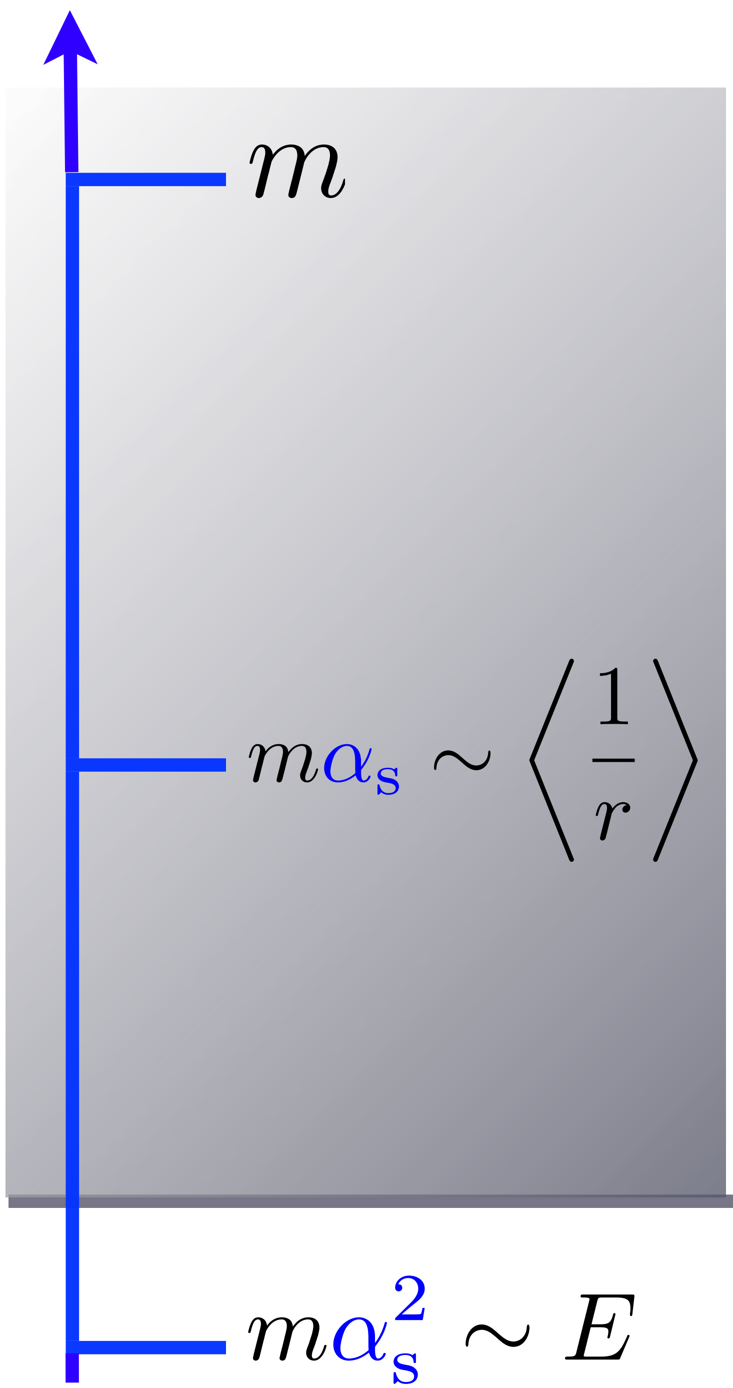
- Non-relativistic $Q\overline{Q}$ bound states are characterized by the hierarchy of the mass, momentum and energy scales
- One can then expand observables in terms of the ratio of the scales and construct a *hierarchy of EFTs* that are equivalent to QCD order-by-order in the expansion parameter



T=0 Scales

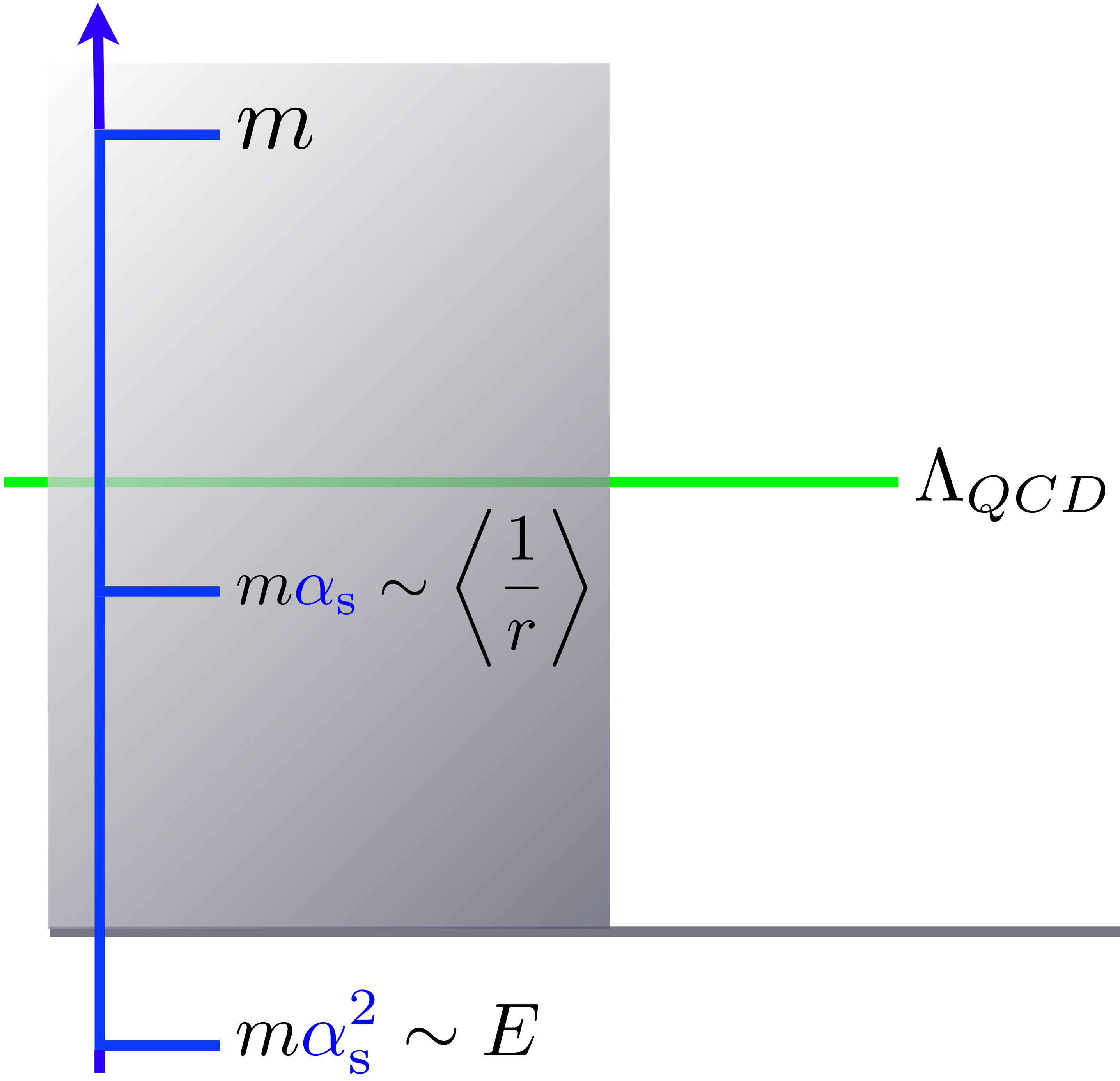

m
$m\alpha_s \sim \left\langle \frac{1}{r} \right\rangle$
$m\alpha_s^2 \sim E$
Integration of the mass scale:
NRQCD

T=0 Scales



Integration of the soft (momentum transfer) scale: pNRQCD

T=0 Scales



Integration of the soft (momentum transfer) scale: pNRQCD

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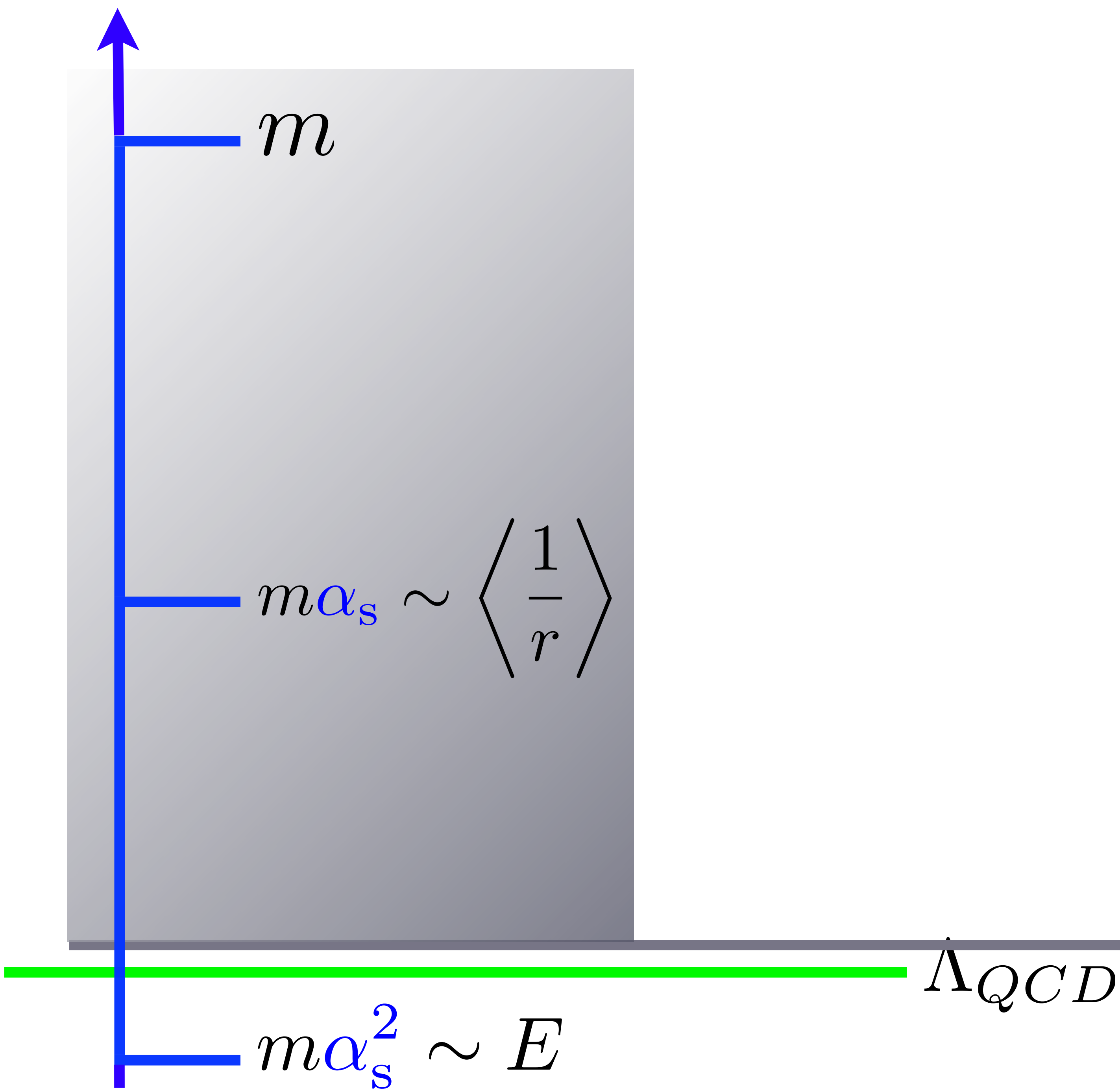
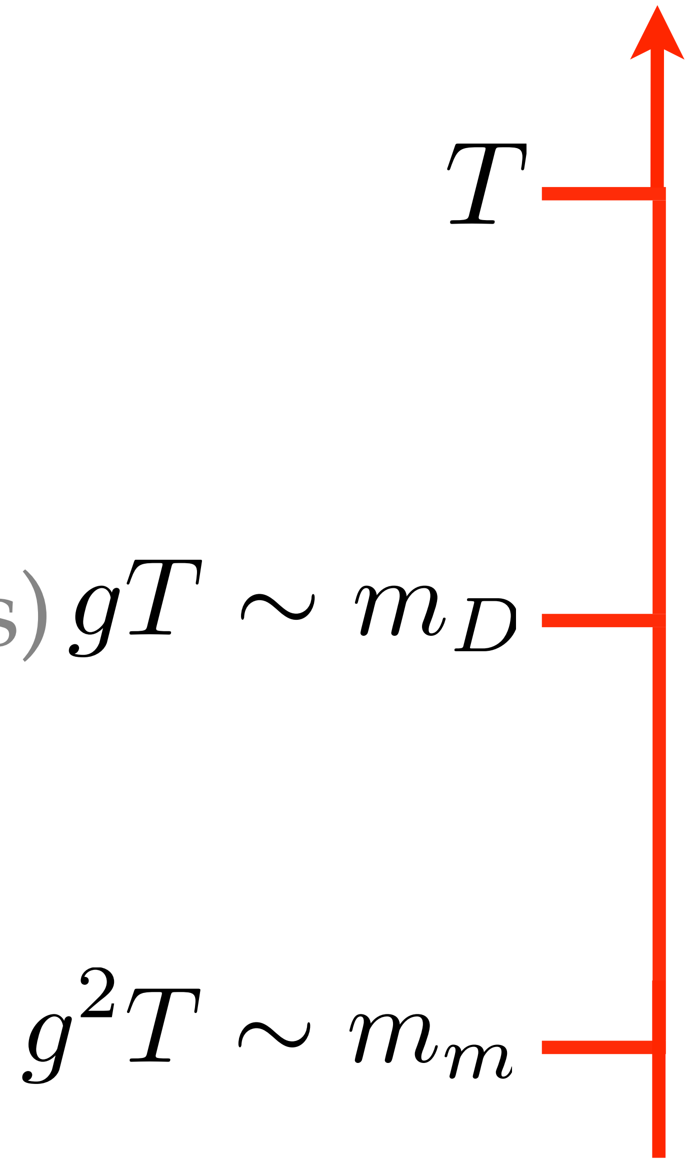


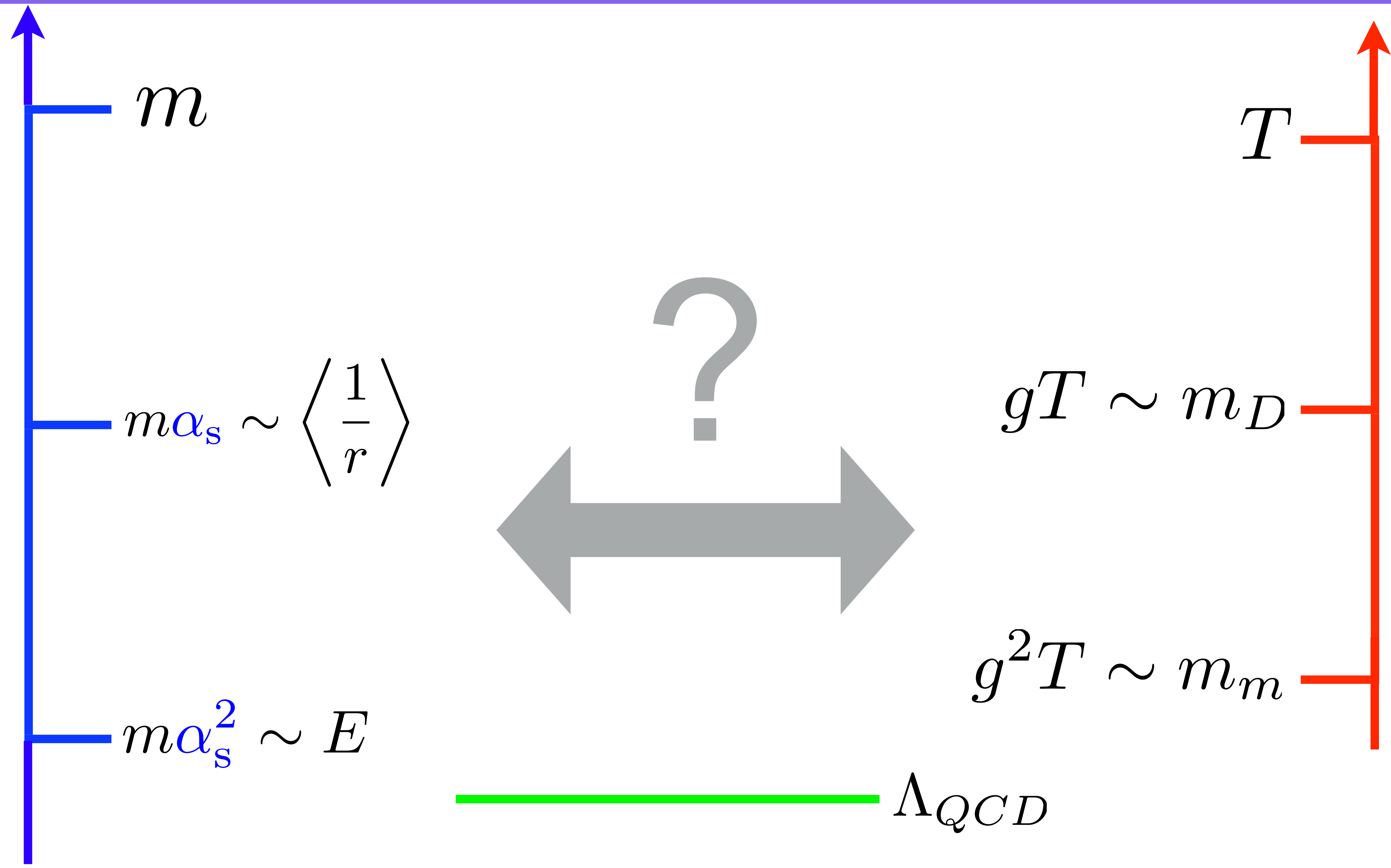
Integration of the soft (momentum transfer) scale: pNRQCD

Thermodynamical scales

- The thermal medium introduces new scales in the physical problem
  - The temperature
  - The electric screening scale (Debye mass)
  - The magnetic screening scale (magnetic mass)
- In the weak coupling assumption these scales develop a hierarchy



Scales of the problem



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- In the regime $T \gg \frac{1}{r} \sim m_D$ the result of Laine et al 2007 is reobtained (also in the Abelian case)

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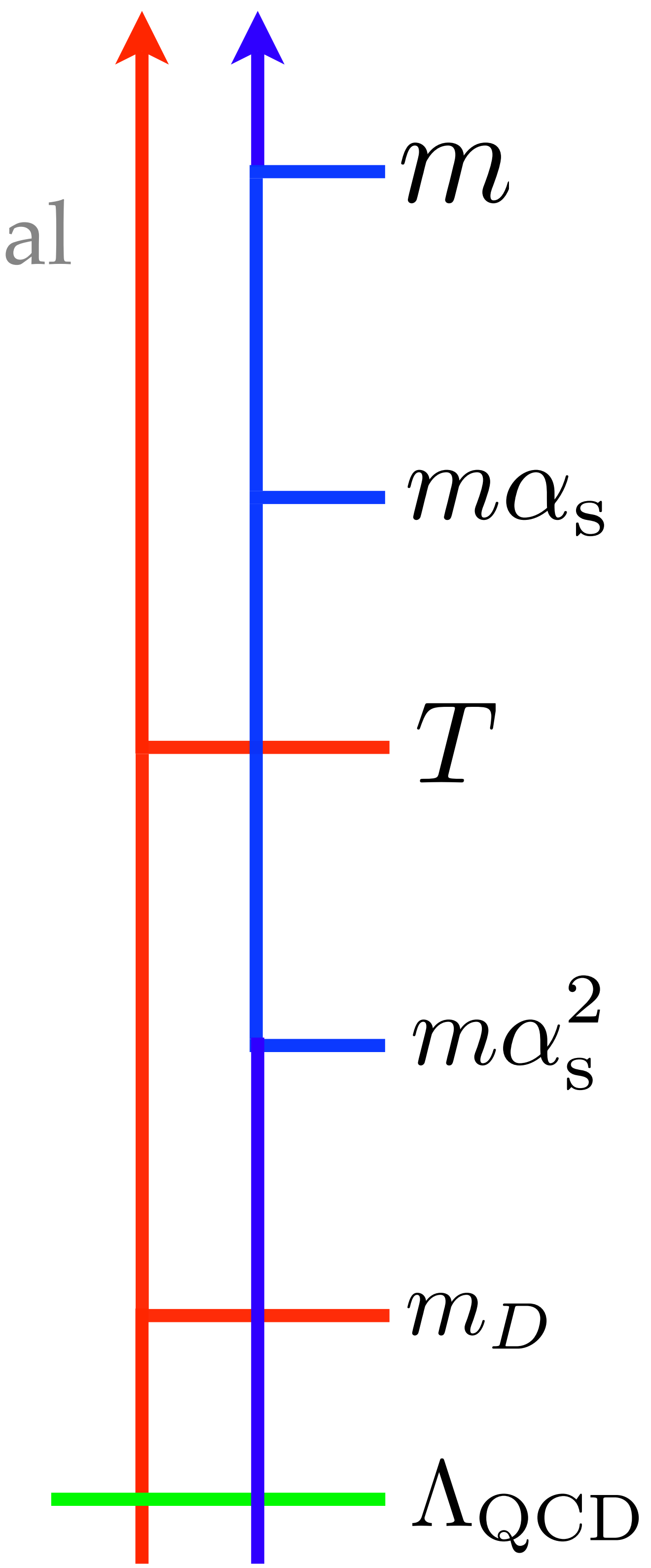
$$m \gg m\alpha_{\rm s} \gg T \gg m\alpha_{\rm s}^2 \gg m_D$$



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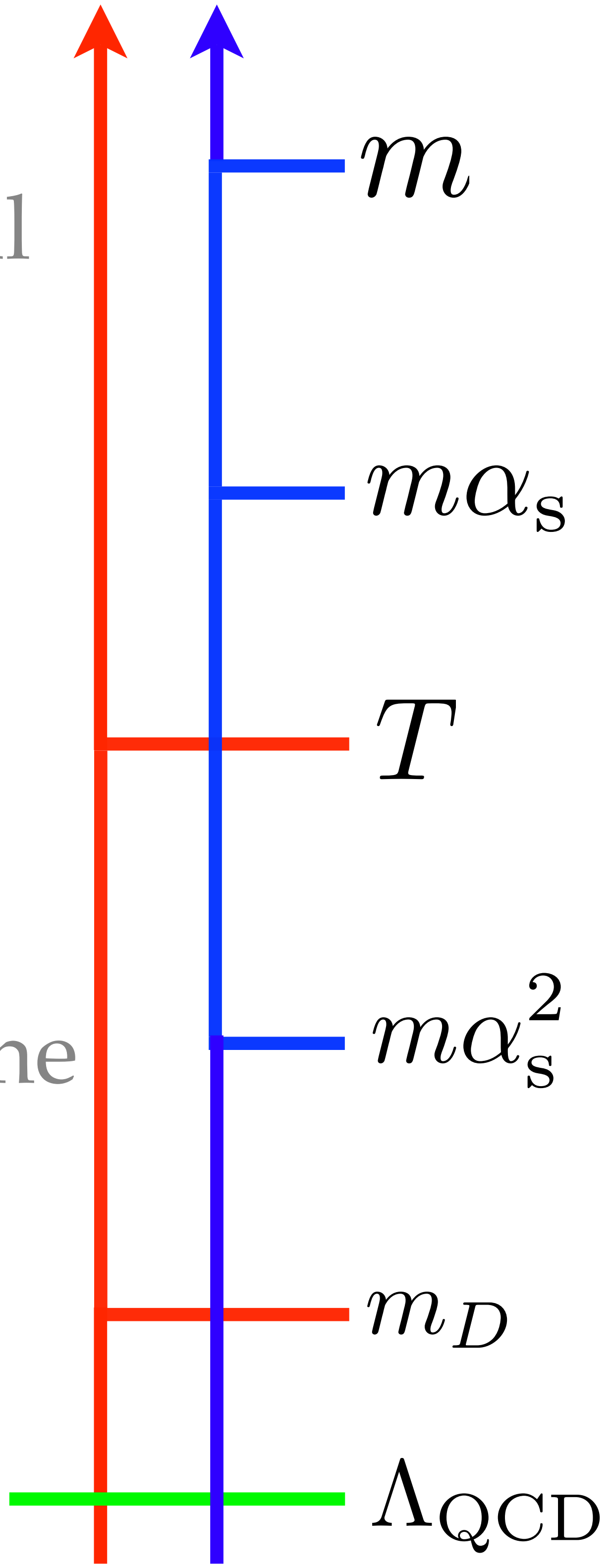

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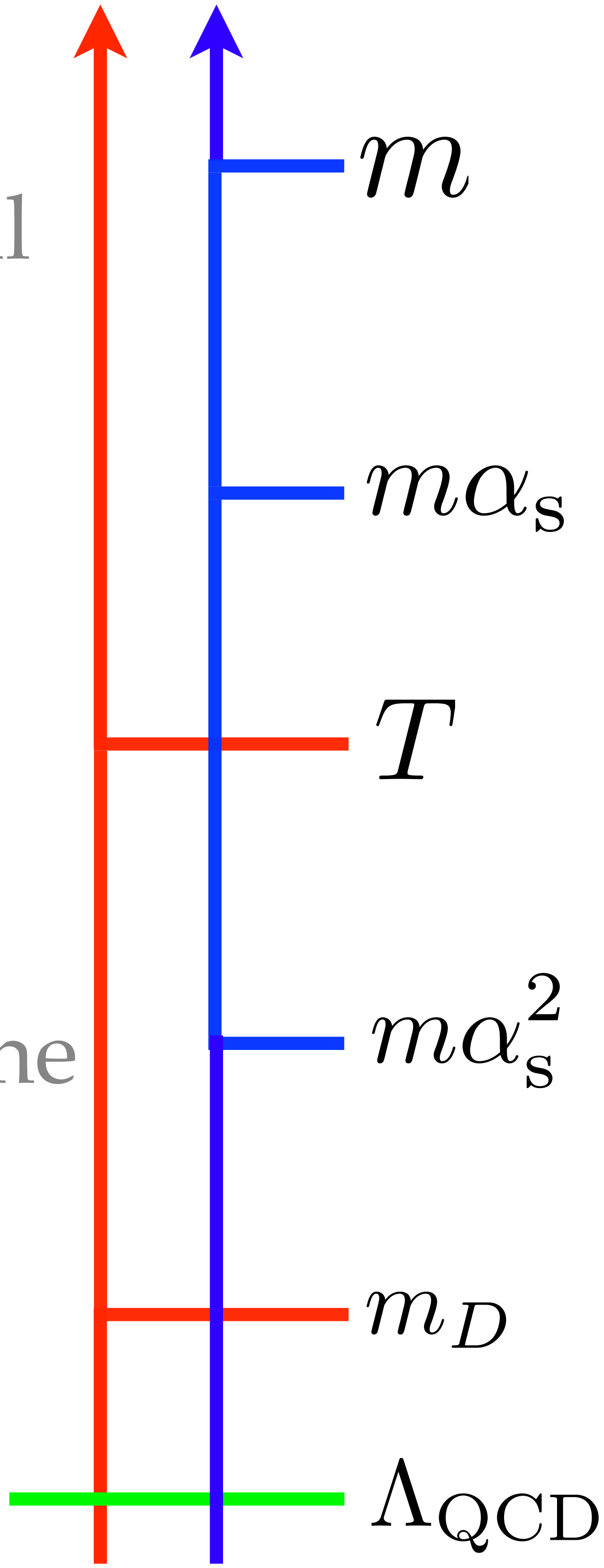

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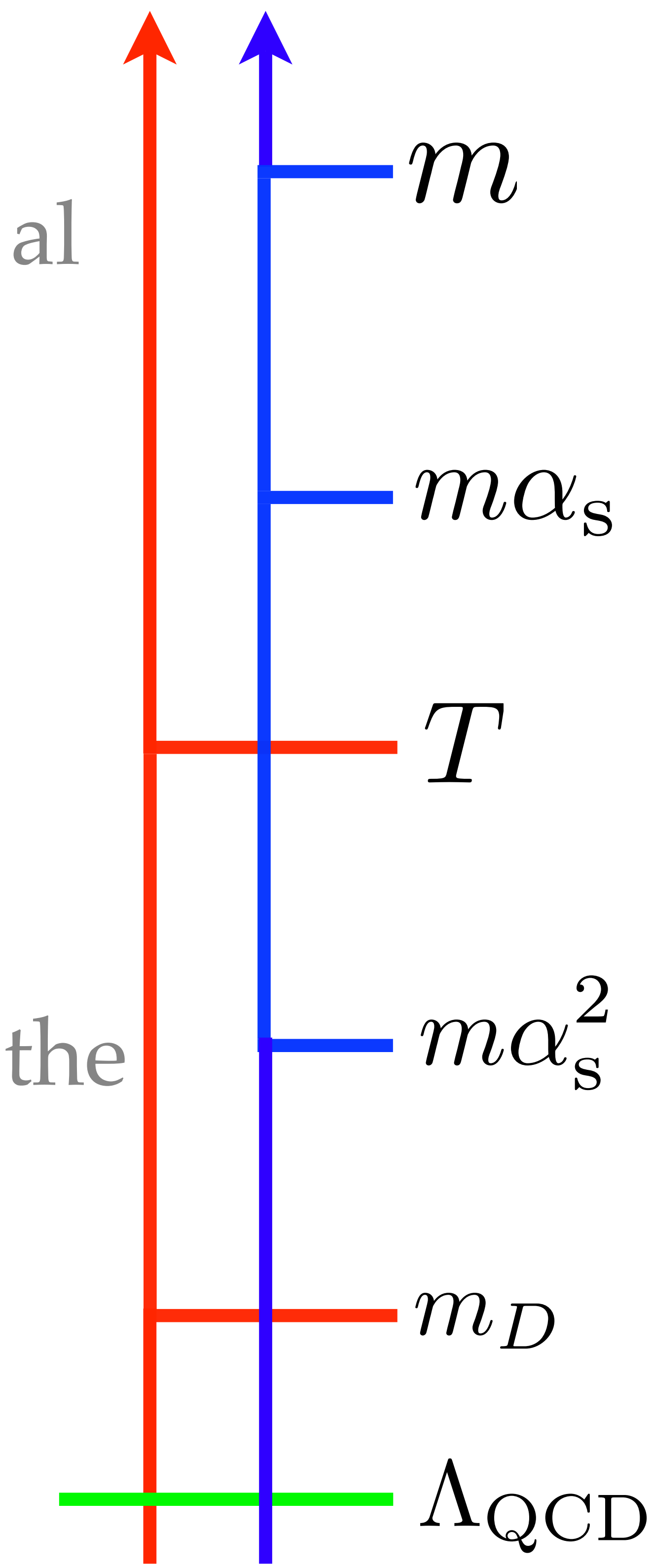
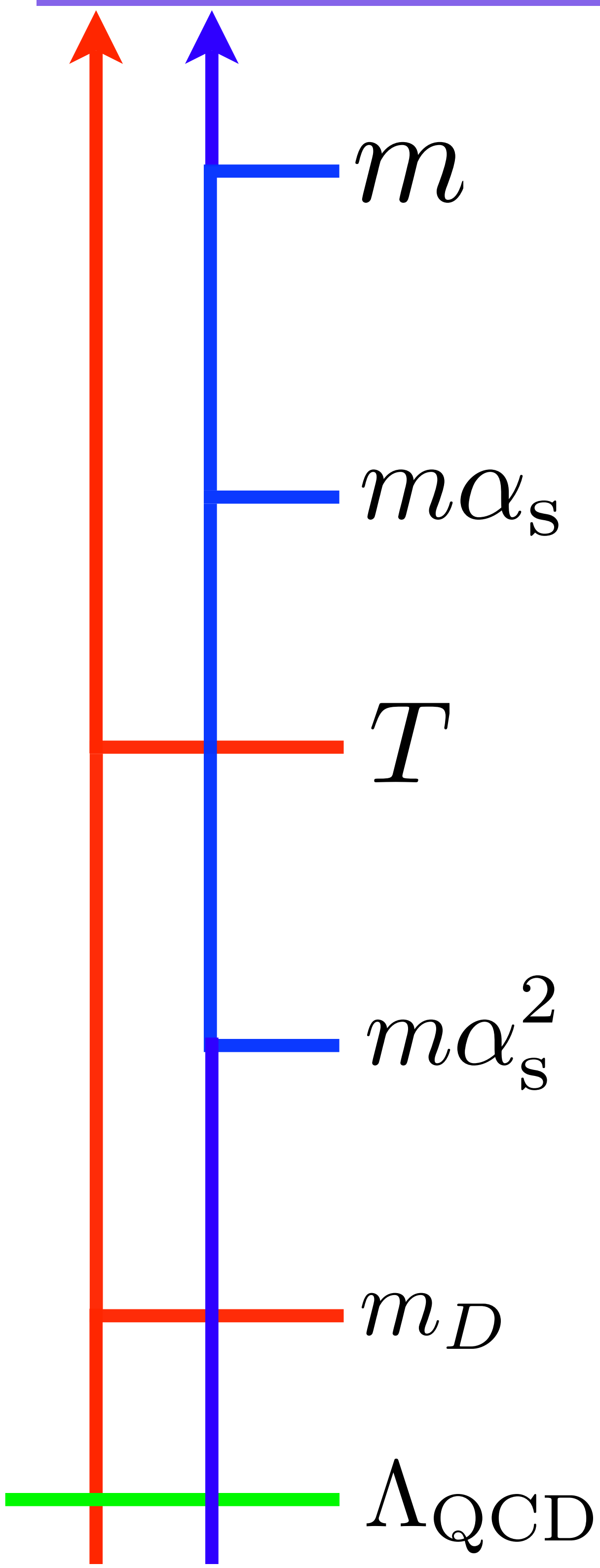
- The temperature is below the *dissociation temperature* $T_d \sim m\alpha_{\mathrm{s}}^{\frac{2}{3}}$

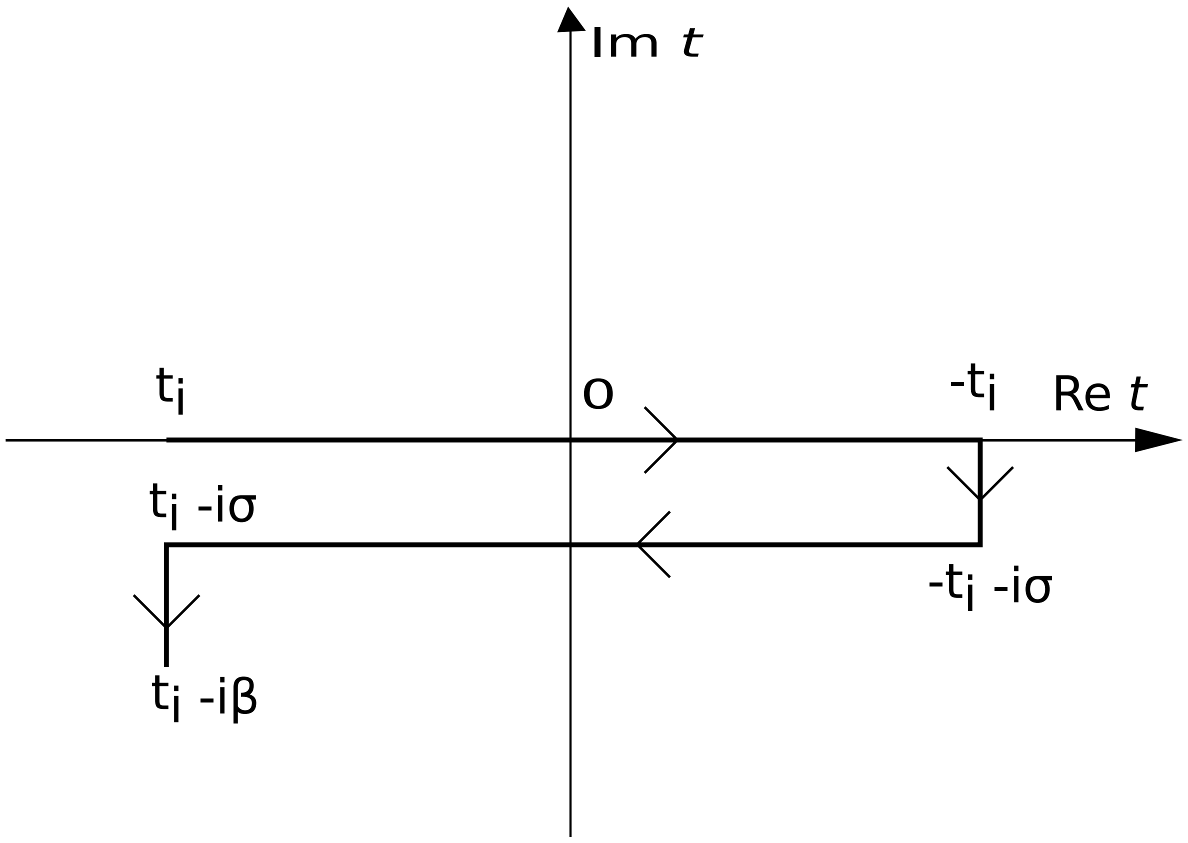


Our goal



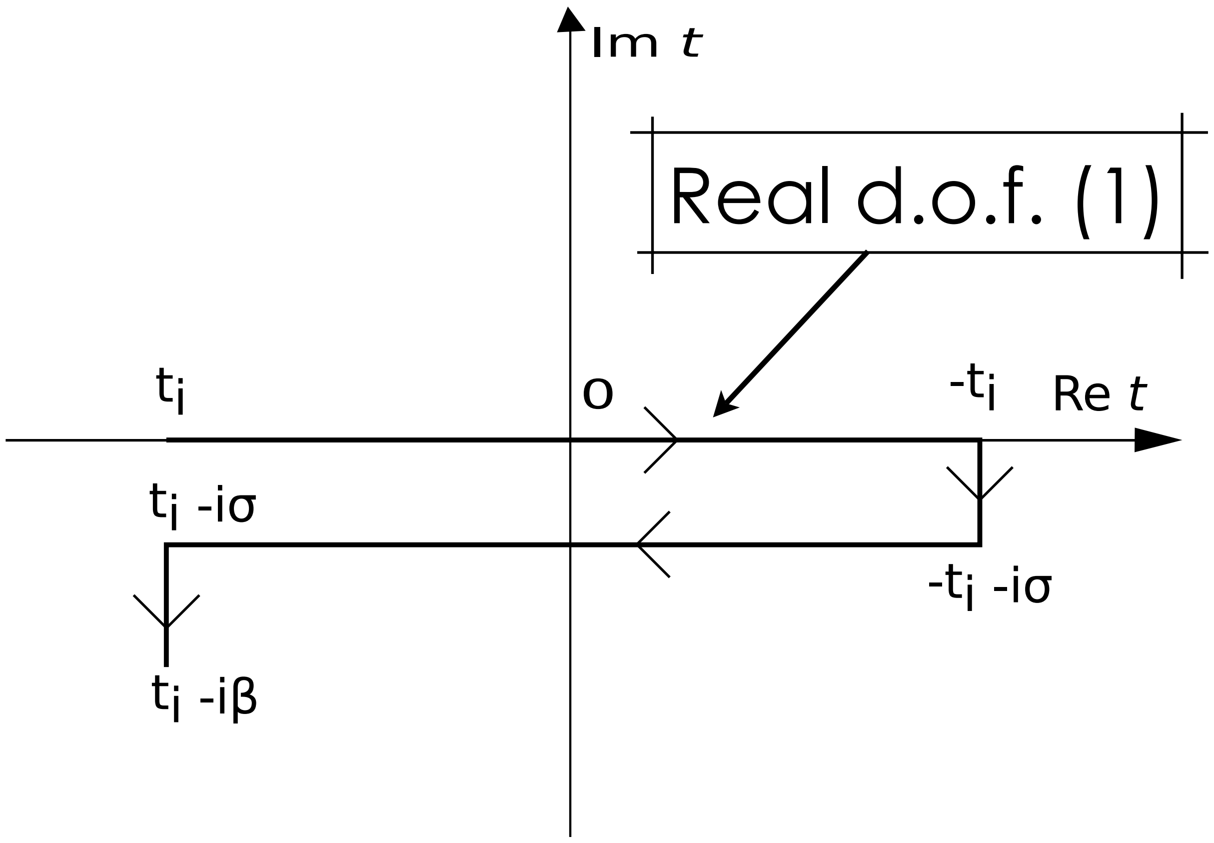
- Our goal is then to compute the spectrum (or thermal corrections to it) and the thermal width in this hierarchy and power counting, up to order $m\alpha_s^5$
- This will be achieved by integrating out the heavier scales in succession and creating a series of EFTs
- Before starting, a reminder of the RTF

The real-time formalism



- Time evolution along this Schwinger-Keldysh path
- $t_i \to -\infty$ limit
- Doubling of the degrees of freedom

The real-time formalism



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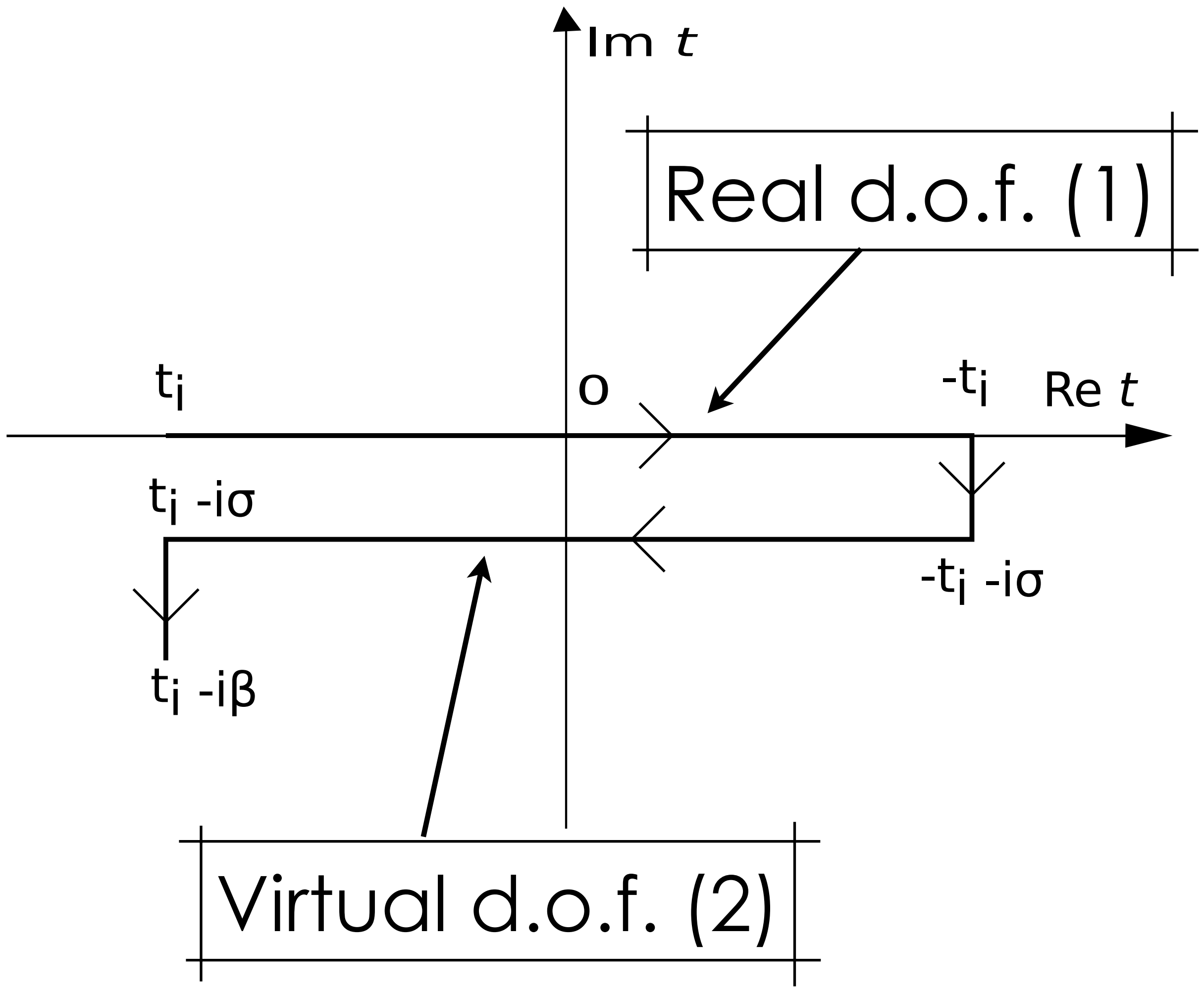


- Time evolution along this Schwinger-Keldysh path
- $t_i \to -\infty$ limit
- Doubling of the degrees of freedom

The real-time formalism

- Closer to the T=0 EFT approach
- In the static quark sector the virtual DOF decouple
- Simpler than the imaginary-time + analytical continuation (potential as pole of the propagator in the large real-time limit)
- Propagators as 2X2 matrices, vertices of type "2" have opposite sign

Free propagators

- Propagators written as $D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$
- Static quark

$$S^{(0)}_{\alpha\beta}(k_0) = \delta_{\alpha\beta} \begin{pmatrix} \dfrac{i}{k_0 + i\eta} & 0 \\ 2\pi\delta(k_0) & \dfrac{-i}{k_0 - i\eta} \end{pmatrix}$$

(static quark of type 2 decouple)

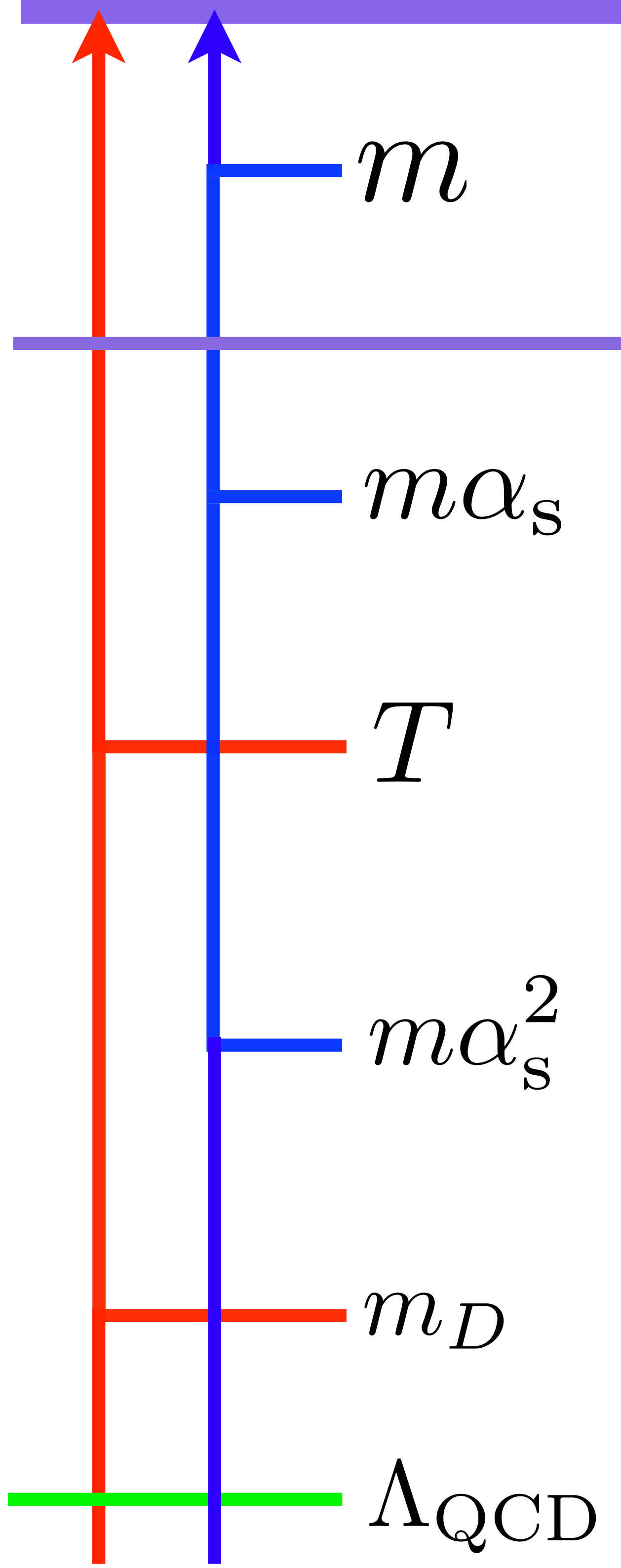
- Gluons (Coulomb gauge)

$$D^{(0)}_{00}(k) = \begin{pmatrix} \dfrac{i}{k^2} & 0 \\ 0 & -\dfrac{i}{k^2} \end{pmatrix}$$

$$D^{(0)}_{ij}(k_0, k) = \left(\delta_{ij} - \frac{k^i k^j}{k^2}\right) \left\{ \begin{pmatrix} \dfrac{i}{k_0^2 - k^2 + i\eta} & \theta(-k_0)\, 2\pi\delta(k_0^2 - k^2) \\ \theta(k_0)\, 2\pi\delta(k_0^2 - k^2) & -\dfrac{i}{k_0^2 - k^2 - i\eta} \end{pmatrix} + 2\pi\delta(k_0^2 - k^2)\, n_{\mathrm{B}}(|k_0|) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

Integrating out m and $m\alpha_s$.

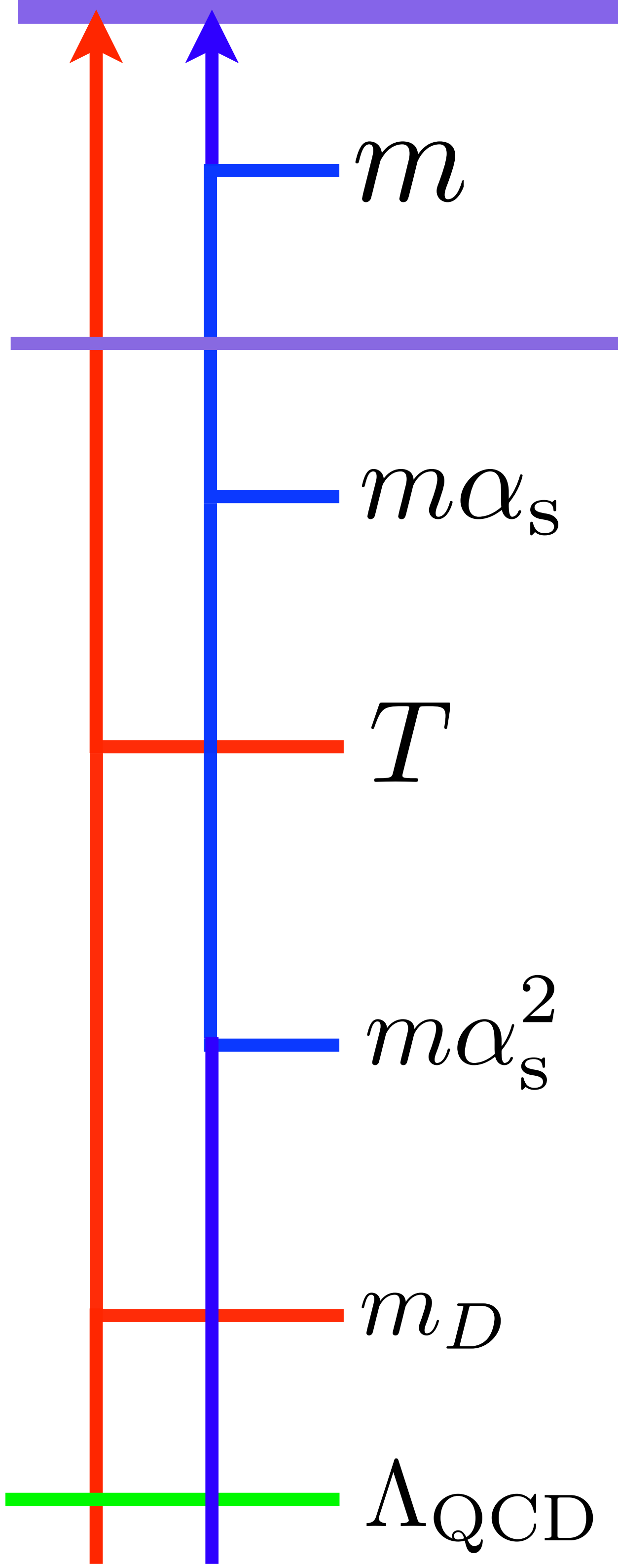
Mass scale: NRQCD



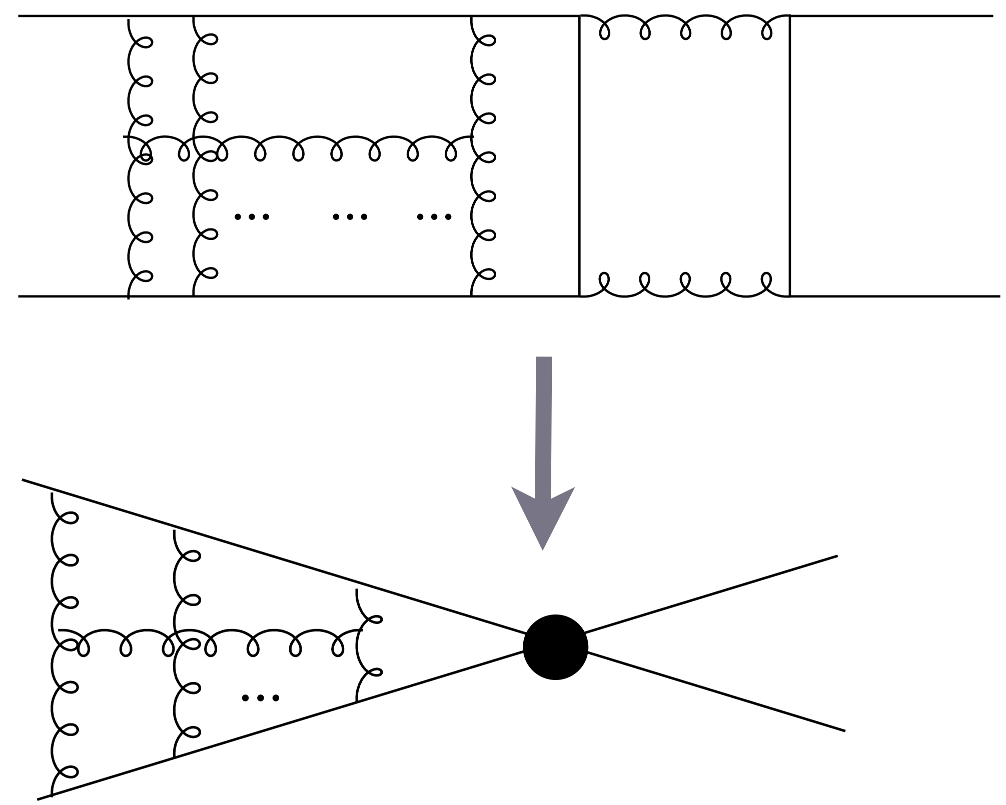
- Smaller scales are put to zero in the matching. This means that the thermal scales do not affect the result of the integration, which yields standard NRQCD

- NRQCD is organized as an expansion in $1/m$
  Caswell Lepage **PLB167** (1986)
  Bodwin Braaten Lepage **PRD51** (1995)

Mass scale: NRQCD



$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm light} + \psi^\dagger\left(iD_0 + \frac{D^2}{2m} + \dots\right)\psi + \chi^\dagger(iD_0 - \frac{D^2}{2m} + \dots)\chi + \dots$$



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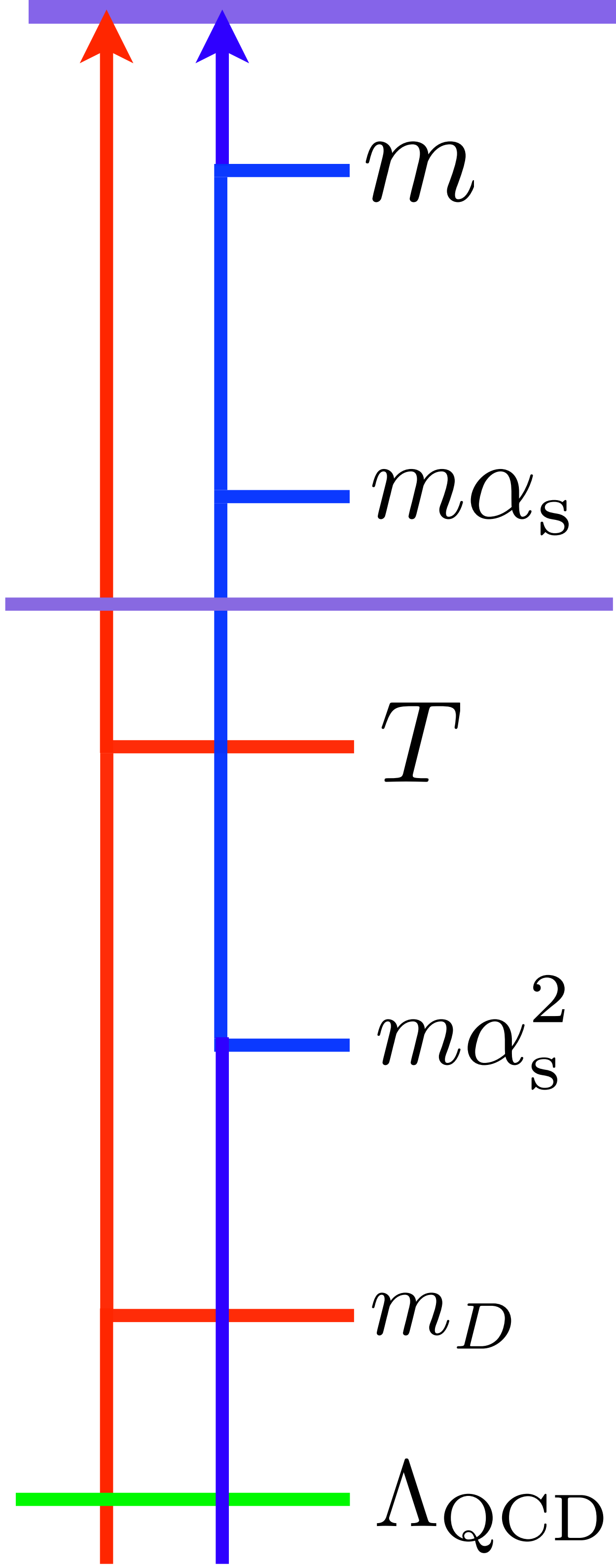
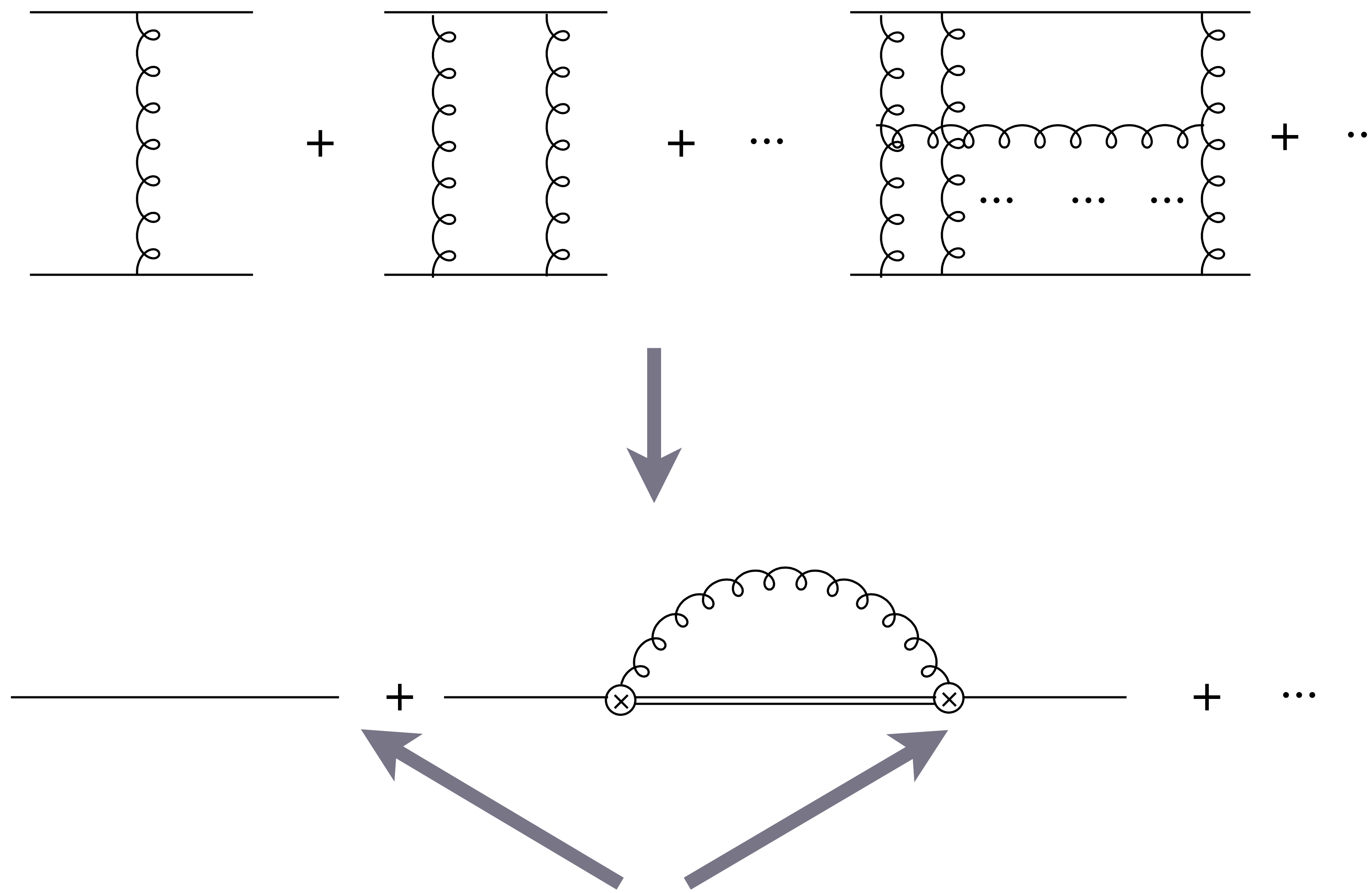
Relative momentum scale: pNRQCD





$$\frac{1}{E - p^2/m - V(r)}$$

Pineda Soto Nucl. Phys. Proc. Suppl. 64(1998)

Brambilla Pineda Soto Vairo **NPB566** (2000)

Weakly coupled pNRQCD

$$\mathcal{L}_{\text{pNRQCD}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{light}} + \text{Tr}\left\{ \text{S}^\dagger \left[i\partial_0 - H_s \right] \text{S} + \text{O}^\dagger \left[iD_0 - H_o \right] \text{O} \right\}$$

$$+ V_A \, \text{Tr}\left\{ \text{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \text{S} + \text{S}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \text{O} \right\} + \frac{V_B}{2} \text{Tr}\left\{ \text{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \, \text{O} + \text{O}^\dagger \text{O} \mathbf{r} \cdot g\mathbf{E} \right\} + \ldots$$

- Degrees of freedom: $Q\overline{Q}$ states with energy $E \sim \Lambda_{QCD}, mv^2$ and momentum $p \lesssim mv$
  Singlet and octet color states
- US gluons with energy / momentum $\lesssim mv$
- Expansion in α_s , $\frac{1}{m}$ and r
- Potentials are Wilson coefficients, receive contributions from all scales higher than the energy

Contribution to the spectrum

- Power-counting in the singlet Hamiltonian

$$H_s = \frac{\mathbf{p}^2}{m} - C_F \frac{\alpha_{\rm s}}{r} + \mathcal{O}(m\alpha_{\rm s}^3)$$

Contribution to the spectrum

- Power-counting in the singlet Hamiltonian

$$H_s = \frac{\mathbf{p}^2}{m} - C_F \frac{\alpha_s}{r} + \mathcal{O}(m\alpha_s^3)$$

- Solve the singlet EOM (Schrödinger Eq.) with this Hamiltonian and obtain the Coulomb levels

$$E_n = -\frac{mC_F^2\alpha_s^2}{4n^2} = \frac{1}{ma_0^2 n^2}\,, \qquad a_0 \equiv \frac{2}{mC_F\alpha_s}$$

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- Treat further terms in the expansion in QM perturbation theory, obtaining the $m\alpha_\mathrm{s}^3 - m\alpha_\mathrm{s}^5$ contributions to the spectrum from this scale

  Brambilla Pineda Soto Vairo **PLB470** (1999) Kniehl Penin **NPB563** (1999) Kniehl Penin Smirnov Steinhauser **NPB635** (2002)

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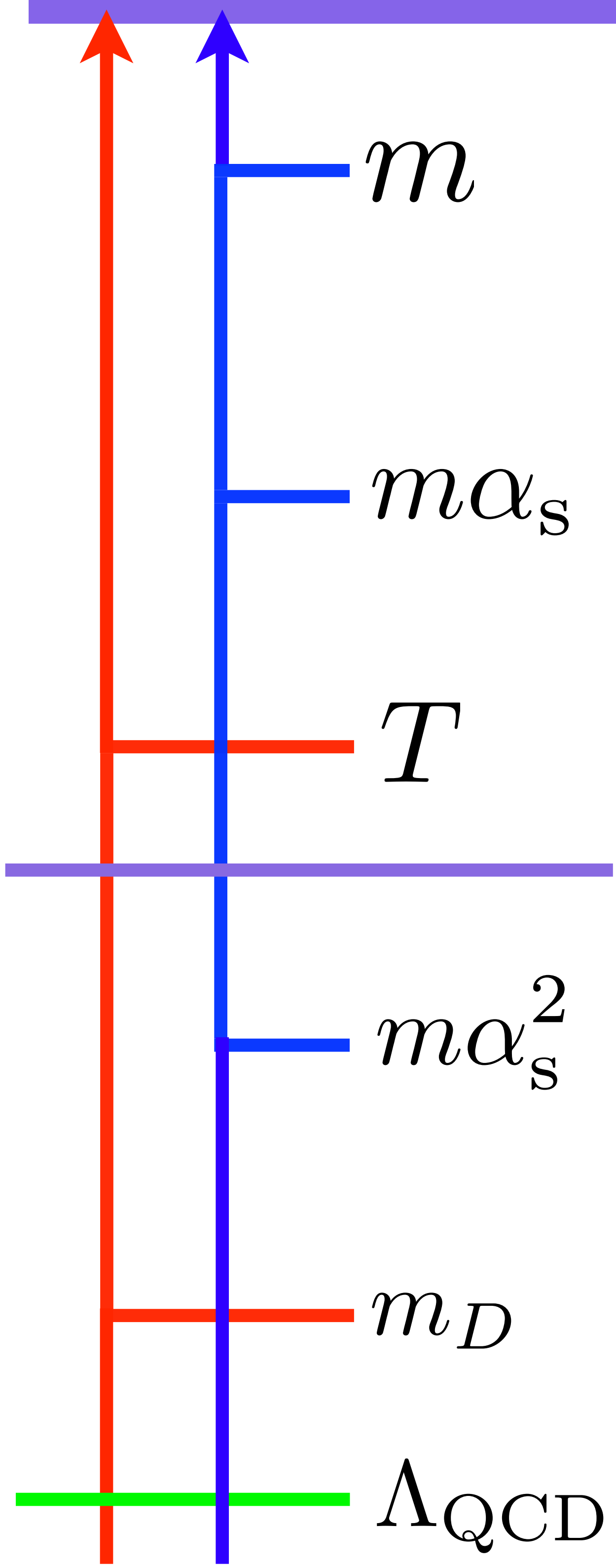
- IR divergences manifest themselves at $m\alpha_s^5$

Divergent contributions to the spectrum

Scale	Vacuum	Thermal
$m\alpha_s$	$\sim m\alpha_s^5 \frac{1}{\epsilon_{IR}}$	/
T		
$m\alpha_s^2$		

Integrating out the temperature

pNRQCD$_{\text{HTL}}$



- Integrating out the temperature brings us to a new EFT called pNRQCD$_{\text{HTL}}$
- In this EFT modes with energy and momenta of the order of the temperature are no longer present. This amounts to:
  - Hard Thermal Loop (HTL) resummation in the gauge and light quark sectors
    Braaten Pisarski PRD45 (1992)
  - Thermal modifications to the potentials in the singlet and octet sector
- This EFT and its matching coefficients are independent of the hierarchy of low-lying scales

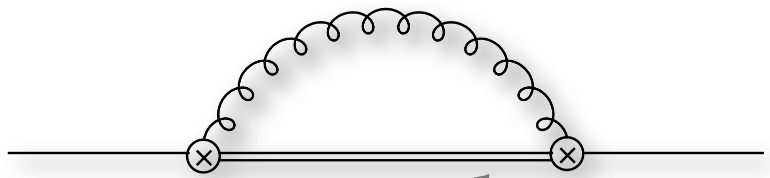
The pNRQCD$_{\mathrm{HTL}}$ Lagrangian

$$\mathcal{L}_{\mathrm{pNRQCD_{HTL}}} = \mathcal{L}_{\mathrm{HTL}} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left[i\partial_0 - H_s - \delta V_s \right] \mathrm{S} + \mathrm{O}^\dagger \left[iD_0 - H_o - \delta V_o \right] \mathrm{O} \right\}$$
$$+\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{S} + \mathrm{S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} \right\} + \frac{1}{2}\mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,\mathrm{O} + \mathrm{O}^\dagger \mathrm{O}\mathbf{r}\cdot g\mathbf{E} \right\} + \dots$$

- The new matching coefficients in the singlet and octet sectors have to be computed by matching pNRQCD to pNRQCD$_{\mathrm{HTL}}$
- We only do this for the singlet sector, which is the only one relevant for the spectrum at our accuracy
- In the matching procedure we have to single out the contribution from the scale T: this will be achieved by appropriate expansions in E/T and m_D/T

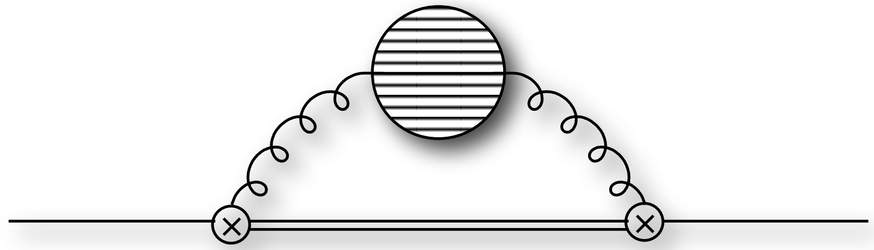
Matching pNRQCD to pNRQCD$_{HTL}$

- The leading contribution to δV_s comes from the dipole vertices



It gives rise to a correction to the potential and to the spectrum

- Subleading contributions come from the radiative correction to that diagram



It gives rise to an imaginary part, corresponding to a thermal width. The phenomenon is called Landau damping

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  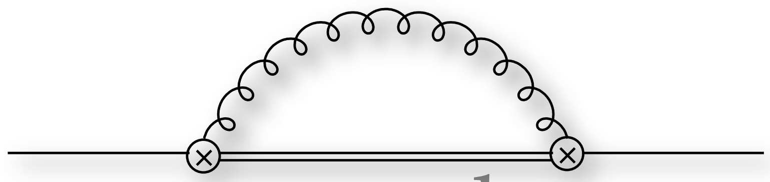

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  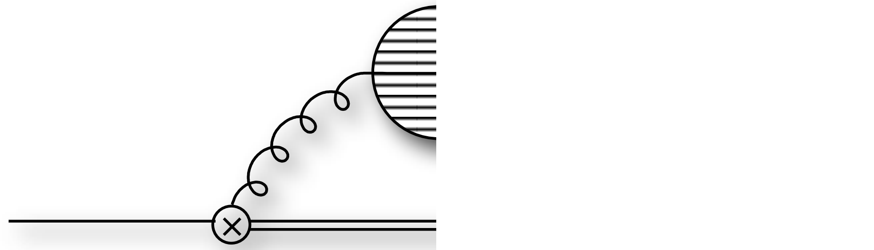

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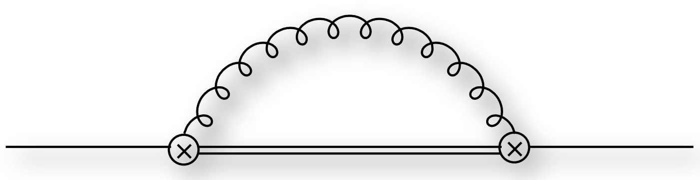

$$\sim r^i \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - H_o - k_0 + i\eta} \left[k_0^2 \, D_{ii}^{(0)} + k^2 \, D_{00}^{(0)} \right] r^i$$

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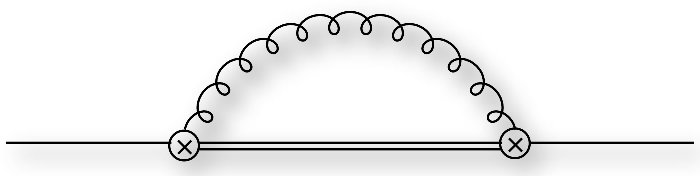

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$$-ig^2 C_F \frac{D-2}{D-1} r^i \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - H_o - k_0 + i\eta} k_0^2 \left[\frac{i}{k_0^2 - k^2 + i\eta} + 2\pi\delta\left(k_0^2 - k^2\right) n_{\mathrm{B}}\left(|k_0|\right) \right] r^i$$

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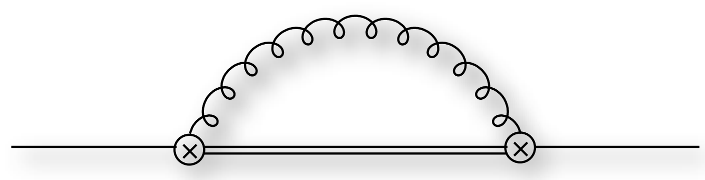

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- The octet propagator must be expanded in $k_0 \sim T \gg (E - H_o)$

Matching pNRQCD to pNRQCD$_{\rm HTL}$

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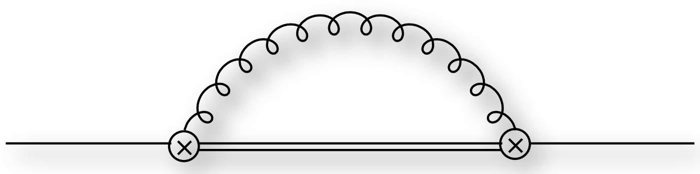

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- The octet propagator must be expanded in $k_0 \sim T \gg (E - H_o)$
- The vacuum part then yields a series of scaleless integrals
- In the thermal part the linear and cubic terms (in the energy) contribute within our accuracy, whereas the quadratic term vanishes in D.R.

Results: the potential

- The correction to the potential from the temperature scale is, when $T \ll m\alpha_s$

$$\delta V_s = \frac{\pi}{9} N_c C_F \, \alpha_s^2 \, T^2 \, r + \frac{2\pi}{3m} C_F \, \alpha_s \, T^2 + \frac{\alpha_s C_F I_T}{3\pi} \left[-\frac{N_c^3}{8} \frac{\alpha_s^3}{r} - (N_c^2 + 2N_c C_F) \frac{\alpha_s^2}{mr^2} + 4(N_c - 2C_F) \frac{\pi \alpha_s}{m^2} \delta^3(\mathbf{r}) + N_c \frac{\alpha_s}{m^2} \left\{ \nabla_{\mathbf{r}}^2, \frac{1}{r} \right\} \right]$$
$$- \frac{3}{2} \zeta(3) \, C_F \, \frac{\alpha_s}{\pi} \, r^2 \, T \, m_D^2 + \frac{2}{3} \zeta(3) \, N_c C_F \, \alpha_s^2 \, r^2 \, T^3$$
$$+ i \left[\frac{C_F}{6} \alpha_s \, r^2 \, T \, m_D^2 \left(-\frac{2}{\epsilon} + \gamma_E + \ln \pi - \ln \frac{T^2}{\mu^2} + \frac{2}{3} - 4 \ln 2 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{4\pi}{9} \ln 2 \; N_c C_F \, \alpha_s^2 \, r^2 \, T^3 \right]$$

$$m_D^2 = \frac{g^2 T^2}{3} \left(N + \frac{n_f}{2} \right)$$

- Both the real and the imaginary part are IR divergent; the divergences cancel in the related physical observables, the spectrum and the width, against UV divergences from some lower scale

Divergent contributions to the spectrum

Scale	Vacuum	Thermal
$m\alpha_{\rm s}$	$\sim m\alpha_{\rm s}^5 \frac{1}{\epsilon_{\rm IR}}$	/
T	scaleless	$\sim -m\alpha_{\rm s}^5 \frac{1}{\epsilon_{\rm IR}}$
$m\alpha_{\rm s}^2$		

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Scale	Vacuum	Thermal
$m\alpha_{\rm s}$	$\sim m\alpha_{\rm s}^5 \frac{1}{\epsilon_{\rm IR}}$	/
T	$\sim m\alpha_{\rm s}^5 \left(\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right)$	$\sim -m\alpha_{\rm s}^5 \frac{1}{\epsilon_{\rm IR}}$
$m\alpha_{\rm s}^2$		

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T	$\sim m\alpha_s^5 \left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right)$	$\sim -m\alpha_s^5 \frac{1}{\epsilon_{IR}}$
$m\alpha_s^2$		

Results: spectrum and width

- The corresponding correction to the spectrum is

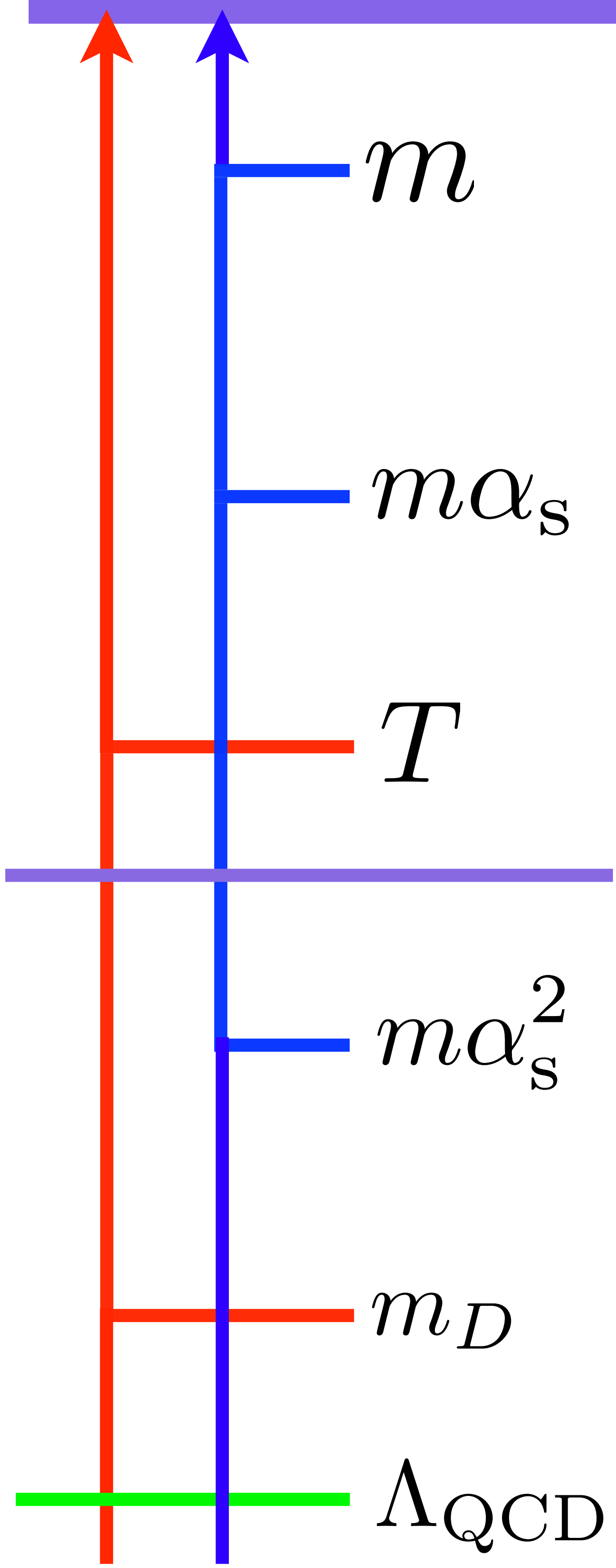
$$\delta E_{n,l}^{(T)} = \frac{\pi}{9} N_c C_F \,\alpha_s^2\, T^2 \frac{a_0}{2}(3n^2 - l(l+1)) + \frac{2\pi}{3m} C_F \,\alpha_s\, T^2$$
$$+ \frac{E_n I_T \alpha_s^3}{3\pi} \left\{ \frac{4C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left(\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right) + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\}$$
$$+ \left(-\frac{3}{2}\zeta(3)\, C_F \,\frac{\alpha_s}{\pi}\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F \,\alpha_s^2\, T^3 \right) \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right].$$

- The width is

$$\Gamma_{n,l} = \left[-\frac{C_F}{6} \alpha_s T m_D^2 \left(-\frac{2}{\epsilon} + \gamma_E + \ln\pi - \ln\frac{T^2}{\mu^2} + \frac{2}{3} - 4\ln 2 - 2\frac{\zeta'(2)}{\zeta(2)} \right) - \frac{4\pi}{9} \ln 2 \; N_c C_F \,\alpha_s^2\, T^3 \right] a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1)\right].$$

Calculating the contribution from the energy scale

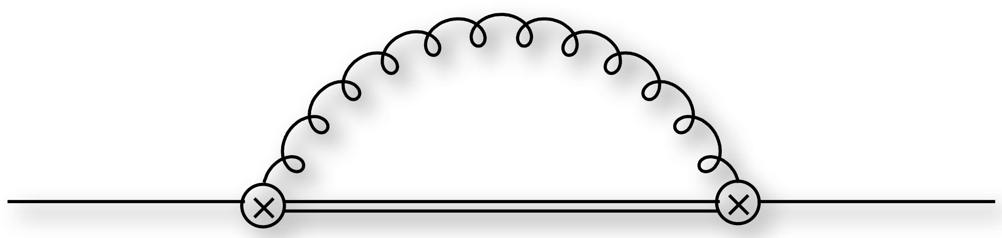
Calculating at the energy scale



- Since we have reached the binding energy scale we cannot proceed further with EFTs in the heavy quark-antiquark sector
- We thus calculate the contributions to the spectrum and the width *within* pNRQCD$_{\mathrm{HTL}}$
- We thus have to use the HTL-resummed propagators
- We employ the expansions $T \gg m\alpha_s^2 \gg m_D$

The calculation

- Only the leading order diagram contributes within our accuracy



$$\sim r^i \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - H_o - k_0 + i\eta} \left[k_0^2\, D_{ii}^{\mathrm{HTL}} + k^2\, D_{00}^{\mathrm{HTL}}\right] r^i$$

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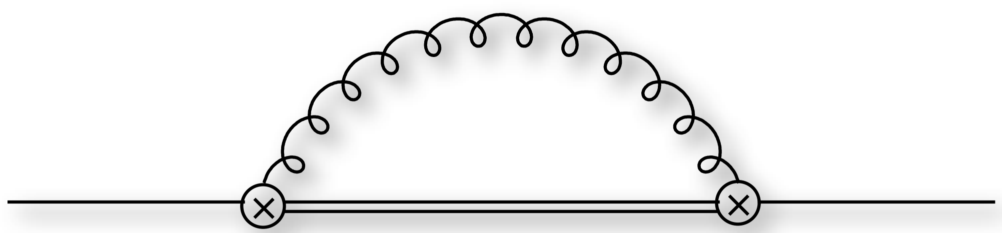

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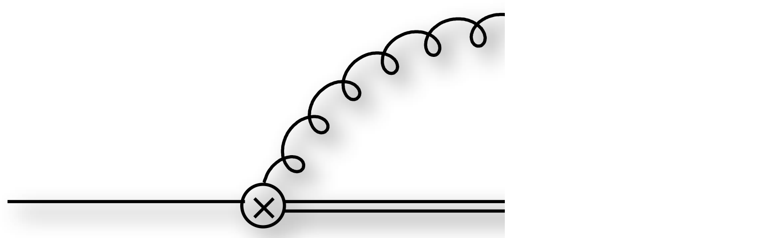
- This diagram introduces a second source of imaginary parts $\Rightarrow$ thermal width. It is the singlet-to-octet decay width

The calculation

- Only the leading order diagram contributes within our accuracy



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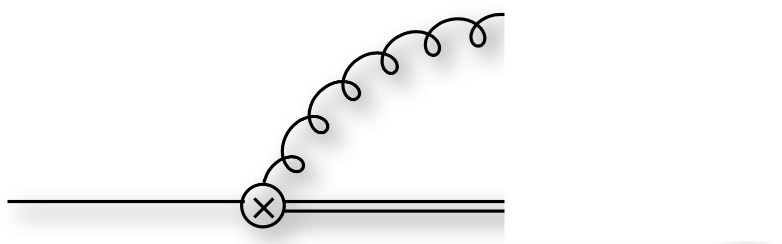
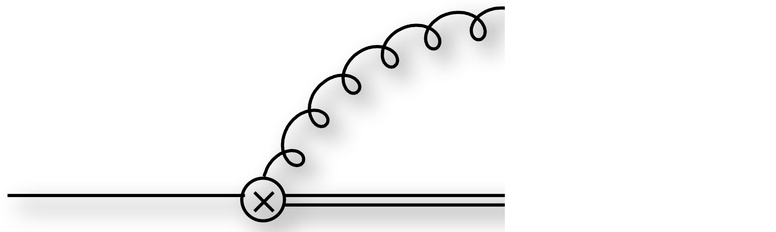


$$\sim r^i \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - H_o - k_0 + i\eta} \left[k_0^2 \, D_{ii}^{\mathrm{HTL}} + k^2 \, D_{00}^{\mathrm{HTL}}\right] r^i$$

- This diagram introduces a second source of imaginary parts $\Rightarrow$ thermal width. It is the singlet-to-octet decay width
- The calculation is quite involved, requiring expansions in the propagators and in the Bose distribution

The calculation

- Only the leading order diagram contributes within our accuracy



$$\sim r^i \int \frac{d^D k}{(2\pi)^D} \frac{i}{E - H_o - k_0 + i\eta} \left[k_0^2 \, D_{ii}^{\rm HTL} + k^2 \, D_{00}^{\rm HTL}\right] r^i$$

- This diagram introduces a second source of imaginary parts $\Rightarrow$ thermal width. It is the singlet-to-octet decay width
- The calculation is quite involved, requiring expansions in the propagators and in the Bose distribution
- UV divergences appear and cancel the IR divergencies from higher scales

Divergent contributions to the spectrum

Scale	Vacuum	Thermal
$m\alpha_s$	$\sim m\alpha_s^5 \frac{1}{\epsilon_{IR}}$	/
T	$\sim m\alpha_s^5 \left(\frac{1}{\epsilon_{IR}} - \frac{1}{\epsilon_{UV}}\right)$	$\sim -m\alpha_s^5 \frac{1}{\epsilon_{IR}}$
$m\alpha_s^2$	$\sim -m\alpha_s^5 \frac{1}{\epsilon_{UV}}$	$\sim m\alpha_s^5 \frac{1}{\epsilon_{UV}}$

Summary and conclusions

The spectrum

- The contribution *of the thermal medium only* to the spectrum is

$$\delta E_{n,l} = \frac{\pi}{9} N_c C_F \, \alpha_s^2 \, T^2 \frac{a_0}{2} \left[3n^2 - l(l+1)\right] + \frac{\pi}{3} C_F^2 \, \alpha_s^2 \, T^2 \, a_0$$
$$+ \frac{E_n \alpha_s^3}{3\pi} \left[\log\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E\right] \left\{ \frac{4 C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] \right.$$
$$\left. + \frac{2 N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} + \frac{2 E_n C_F^3 \alpha_s^3}{3\pi} L_{n,l}$$
$$+ \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right] \left\{ - \left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F \, \alpha_s \, T \, m_D^2 \right.$$
$$\left. + \frac{2}{3}\zeta(3) \, N_c C_F \, \alpha_s^2 \, T^3 \right\}$$

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$$m\alpha_s^5 \sim \left\{ \begin{array}{l} + \frac{E_n \alpha_s^3}{3\pi} \left[\log\left(\frac{2\pi T}{E_1}\right)^2 - 2\gamma_E \right] \left\{ \frac{4 C_F^3 \delta_{l0}}{n} + N_c C_F^2 \left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n} \right] \right. \\ \left. + \frac{2 N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4} \right\} + \frac{2 E_n C_F^3 \alpha_s^3}{3\pi} L_{n,l} \end{array} \right.$$

$$+ \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right] \left\{ - \left[\frac{3}{2\pi} \zeta(3) + \frac{\pi}{3} \right] C_F \, \alpha_s \, T \, m_D^2 + \frac{2}{3} \zeta(3) \, N_c C_F \, \alpha_s^2 \, T^3 \right\}$$

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$$m\alpha_s^6 \frac{T^3}{E^3} \sim \; + \frac{a_0^2 n^2}{2}\left[5n^2 + 1 - 3l(l+1)\right]\left\{ - \left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3}\right] C_F\,\alpha_s\, T\, m_D^2 + \frac{2}{3}\zeta(3)\, N_c C_F\, \alpha_s^2\, T^3 \right\}$$

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$$m\alpha_s^6 \frac{T^3}{E^3} \sim \; + \frac{a_0^2 n^2}{2} \left[5n^2 + 1 - 3l(l+1)\right] \left\{ - \left[\frac{3}{2\pi}\zeta(3) + \frac{\pi}{3} \right] C_F \, \alpha_s \, T \, m_D^2 \right. \\ \left. + \frac{2}{3}\zeta(3) \, N_c C_F \, \alpha_s^2 \, T^3 \right\}$$

- The leading terms thus indicate that the mass of the bound state increases quadratically with the temperature.

- Up to this order the thermal shift to the spectrum is independent of the spin

The thermal width

- The contribution of the thermal medium to the width is

$$
\begin{aligned}
\Gamma_{n,l} &= \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3 T}{n^2}(C_F+N_c) \\
&+ \frac{2E_n\alpha_s^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\} \\
&- \left[\frac{C_F}{6}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\; N_c C_F\,\alpha_s^2\,T^3\right] \\
&\qquad\qquad\qquad\qquad\qquad\qquad \times\, a_0^2 n^2\left[5n^2 + 1 - 3l(l+1)\right] \\
&+ \frac{8}{3}C_F\alpha_s\,T m_D^2\,a_0^2 n^4\,I_{n,l}
\end{aligned}
$$

The thermal width

- The contribution of the thermal medium to the width is

$$\Gamma_{n,l} = \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3 T}{n^2}(C_F + N_c) + \frac{2E_n\alpha_s^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\}$$

$$- \left[\frac{C_F}{6}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\; N_c C_F\,\alpha_s^2\, T^3\right] \times a_0^2 n^2\left[5n^2 + 1 - 3l(l+1)\right] + \frac{8}{3}C_F\alpha_s\, T m_D^2\, a_0^2 n^4\, I_{n,l}$$

- Two sources of decay: singlet-to-octet thermal breakup and Landau damping

The thermal width

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$$\Gamma_{n,l} = \frac{1}{3}N_c^2 C_F \alpha_s^3 T + \frac{4}{3}\frac{C_F^2\alpha_s^3 T}{n^2}(C_F + N_c) \;\; \sim m\alpha_s^5 \frac{T}{E}$$
$$+\frac{2E_n\alpha_s^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\}$$
$$-\left[\frac{C_F}{6}\alpha_s T m_D^2\left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2\; N_c C_F\,\alpha_s^2\,T^3\right]$$
$$\times\, a_0^2 n^2\left[5n^2 + 1 - 3l(l+1)\right]$$
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$$\times\, a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1)\right]$$

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- Two sources of decay: singlet-to-octet thermal breakup and Landau damping

- In our hiearchy the former is larger than the latter

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$$m\alpha_s^5 \sim \;\; +\frac{2E_n\alpha_s^3}{3}\left\{\frac{4C_F^3\delta_{l0}}{n} + N_c C_F^2\left[\frac{8}{n(2l+1)} - \frac{1}{n^2} - \frac{2\delta_{l0}}{n}\right] + \frac{2N_c^2 C_F}{n(2l+1)} + \frac{N_c^3}{4}\right\}$$

$$m\alpha_s^6\frac{T^3}{E^3} \sim \begin{cases} -\left[\frac{C_F}{6}\alpha_s T m_D^2 \left(\ln\frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2\frac{\zeta'(2)}{\zeta(2)}\right) + \frac{4\pi}{9}\ln 2 \; N_c C_F \, \alpha_s^2 \, T^3\right] \\ \qquad\qquad \times \, a_0^2 n^2 \left[5n^2 + 1 - 3l(l+1)\right] \\ +\frac{8}{3}C_F\alpha_s \, T m_D^2 \, a_0^2 n^4 \, I_{n,l} \end{cases}$$

- Two sources of decay: singlet-to-octet thermal breakup and Landau damping

- In our hiearchy the former is larger than the latter

- Up to this order the width as well is independent of the spin

Conclusions

- The method: we have shown how to construct a series of EFT to integrate out in succession the many scales that characterize a non-relativistic bound state at finite temperature

- We have obtained the contribution to the singlet potential from the scale T, when $T \ll m\alpha_{\mathrm{s}}$

- Back to the structure: we have obtained the contribution of the thermal bath to the spectrum and the width of heavy quarkonia, in a scale setup that can be relevant for the ground state of bottomonium


PEANUTS
WHEW THAT'S TOO HARD WORK..
I THINK IF I WERE A SALMON, I'D STICK TO SWIMMING DOWNSTREAM!
10-17