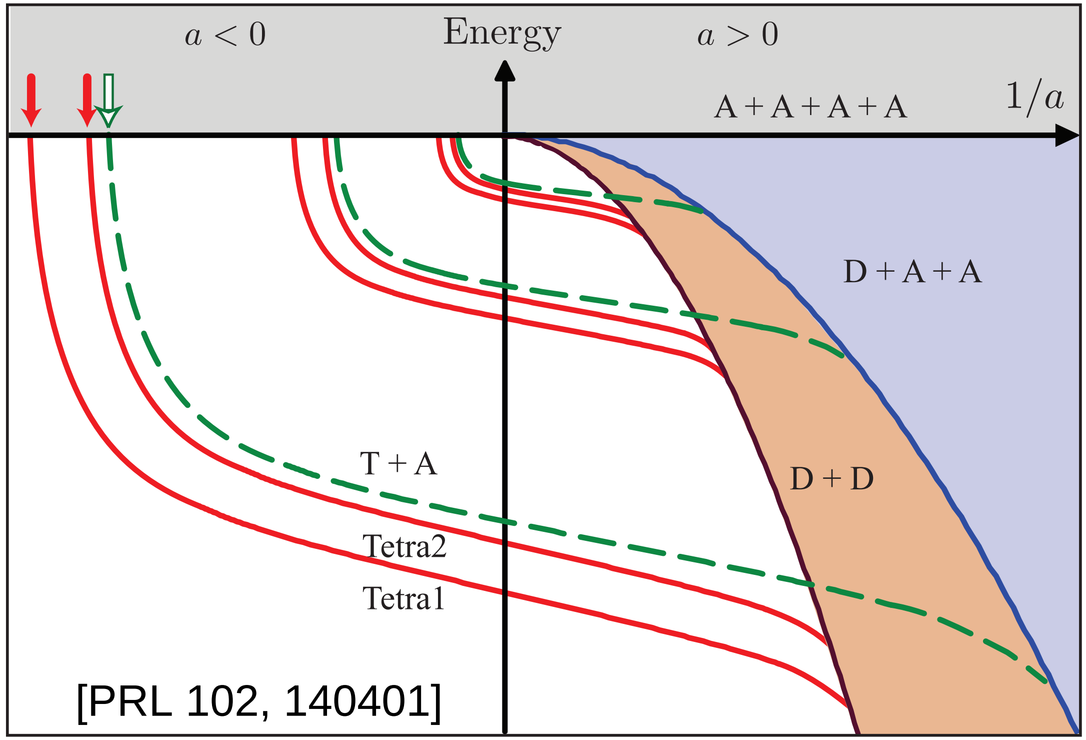
Universality in four-body scattering

A. Deltuva

Centro de Física Nuclear da Universidade de Lisboa

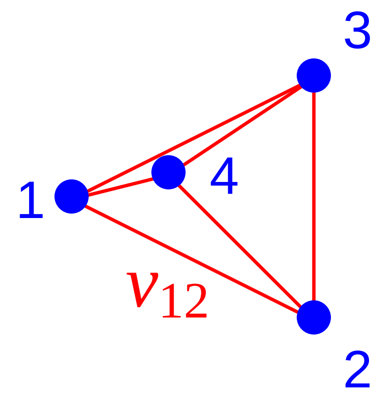
Collaborators: R. Lazauskas and L. Platter

Four-boson Efimov physics



- momentum-space scattering equations
- atom-trimer scattering
- unstable tetramers

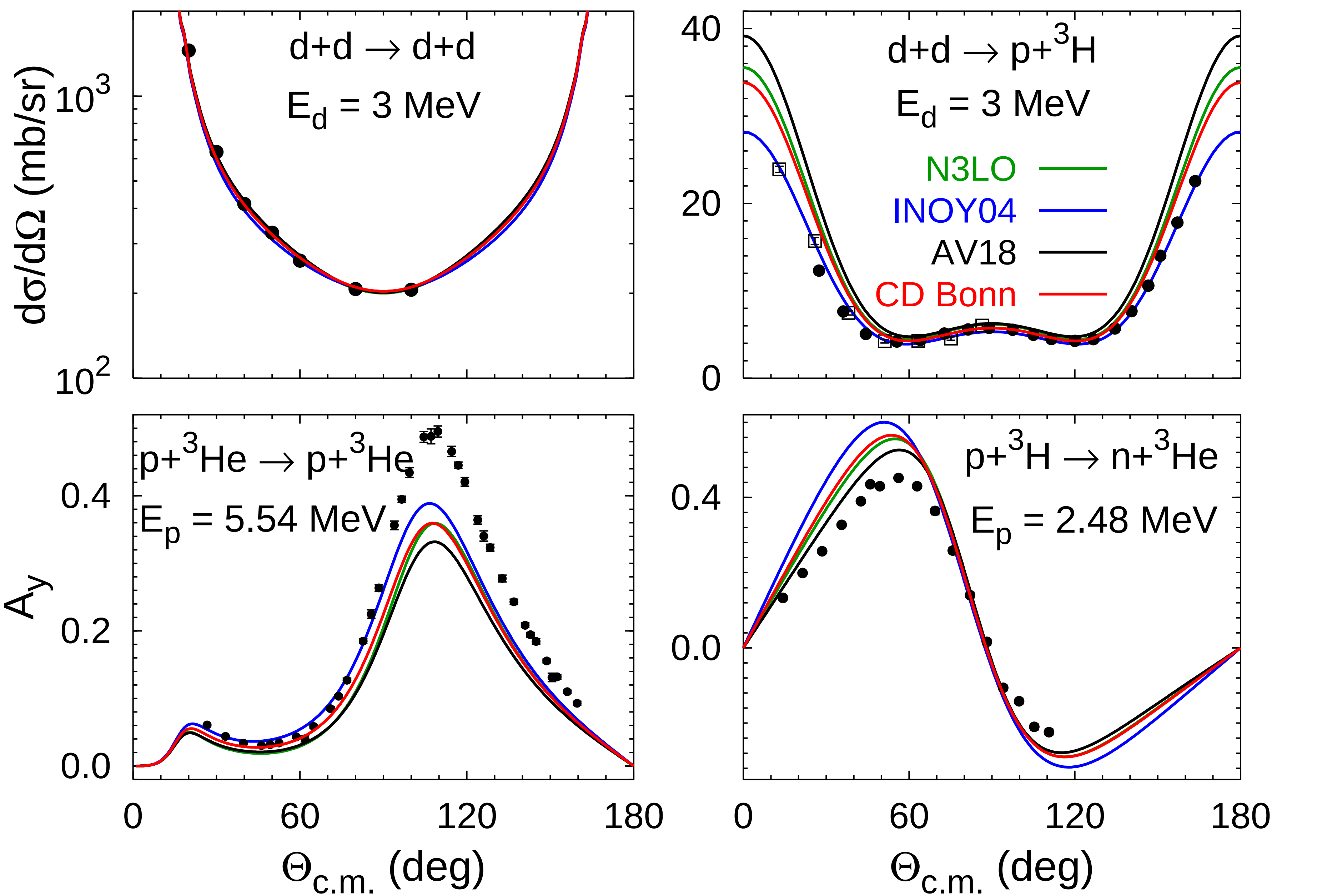
Four-particle scattering



Hamiltonian $H_0 + \sum_{i>j} v_{ij}$

- Wave function:
  Faddeev-Yakubovsky equations
- Transition operators:
  Alt-Grassberger-Sandhas equations

Fermionic system: 4N reactions



[PRC 75, 014005; PRL 98, 162502; PRC 76, 021001 (2007)]

Symmetrized bosonic AGS equations

$$t = v + vG_0 t$$

$$u_j = P_j G_0^{-1} + P_j t G_0 u_j$$

$$3+1: \quad P_1 = P_{12}P_{23} + P_{13}P_{23}$$

$$2+2: \quad P_2 = P_{13}P_{24}$$

$$U_{11} = (G_0 t G_0)^{-1} P_{34} + P_{34} u_1 G_0 t G_0 U_{11} + u_2 G_0 t G_0 U_{21}$$

$$U_{21} = (G_0 t G_0)^{-1}(1 + P_{34}) + (1 + P_{34}) u_1 G_0 t G_0 U_{11}$$

$$U_{12} = (G_0 t G_0)^{-1} + P_{34} u_1 G_0 t G_0 U_{12} + u_2 G_0 t G_0 U_{22}$$

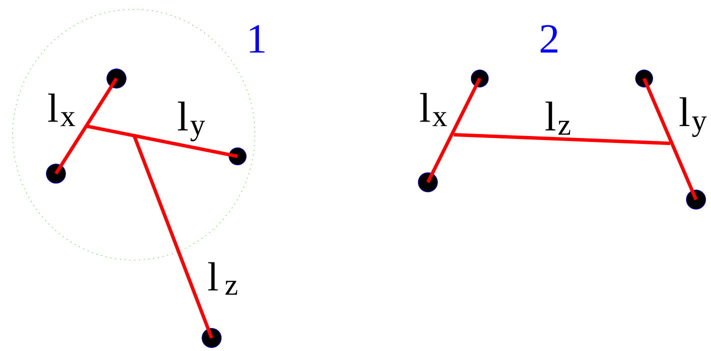
$$U_{22} = (1 + P_{34}) u_1 G_0 t G_0 U_{12}$$

scattering amplitude $\quad T_{fi} = s_{fi} \langle \phi_f | U_{fi} | \phi_i \rangle$

$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$

Solution of AGS equations

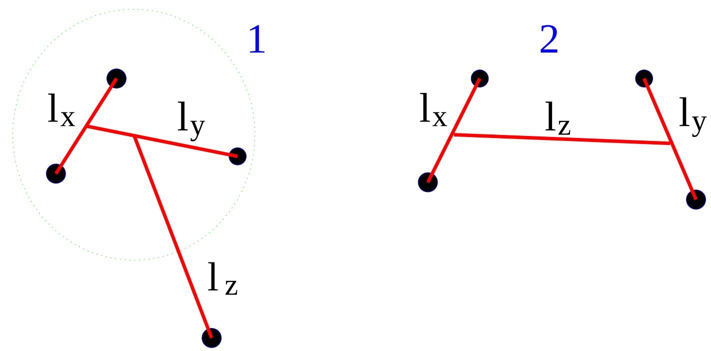
$$U_{11}|\phi_1\rangle = G_0^{-1}P_{34}P_1|\phi_1\rangle + P_{34}u_1G_0\,t\,G_0\,U_{11}|\phi_1\rangle + u_2G_0\,t\,G_0\,U_{21}|\phi_1\rangle$$



- momentum-space partial-wave basis, two types, partially symmetrized: $|k_xk_yk_z[(l_xl_y)Sl_z]J\rangle_1$ & $|k_xk_yk_z[(l_xl_y)Sl_z]J\rangle_2$
- set of coupled integral equations in 3 variables: ${}_j\langle k_xk_yk_z[(l_xl_y)Sl_z]J|U_{ji}|\phi_i\rangle$

Solution of AGS equations

$$U_{11}|\phi_1\rangle = G_0^{-1} P_{34} P_1 |\phi_1\rangle + P_{34} u_1 G_0 t G_0 U_{11} |\phi_1\rangle + u_2 G_0 t G_0 U_{21} |\phi_1\rangle$$



- momentum-space partial-wave basis,
  two types, partially symmetrized:
  $|k_x k_y k_z [(l_x l_y) S l_z] J\rangle_1$ & $|k_x k_y k_z [(l_x l_y) S l_z] J\rangle_2$
- set of coupled integral equations in 3 variables:
  $_j\langle k_x k_y k_z [(l_x l_y) S l_z] J | U_{ji} | \phi_i\rangle$
- subsystem bound state poles:
  $u_j(E_j \to -B_j) \to P_j G_0^{-1} |\phi_j\rangle s_{jj} (E_j + i0 + B_j)^{-1} \langle \phi_j | G_0^{-1} P_j$
- real equations below 3b breakup threshold (K-matrix):
  $u_j \to \mathcal{P} u_j$: $U_{ji} \to K_{ji}$

Separable potential

- universal — independent of short-range details

$$v = |g\rangle \, \lambda \, \langle g| \; \delta_{l_x 0}$$

$$t = |g\rangle \, \tau \, \langle g| \; \delta_{l_x 0}$$

$$\langle k_x | g \rangle = [1 + c_2 (k_x/\Lambda)^2] e^{-(k_x/\Lambda)^2}$$

- small system of 2-variable integral equations for ${}_j\langle g k_y k_z [(l_x l_y) S l_z] J | G_0 K_{ji} | \phi_i \rangle$:

$$\begin{aligned} \langle g | G_0 K_{11} | \phi_1 \rangle = & \; P_{34} \langle g | P_1 | \phi_1 \rangle \\ & + P_{34} \mathcal{P} \, \langle g | G_0 u_1 G_0 | g \rangle \tau \langle g | G_0 K_{11} | \phi_1 \rangle \\ & + \mathcal{P} \, \langle g | G_0 u_2 G_0 | g \rangle \tau \langle g | G_0 K_{21} | \phi_1 \rangle \end{aligned}$$

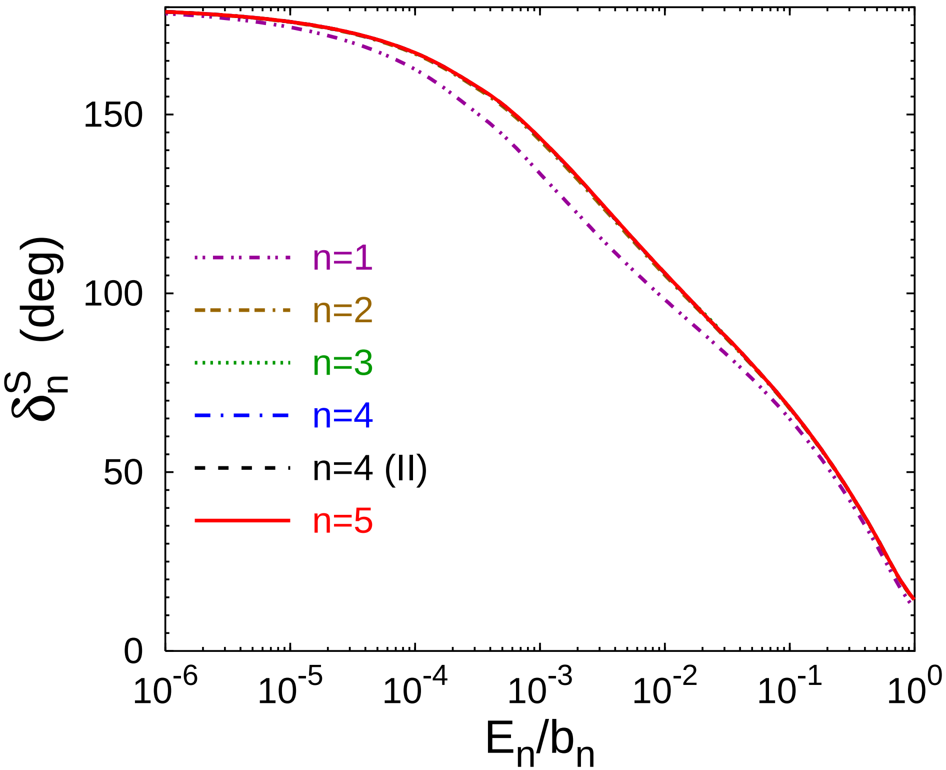
1+(3)$_n$ scattering length and effective range $(a \to \infty)$

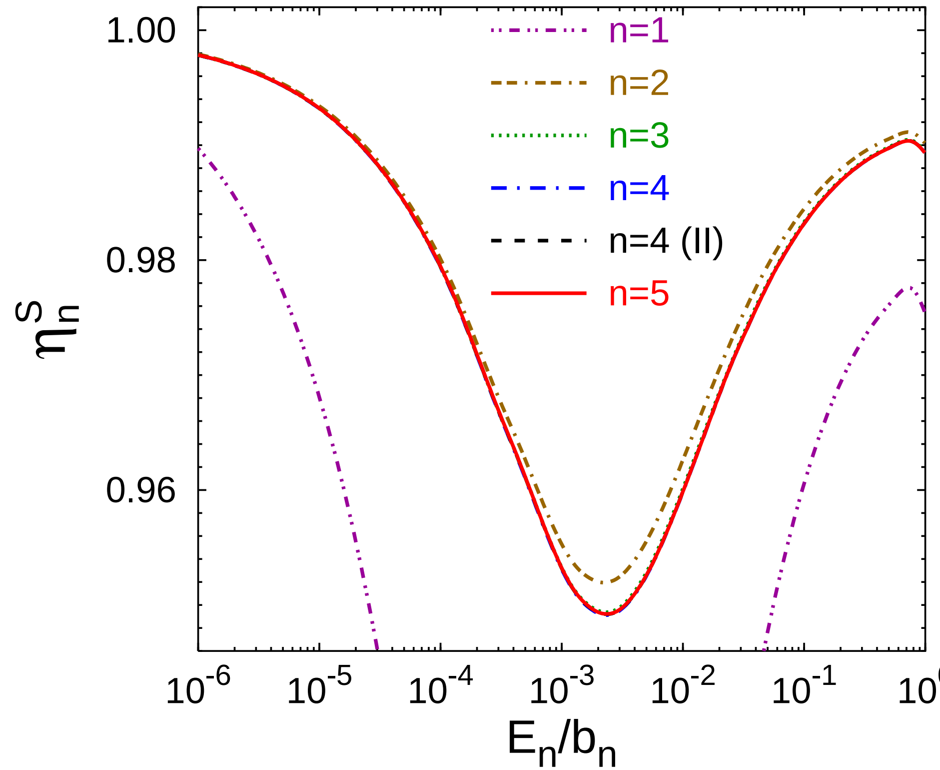
I. $c_2 = 0$ II. $c_2 = -9.17$

$$L_n = \hbar / \sqrt{2\mu_1 b_n}$$

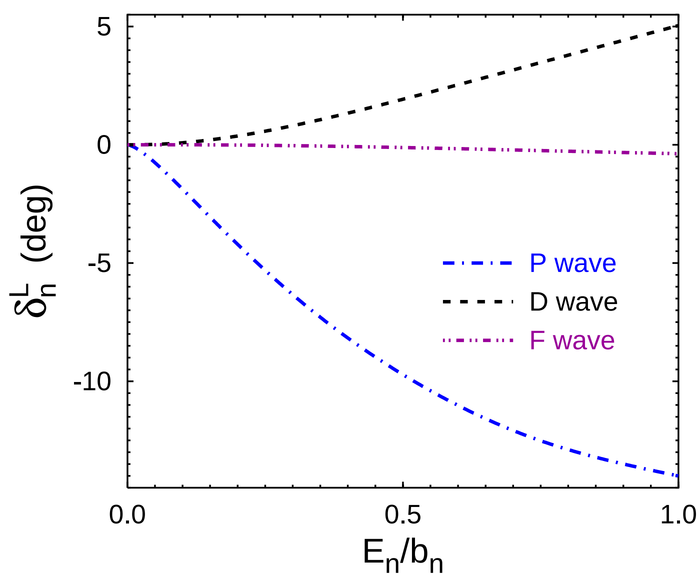
n	b_{n-1}/b_n	$\mathrm{Re}\,A_n/L_n$	$\mathrm{Im}\,A_n/L_n$	$\mathrm{Re}\,r_n/L_n$	$\mathrm{Im}\,r_n/L_n$
I. 0		6.79		2.81	
1	548.114	31.1	-5.18	3.35	-0.043
2	515.214	23.1	-1.05	3.23	-0.016
3	515.036	22.7	-1.08	3.22	-0.017
4	515.035	22.6	-1.09	3.22	-0.017
5	515.035	22.6	-1.09	3.22	-0.017
II. 1	2126.36	11.7	-0.21	2.93	-0.012
2	518.570	22.5	-1.52	3.22	-0.024
3	515.042	22.7	-1.09	3.22	-0.017
4	515.035	22.6	-1.09	3.22	-0.017

1+(3)$_n$ scattering: phase shifts and inelasticities $(a \to \infty)$



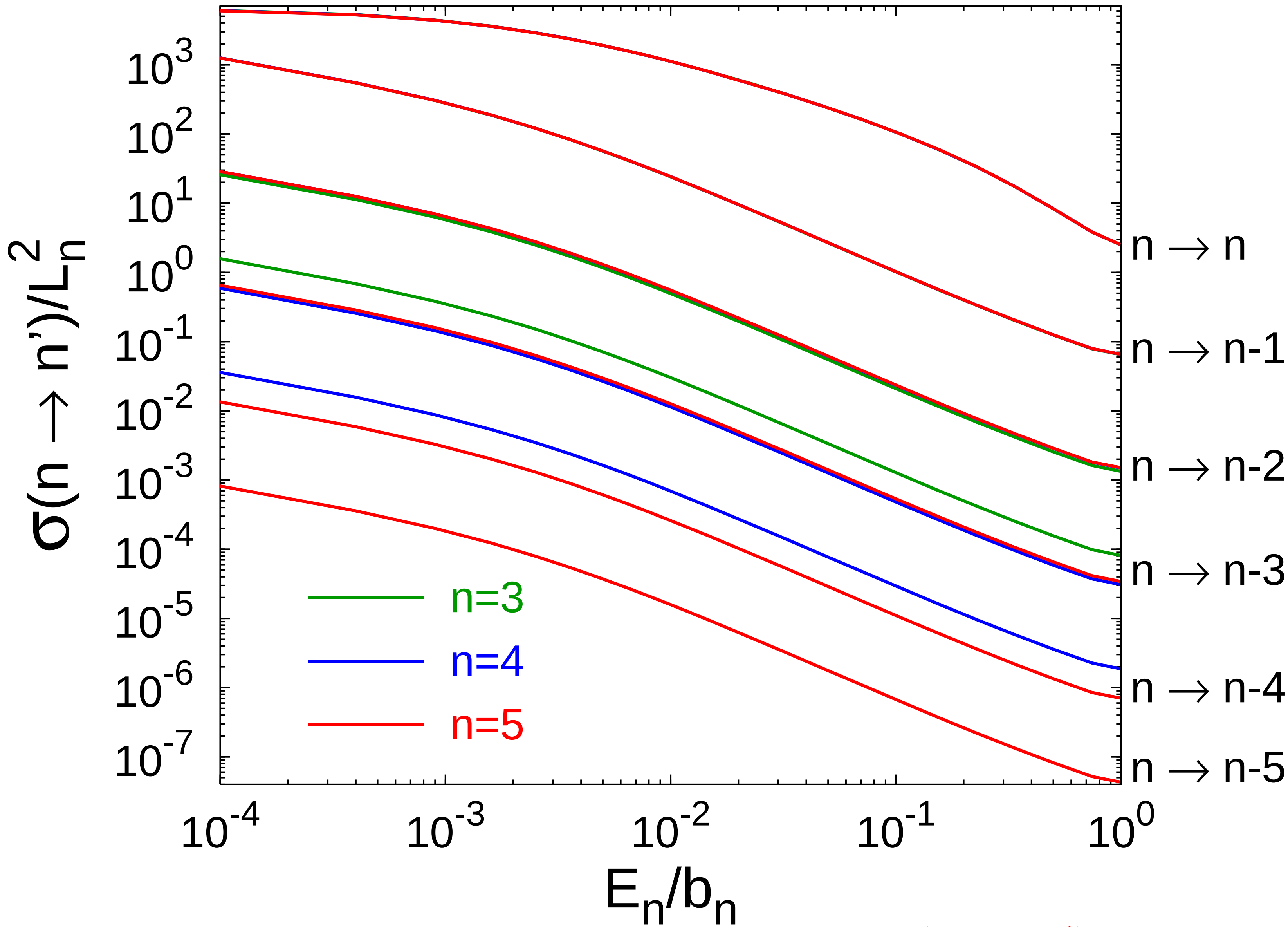


$$E_n = p_n^2/2\mu_1$$



$$1 - \eta_n^P < 10^{-6}$$

1+(3)$_n$ scattering: cross sections $(a \to \infty)$



$$\frac{\sigma(n \to n')}{\sigma(n \to n'\text{-}1)} \approx 43.7$$

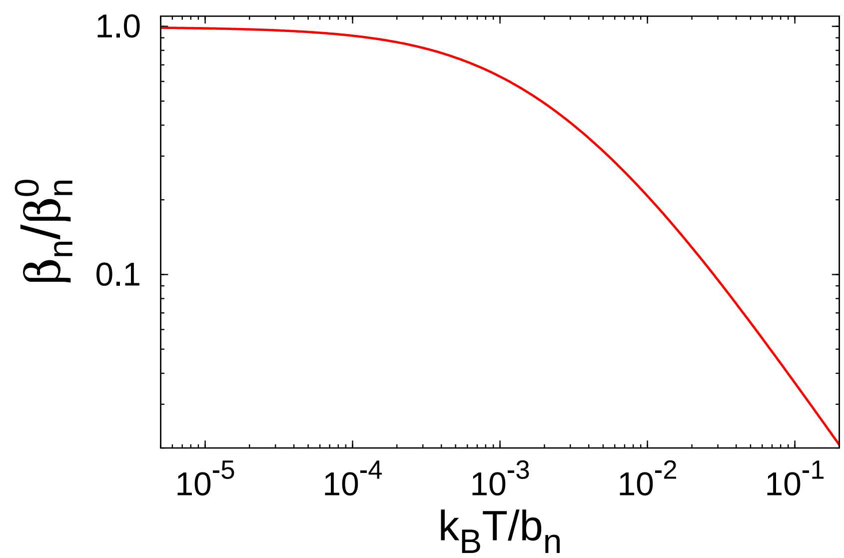
[PRA 82, 040701 (2010)]

Trimer relaxation

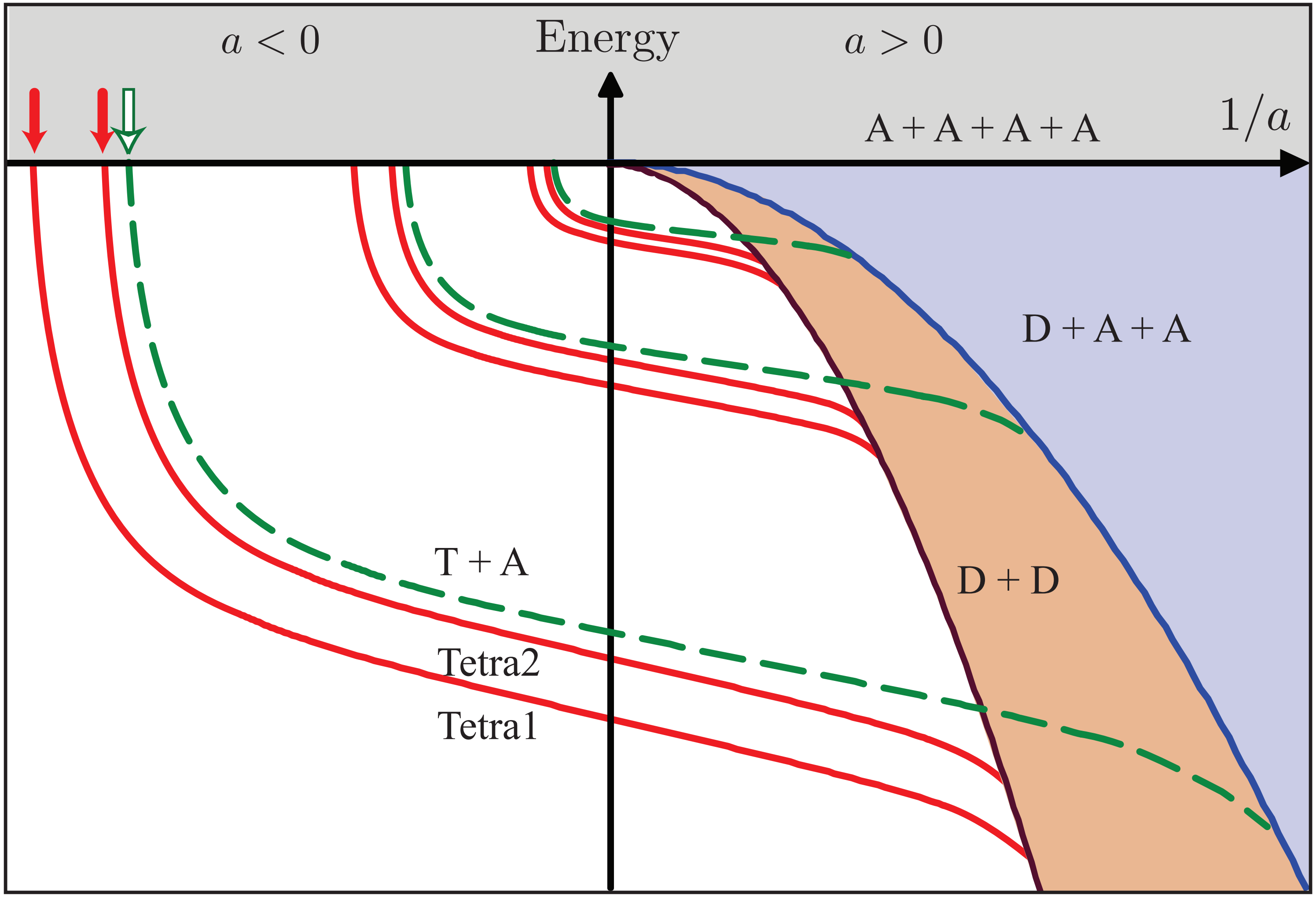
- Ultracold mixture of atoms and excited trimers in a trap with densities $\rho_a(0) >> \rho_n(0)$
- Loss of trimers via inelastic scattering
  $1+(3)_n \longrightarrow 1+(3)_{n'}$ if $(b_{n'} - b_n)/4 > V_{\mathrm{trap}}$
- Time evolution $\rho_n(t) = \rho_n(0) e^{-\beta_n \rho_a(0) t}$
- Relaxation rate constant $\beta_n = (1/\mu_1) \sum_{n'} \langle p_n \sigma(n \rightarrow n') \rangle_T$

Trimer relaxation

- Ultracold mixture of atoms and excited trimers in a trap with densities $\rho_a(0) >> \rho_n(0)$
- Loss of trimers via inelastic scattering
  $1+(3)_n \rightarrow 1+(3)_{n'}$ if $(b_{n'} - b_n)/4 > V_{\mathrm{trap}}$
- Time evolution $\rho_n(t) = \rho_n(0)e^{-\beta_n \rho_a(0) t}$
- Relaxation rate constant $\beta_n = (1/\mu_1)\sum_{n'} \langle p_n \sigma(n \rightarrow n') \rangle_T$
- Zero temperature limit $\beta_n^0 = -(4\pi\bar{h}/\mu_1)\,\mathrm{Im}\,A_n$
- Finite temperature:



Unstable tetramers



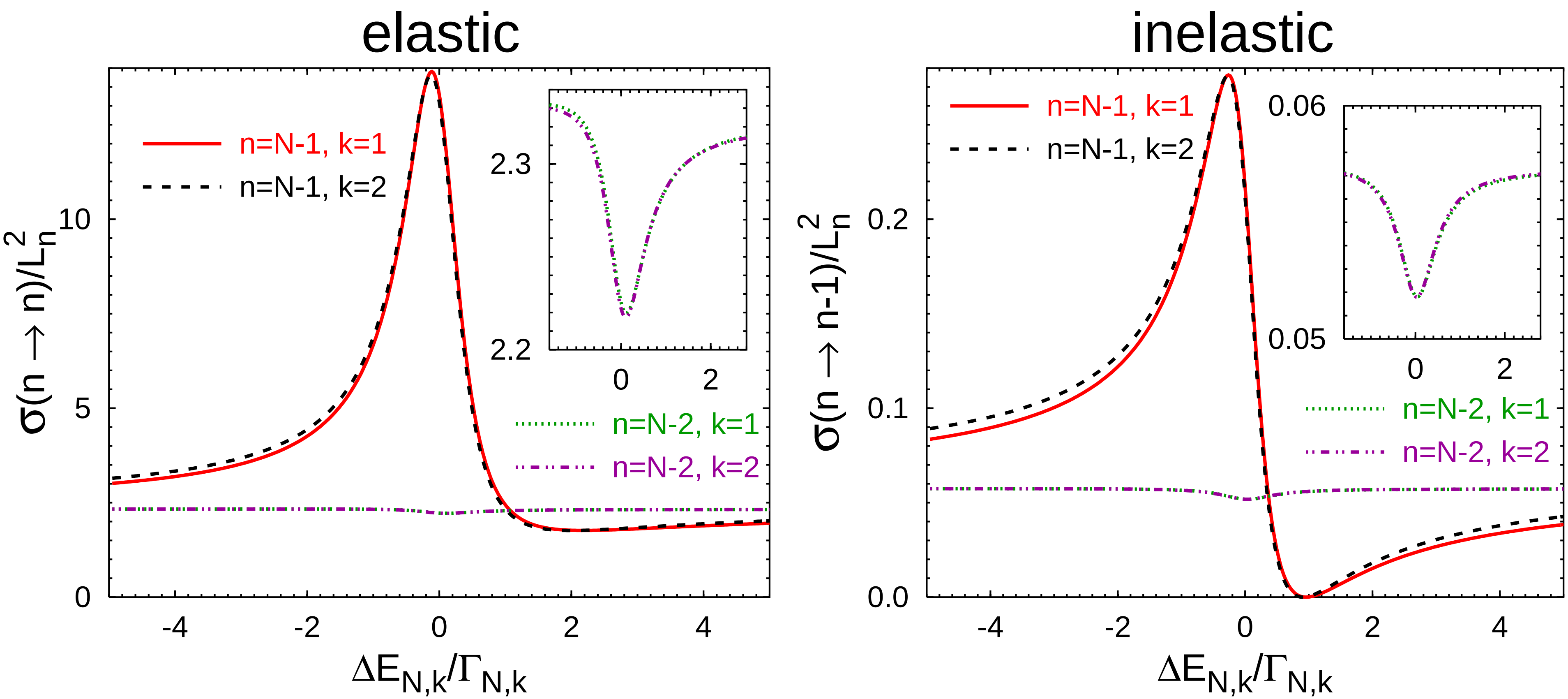
$$U_{ji} \approx \hat{U}_{ji}^{(-1)} (E - E_r)^{-1} + \hat{U}_{ji}^{(0)} + \dots, \quad E_r = -B_{N,k} - i\Gamma_{N,k}/2$$

Unstable tetramers $(a \to \infty)$

N	$B_{N,1}/b_N$	$\Gamma_{N,1}/2b_N$	$B_{N,2}/b_N$	$\Gamma_{N,2}/2b_N$
I. 1	4.5175	0.03364	1.00106	3.82×10^{-4}
2	4.6041	0.01367	1.00217	2.14×10^{-4}
3	4.6104	0.01472	1.00227	2.36×10^{-4}
4	4.6108	0.01485	1.00228	2.38×10^{-4}
5	4.6108	0.01484	1.00228	2.38×10^{-4}
II. 1	4.9929	0.01361	1.00997	4.18×10^{-4}
2	4.6114	0.02085	1.00228	3.34×10^{-4}
3	4.6104	0.01494	1.00227	2.39×10^{-4}
4	4.6108	0.01484	1.00228	2.38×10^{-4}
[UC]	4.58		1.01	

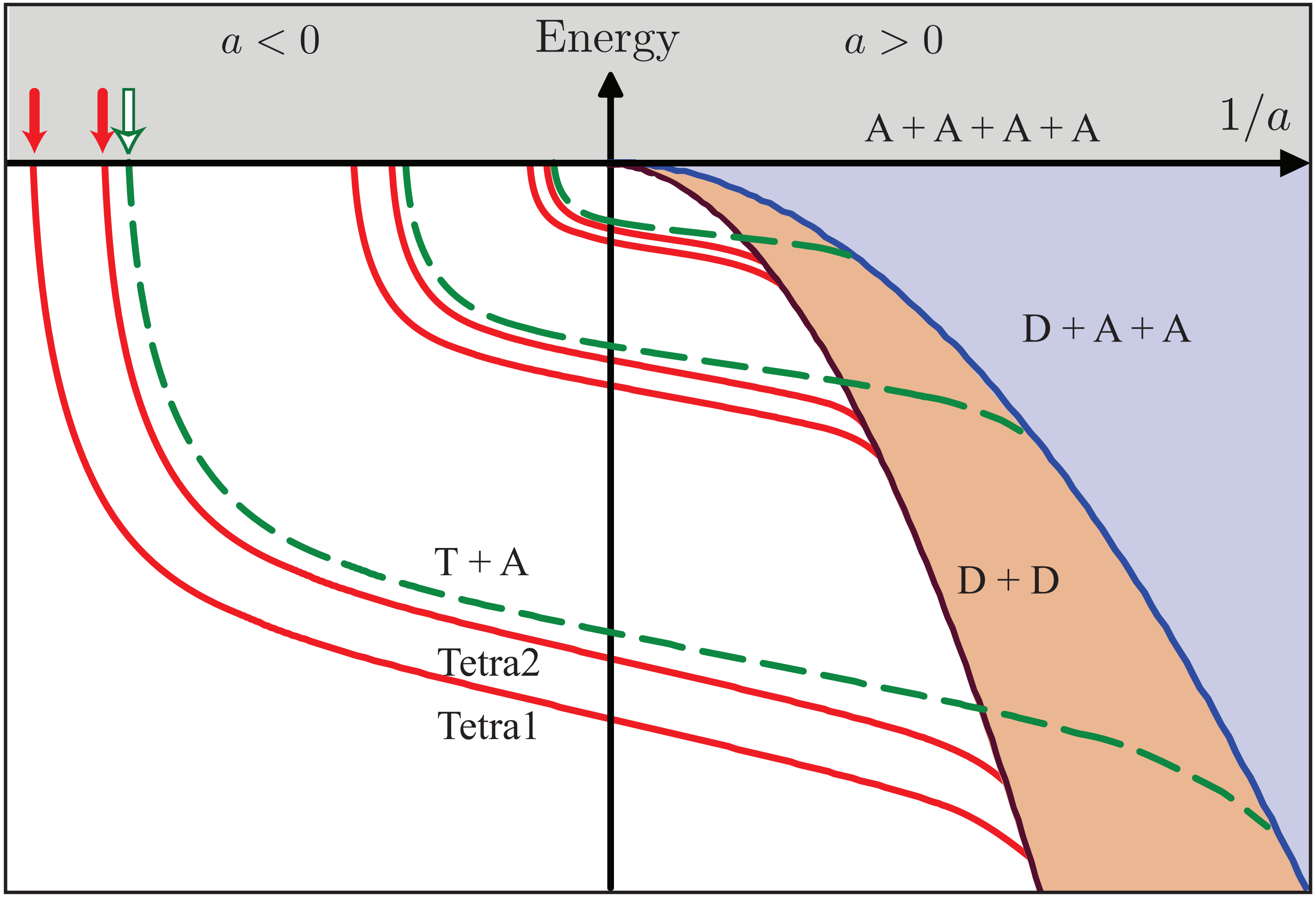
[UC: J. von Stecher et al., Nature Phys. 5, 417 (2009)]

1+(3)$_n$ scattering at $E \approx -B_{N,k} \quad (a \to \infty)$

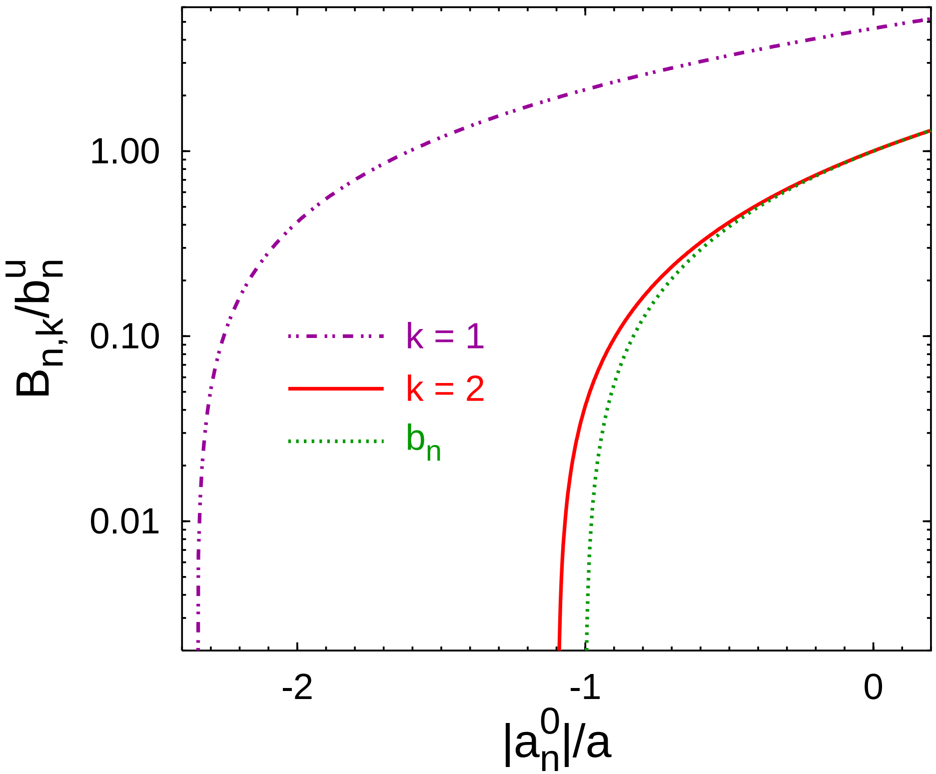


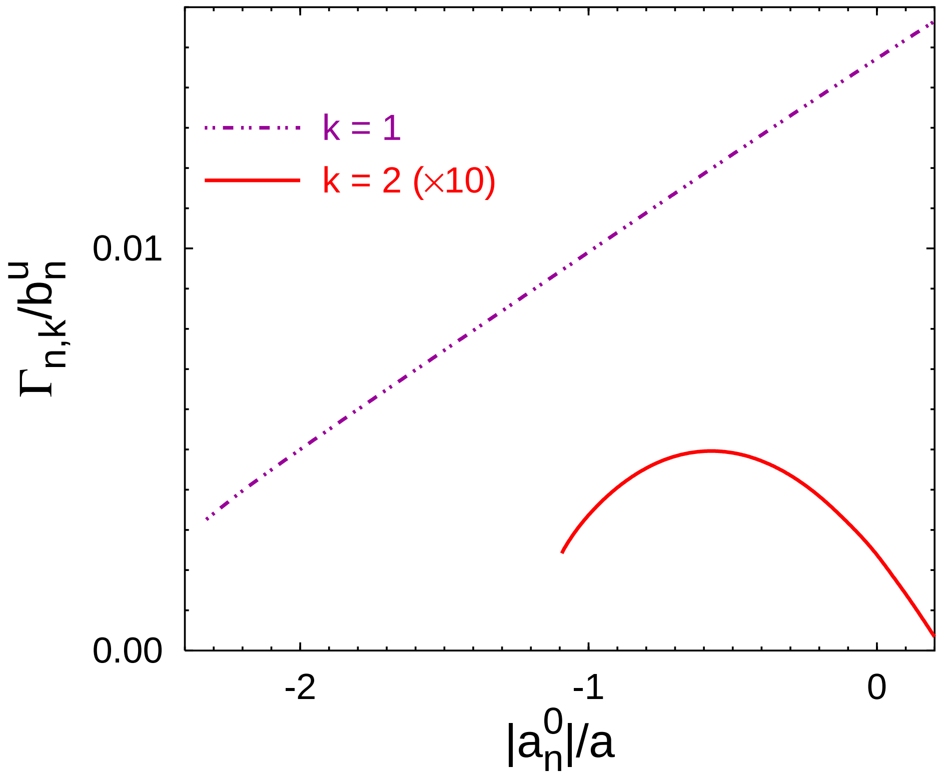
$$\Delta E_{N,k} = E_n - b_n + B_{N,k}$$

Unstable tetramers



Unstable tetramers $(a < 0)$

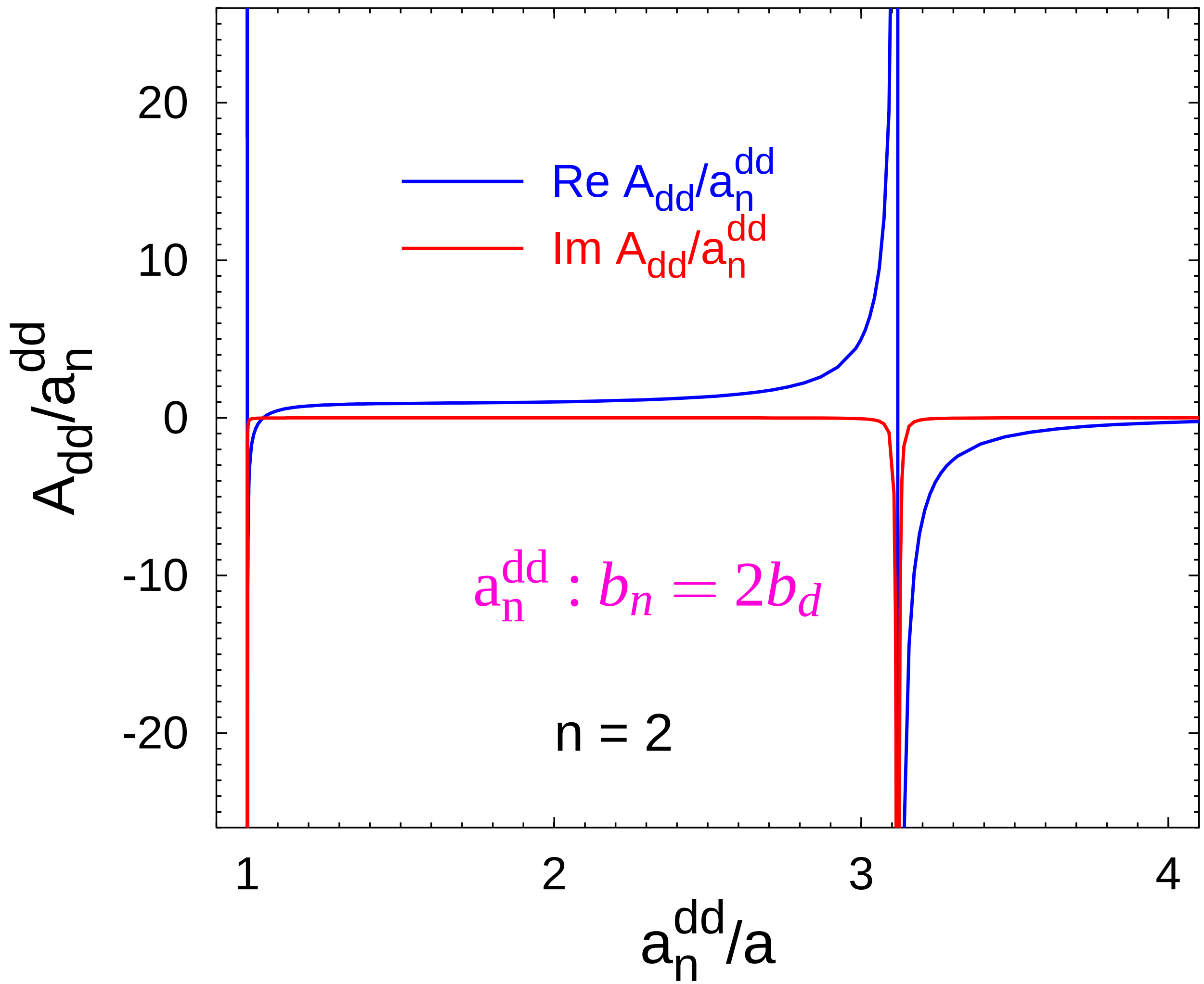




$a_n^0 : \; b_n = 0$ $\qquad$ $a_{n,k}^0 : \; B_{n,k} = 0$

	$a_{n,1}^0/a_n^0$	$a_{n,2}^0/a_n^0$	$a \longrightarrow \infty :$	$B_{n,1}/b_n$	$B_{n,2}/b_n$
	0.426	0.912		4.611	1.0023
[UC]	0.43	0.90		4.58	1.01

Unstable tetramers in (2)+(2) scattering

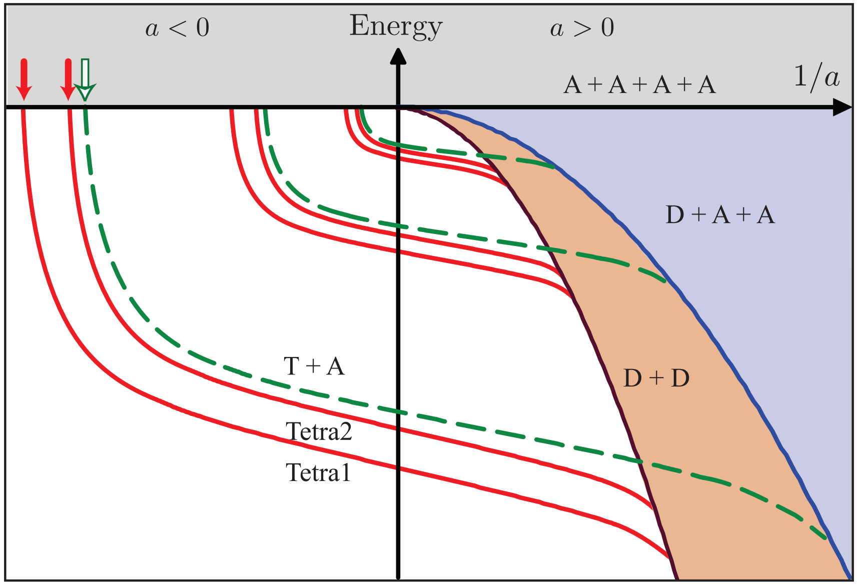


universal limit: $n > 2$ needed

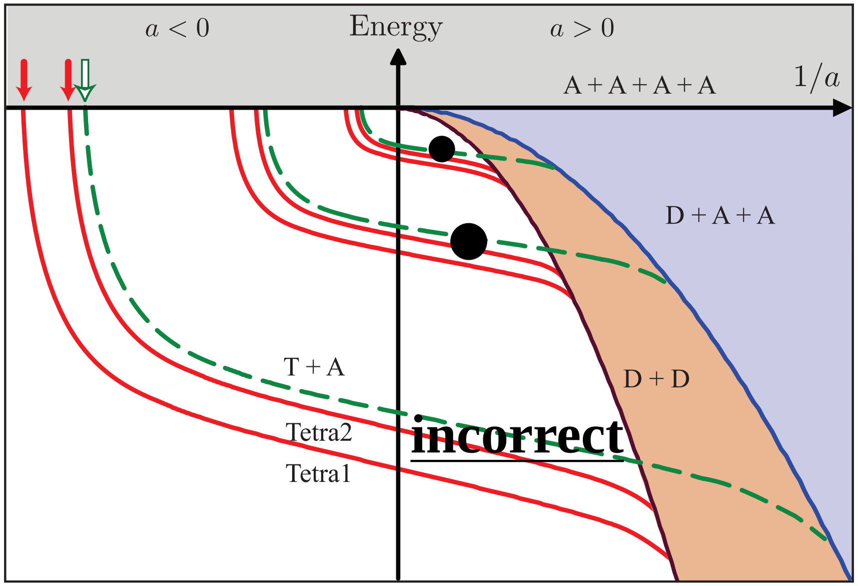
$\mathrm{Re}\,A_{dd}$ consistent with [UC: PRL 103, 033004 (2009)]

$\mathrm{Im}\,A_{dd} \Longrightarrow$ dimer-dimer relaxation rate constant at $T = 0$

Unstable tetramers



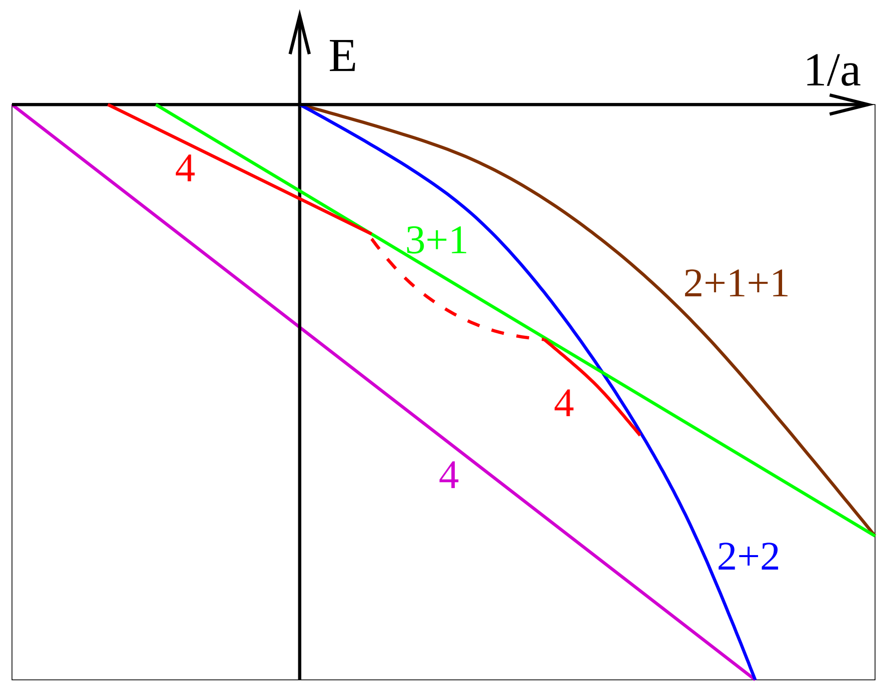
Unstable tetramers



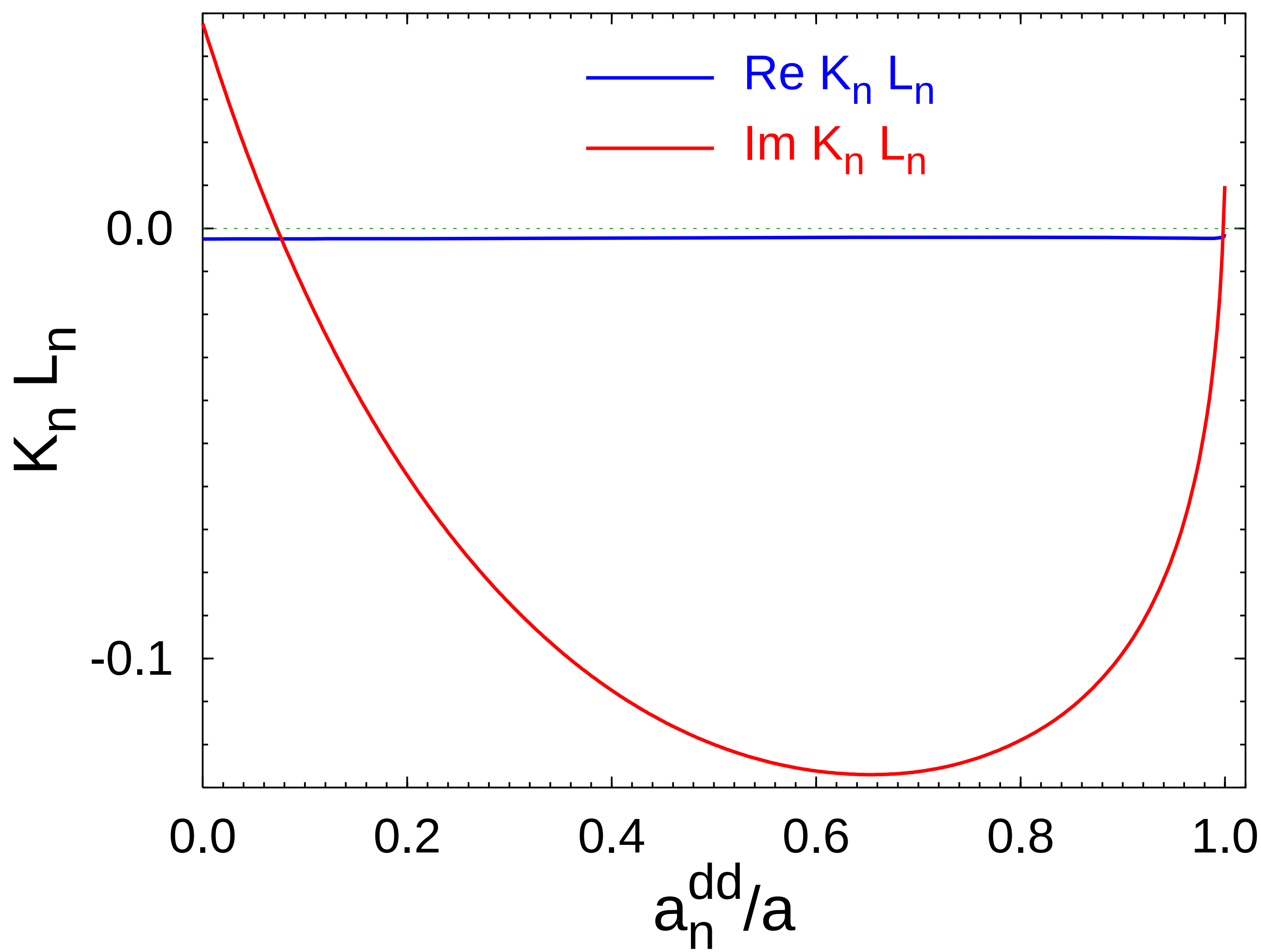
unstable bound state

↑↓

inelastic virtual state

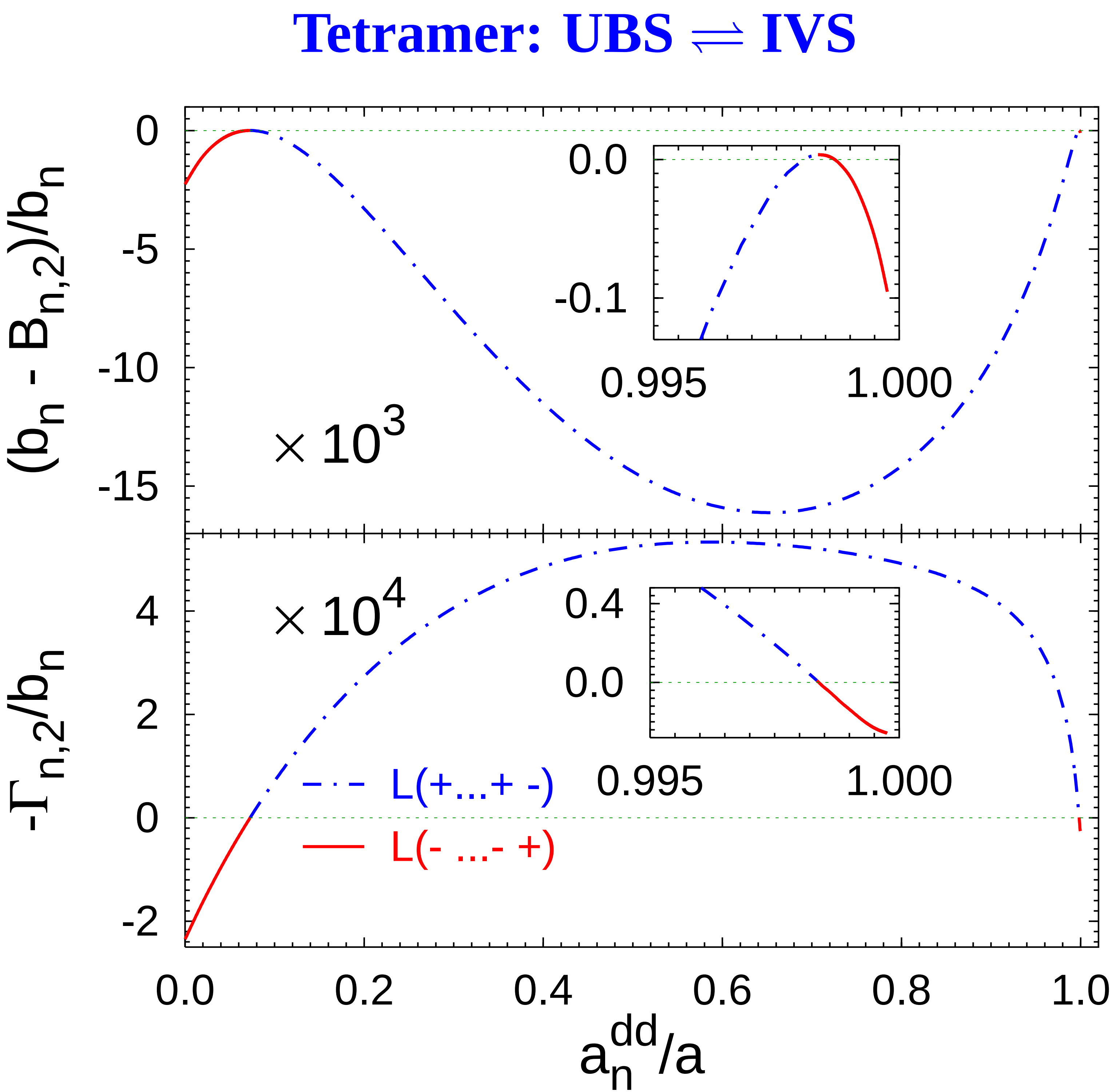


Tetramer: UBS $\rightleftharpoons$ IVS



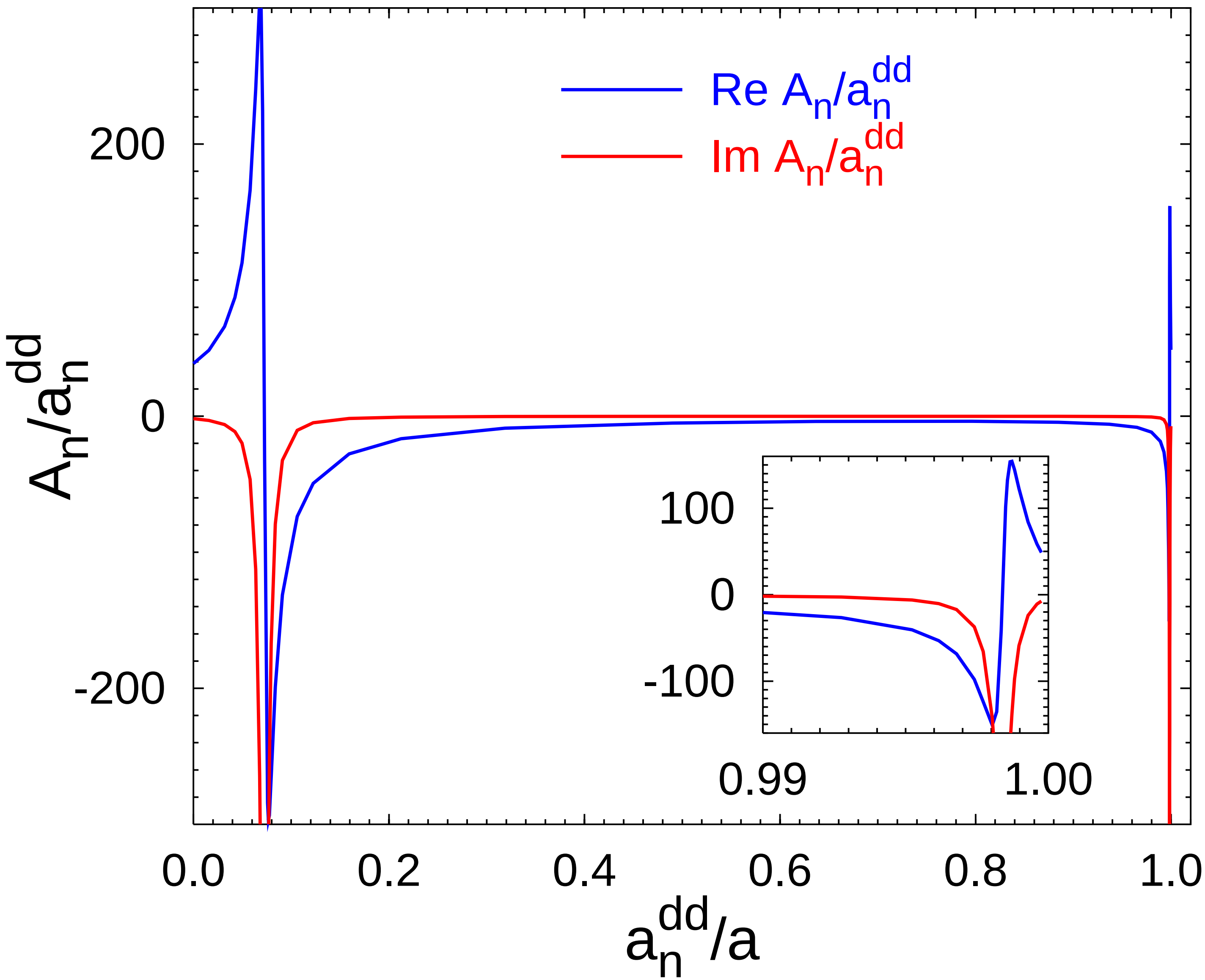
$$-1/A_n + \frac{1}{2} r_n K_n^2 - iK_n = 0$$

$$-B_{N,k} - i\Gamma_{N,k}/2 = -b_n + K_n^2/2\mu_1$$



Tetramer: UBS ⇌ IVS
(b_n - B_n,2)/b_n
0
-5
-10
-15
× 10^3
0.0
-0.1
0.995
1.000
-Γ_n,2/b_n
4
2
0
-2
× 10^4
0.4
0.0
0.995
1.000
L(+...+ -)
L(- ...- +)
0.0
0.2
0.4
0.6
0.8
1.0
a_n^dd/a

$1{+}(3)_n$ scattering length: UBS $\rightleftharpoons$ IVS



Trimer relaxation: $\beta_n^0 = -(4\pi\bar{h}/\mu_1)\,\mathrm{Im}A_n$

Summary

- four-boson AGS scattering equations in momentum space
- universal relations for scattering observables
- properties of unstable tetramers
- further work: dimer-dimer scattering