474th International Wilhelm und Else Heraeus Seminar
Strong interactions: From methods to structures
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Three-boson systems near a Feshbach resonance in a two-channel zero-range model

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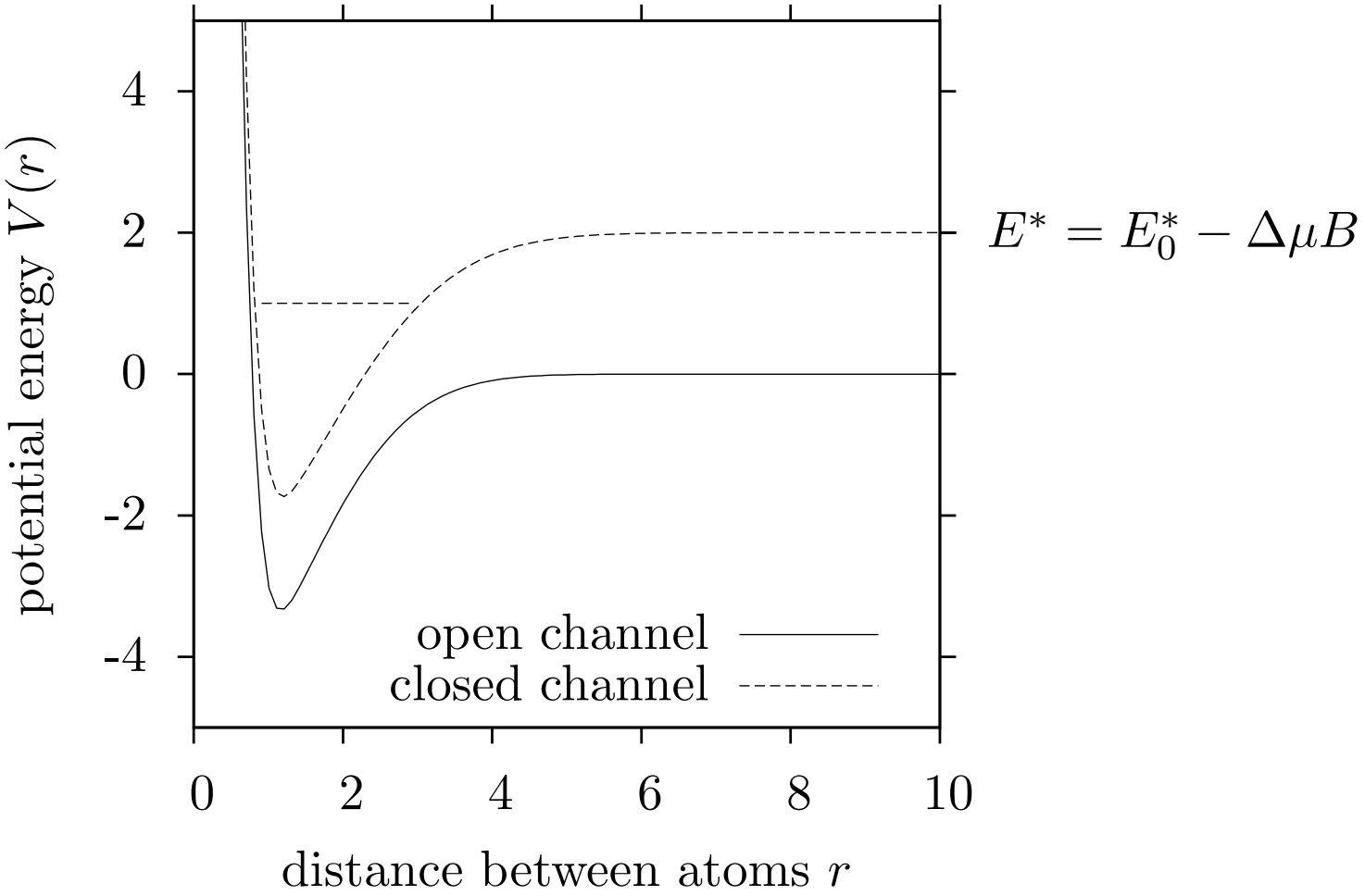
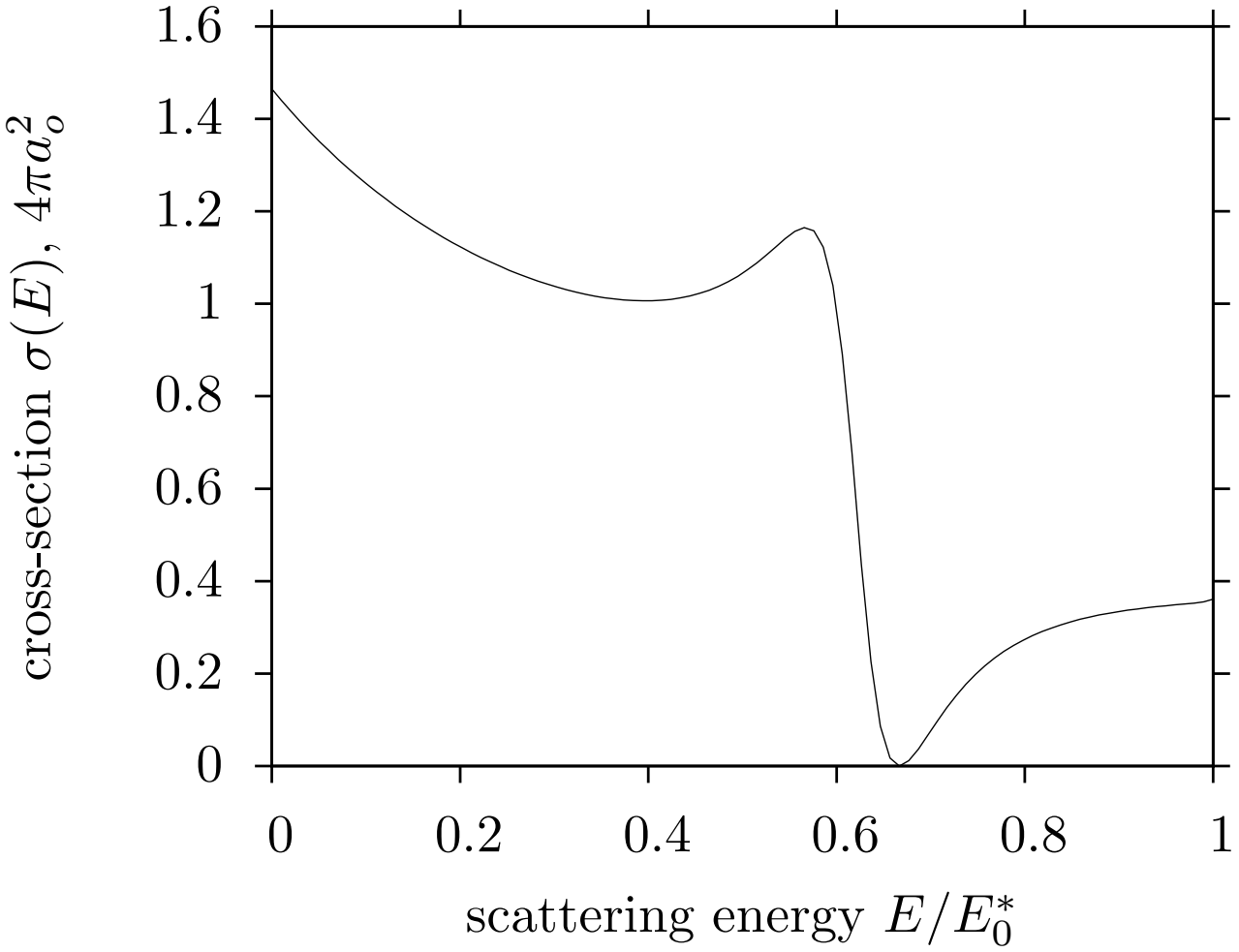

potential energy $V(r)$
4
2
0
-2
-4
0
2
4
6
8
10
distance between atoms r
open channel
closed channel
$E^* = E_0^* - \Delta\mu B$

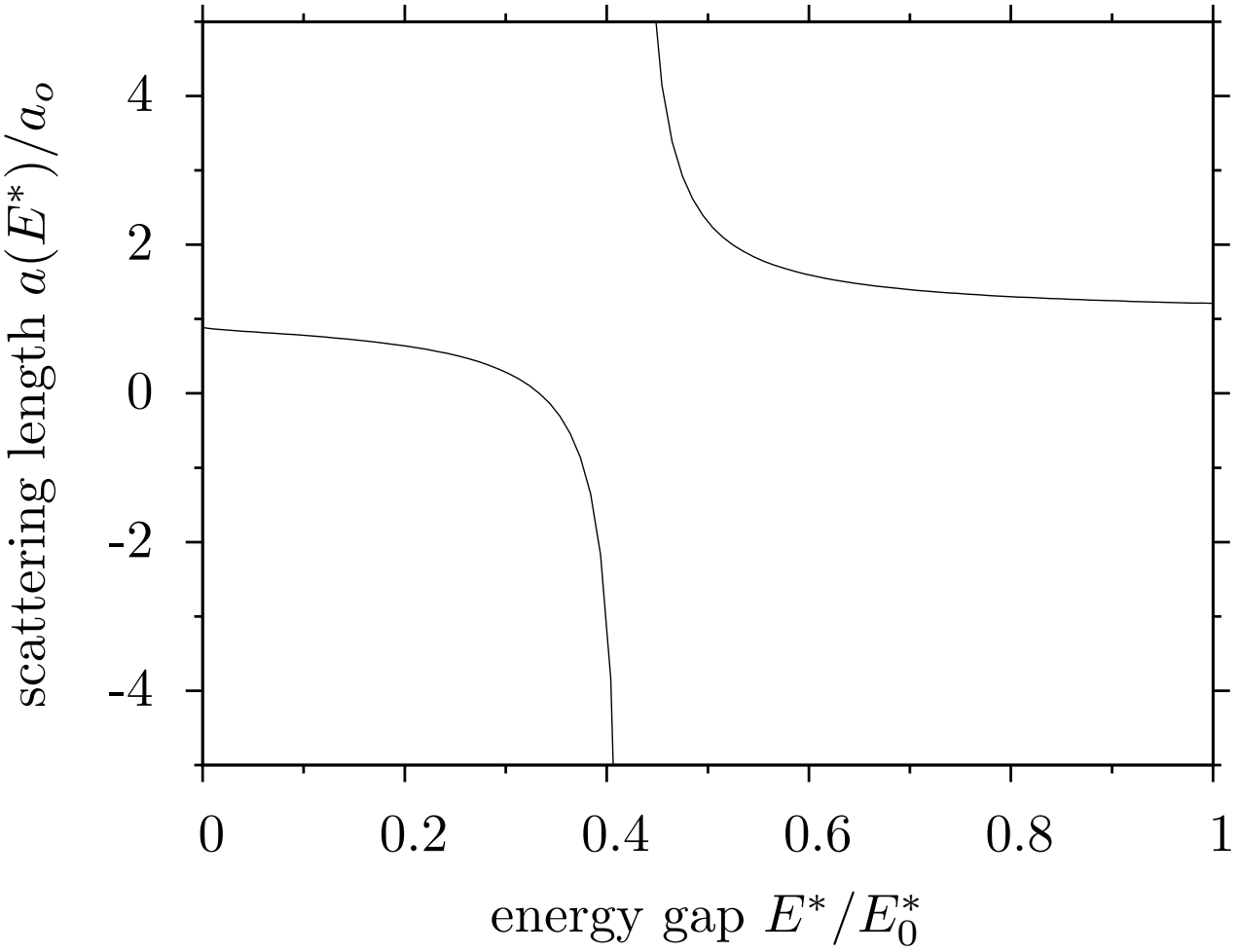
$$\psi(r) = \frac{1}{r} \begin{bmatrix} u_c(r) \\ u_o(r) \end{bmatrix}$$

$$-\frac{\hbar^2}{2m} u_o'' = E u_o \ ,$$

$$-\frac{\hbar^2}{2m} u_c'' + E^* u_c = E u_c \ ,$$

$$\begin{cases} u_o'(0) = \frac{1}{a_o} u_o(0) + \beta u_c(0) \\ u_c'(0) = \frac{1}{a_c} u_c(0) + \beta u_o(0) \end{cases}$$


1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
cross-section $\sigma(E)$, $4\pi a_o^2$
scattering energy E/E_0^*


scattering length $a(E^*)/a_o$
4
2
0
-2
-4
0
0.2
0.4
0.6
0.8
1
energy gap E^*/E_0^*

$$\rho^2 = \frac{1}{m}\sum_i m_i r_i^2$$

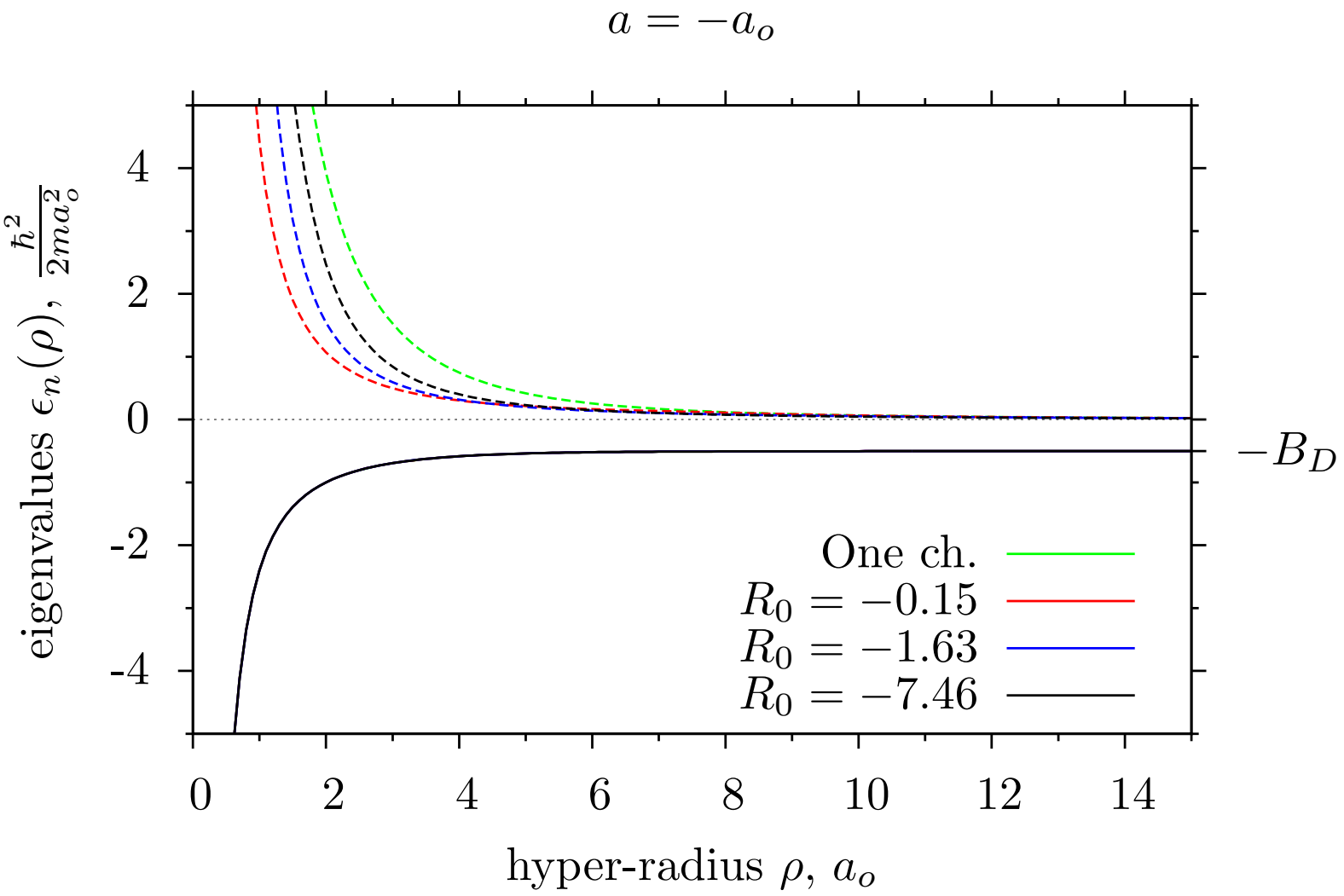
$$\hat{H}\Big|_\rho \Phi_n = \epsilon_n(\rho)\Phi_n$$

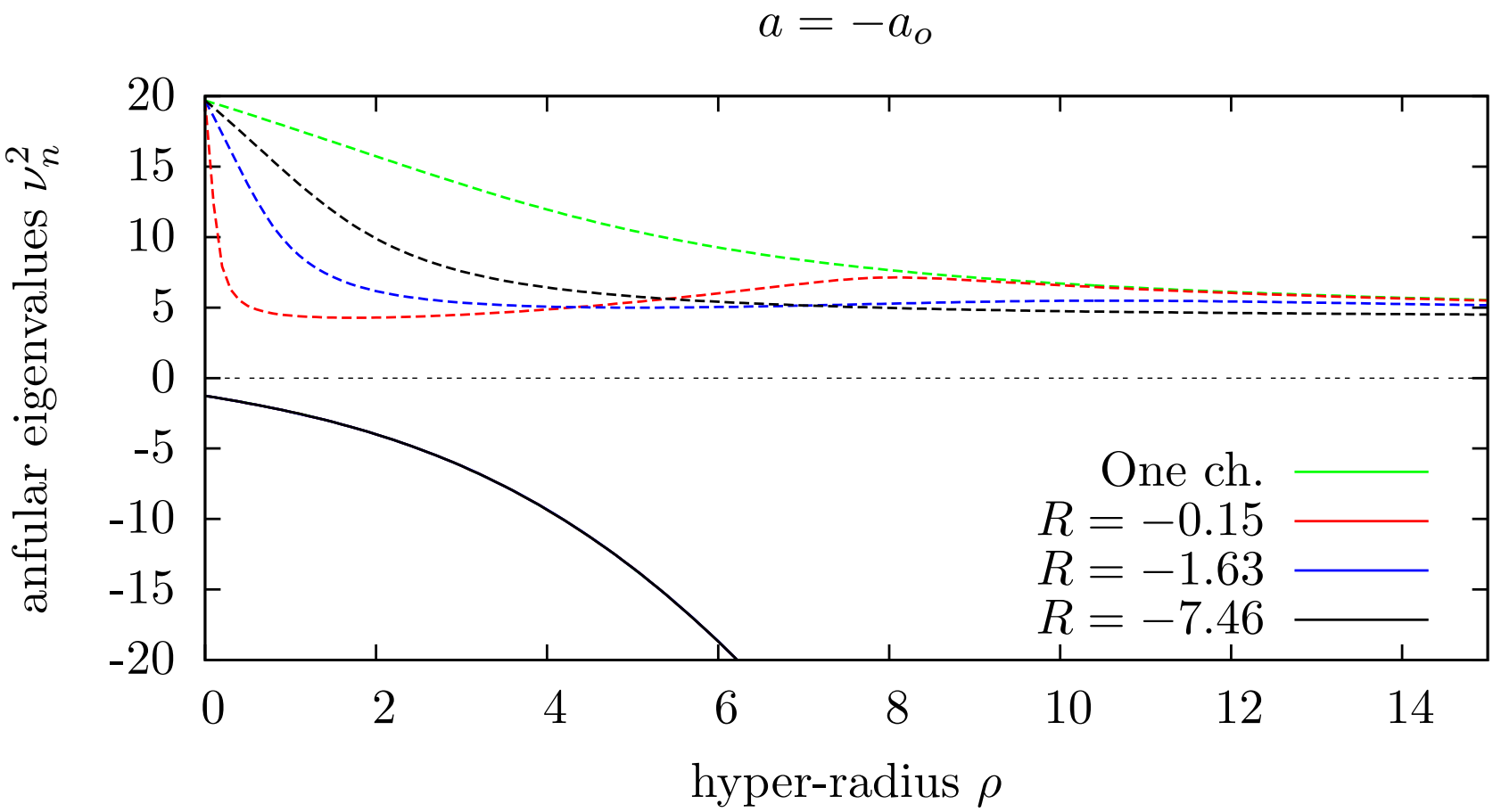
$$\Psi = \frac{1}{\rho^{5/2}}\sum_n f_n(\rho)\Phi_n$$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\rho^2} + \frac{\hbar^2}{2m}\frac{15}{4}\frac{1}{\rho^2} + \epsilon_n(\rho) - E\right) f_n = \frac{\hbar^2}{2m}\sum_{n'}\hat{Q}_{nn'}f_{n'}$$

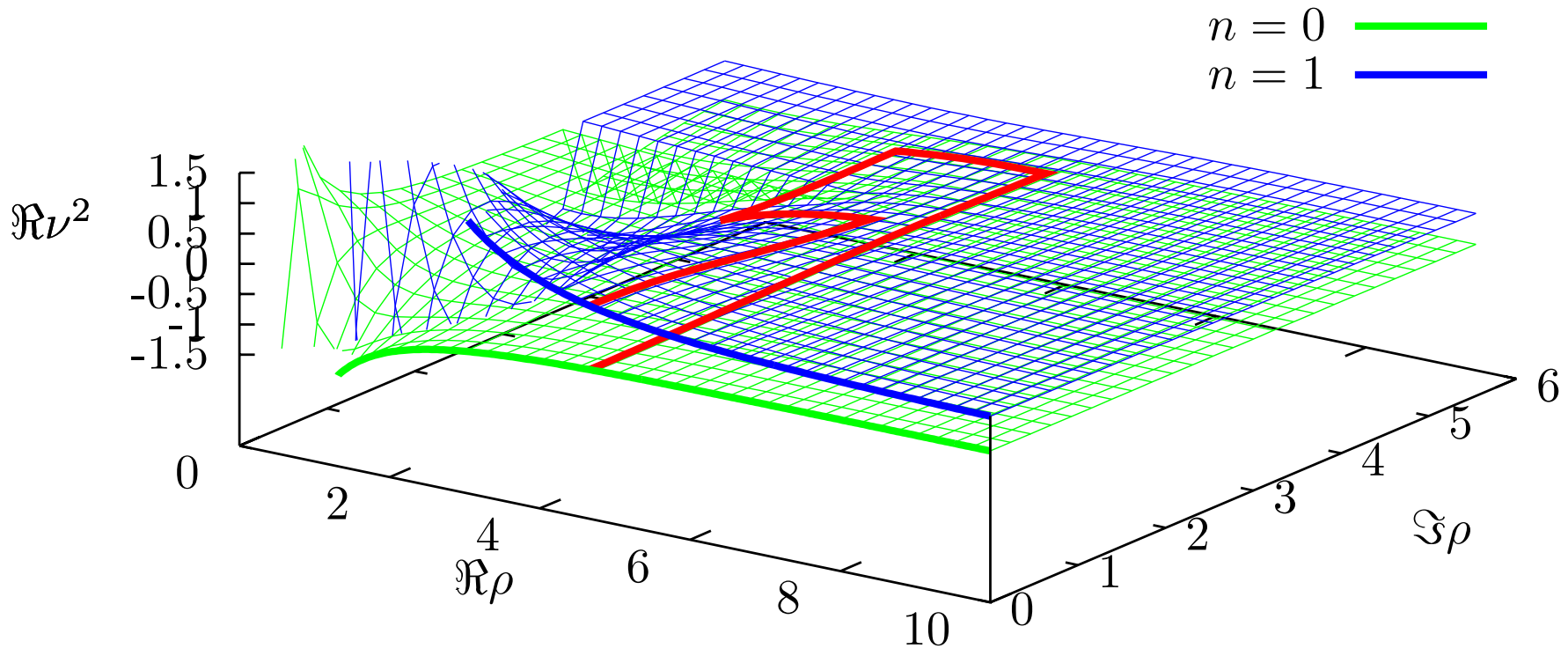
$$\left(-\frac{\partial^2}{\partial\rho^2} + \frac{\nu^2 - \frac{1}{4}}{\rho^2} - k^2\right) f_n = \sum_{n'}\hat{Q}_{nn'}f_{n'}$$

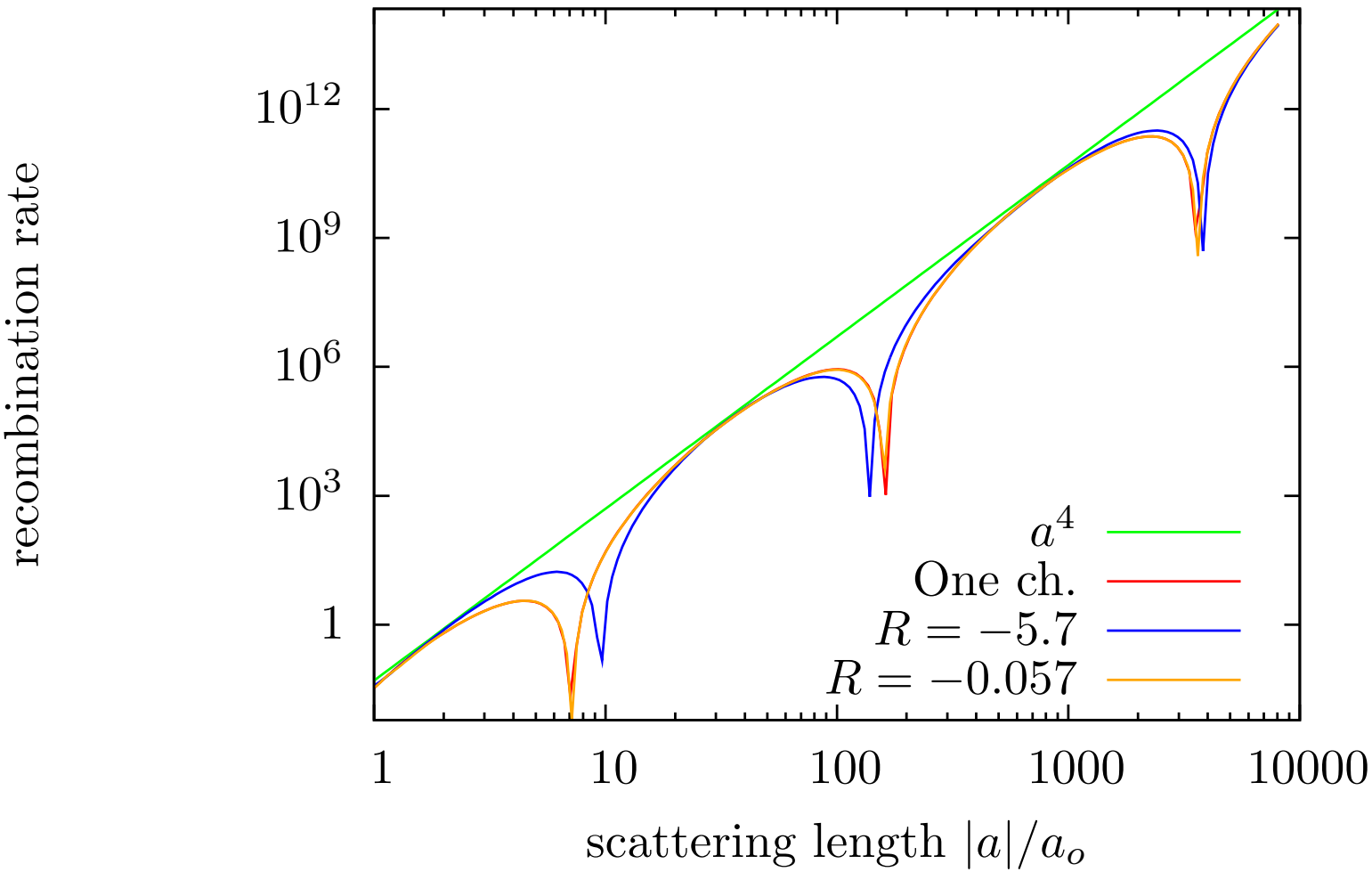
$$\frac{-\nu\cos\left(\nu\frac{\pi}{2}\right)+\frac{8}{\sqrt{3}}\sin\left(\nu\frac{\pi}{6}\right)}{\sin\left(\nu\frac{\pi}{2}\right)}=\frac{\rho}{a}$$


$a = -a_o$
eigenvalues $\epsilon_n(\rho)$, $\frac{\hbar^2}{2ma_o^2}$
4
2
0
-2
-4
0
2
4
6
8
10
12
14
hyper-radius ρ, a_o
$-B_D$
One ch.
$R_0 = -0.15$
$R_0 = -1.63$
$R_0 = -7.46$


$a = -a_o$
anfular eigenvalues ν_n^2
20
15
10
5
0
-5
-10
-15
-20
0
2
4
6
8
10
12
14
hyper-radius ρ
One ch.
$R = -0.15$
$R = -1.63$
$R = -7.46$

$$\text{Rate} \propto e^{-2\text{Im}S} \sin^2(2\text{Re}S)\,, \quad S = \int_c d\rho \sqrt{k^2 - \nu^2(\rho)/\rho^2}$$




recombination rate
10^{12}
10^{9}
10^{6}
10^{3}
1
1
10
100
1000
10000
scattering length $|a|/a_o$
a^4
One ch.
$R = -5.7$
$R = -0.057$