Computing Nucleon Magnetic Moments and Electric Polarizabilities with lattice QCD in Background Electric Fields

André Walker-Loud

ElectroMagnetic Collaboration

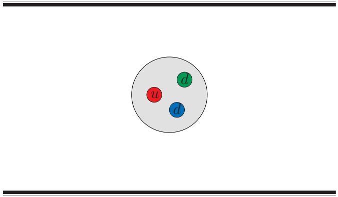
Strong Interactions:
From Methods to Structures
12 - 16 February, 2011

ElectroMagnetic Collaboration

- Will Detmold (William and Mary)
- Brian Tiburzi (MIT)
- André Walker-Loud (LBNL)

Outline

Polarizabilities are staple quantities of hadron structure: they measure the "stiffness" of a hadron immersed in a background electromagnetic field



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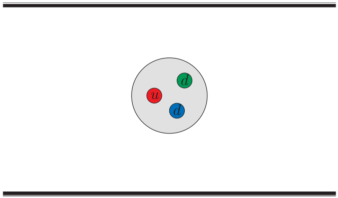
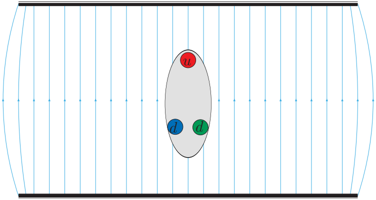




A description of this low-energy hadron structure from QCD is desirable.

comparison of experiment and phenomenological prediction

pion

two-loop ChPT prediction

U.Burgi; NPB 479(1996), PLB 377(1996)
J. Gasser et.al.; NPB 745 (2006)

$$\alpha_E^\pi = 2.4 \pm 0.5$$

$$\beta_E^\pi = -2.1 \pm 0.5$$

experimental determination

Y.M. Antipov et.al.; PLB 121(1983), Z.Phys. C 26 (1985)

$$\alpha_E^\pi = -\beta_M^\pi = 6.8 \pm 1.4 \pm 1.2$$

assumed $(\alpha_E^\pi = -\beta_M^\pi)$

nucleon

Polarizability	Proton	Neutron
$\alpha\,[10^{-4}\,\mathrm{fm}^3]$	11.9 ± 1.4	12.5 ± 1.7
$\beta\,[10^{-4}\,\mathrm{fm}^3]$	1.2 ± 0.9	2.7 ± 1.8
$\gamma_1\,[10^{-4}\,\mathrm{fm}^4]$	*1.1±0.25*	*3.7±0.4*
$\gamma_2\,[10^{-4}\,\mathrm{fm}^4]$	*-1.5±0.36*	*-0.1±0.5*
$\gamma_3\,[10^{-4}\,\mathrm{fm}^4]$	*0.2±0.24*	*0.4±0.5*
$\gamma_4\,[10^{-4}\,\mathrm{fm}^4]$	*3.3±0.11*	*2.3±0.35*
$\gamma_\pi\,[10^{-4}\,\mathrm{fm}^4]$	-38.7 ± 1.8	58.6 ± 4.0

measured

expected

(theoretical disagreements)

- Compass at CERN will measure pion and kaon polarizabilites through Primakoff process
- Compton MAX-lab (Lund) will extract neutron $\mathcal{E}M$ polarizabilities from Compton scattering on deuterium
- HIγS TUNL will make high precision measurements of proton and neutron electromagnetic and spin polarizabilites

Chiral non-analytic physics:

$$\alpha_E^{\pi^\pm} = \frac{8\alpha_{f.s.}}{f_\pi^2}\frac{L_9 + L_{10}}{m_\pi}$$ LO χPT

$$\alpha_E^N = \frac{5\alpha_{f.s.}\, g_A^2}{192\pi f_\pi^2}\frac{1}{m_\pi} + \Delta\text{-contributions}$$ NLO χPT (leading loop)

$$\beta_B^N = \frac{\alpha_{f.s.}\, g_A^2}{384\pi f_\pi^2}\frac{1}{m_\pi} + \Delta\text{-contributions}$$ NLO χPT (leading loop)

$$\gamma_{E_1 E_1}^N = -\frac{5\alpha_{f.s.}\, g_A^2}{192\pi^2 f_\pi^2}\frac{1}{m_\pi^2} + \Delta\text{-contributions}$$ NLO χPT (leading loop)

time varying $\mathcal{E}$-field

For sufficiently low energy ($\omega << m_\pi$), a spin $1/2$ baryon has the effective Hamiltonian

$$H_{eff} = \frac{(\vec{p} - Q\vec{A})^2}{2M} + Q\phi - \frac{1}{2} 4\pi \Big(\alpha \vec{\mathcal{E}}^2 + \beta \vec{\mathcal{B}}^2$$

$$\gamma_{E_1 E_1} \vec{\sigma} \cdot \vec{\mathcal{E}} \times \dot{\vec{\mathcal{E}}} + \gamma_{M_1 M_1} \vec{\sigma} \cdot \vec{\mathcal{B}} \times \dot{\vec{\mathcal{B}}} + \gamma_{M_1 E_2} \sigma_i \mathcal{E}_{ij} \mathcal{B}_j + \gamma_{E_1 M_2} \sigma_i \mathcal{B}_{ij} \mathcal{E}_j \Big)$$

where

$$\mathcal{E}_{ij} = \frac{1}{2} (\nabla_i \mathcal{E}_j + \nabla_j \mathcal{E}_i) \qquad \mathcal{B}_{ij} = \frac{1}{2} (\nabla_i \mathcal{B}_j + \nabla_j \mathcal{B}_i)$$

$$\gamma_{E_1 E_1} = -\gamma_1 - \gamma_3 \qquad \gamma_{M_1 M_1} = \gamma_4$$

$$\gamma_{E_1 M_2} = \gamma_3 \qquad \gamma_{M_1 E_2} = \gamma_2 + \gamma_4$$

For specific choices of A_μ, one can isolate the various (spin) polarizabilites W. Detmold, B.C. Tiburzi, AWL PRD 73 (2006).

For our calculation, we want Euclidean action which respects periodic boundary conditions (hyper-torus)

$$e^{-i\int d^4x_M \frac{1}{4}F_{\mu\nu}F^{\mu\nu}} = e^{i\int d^4x_M \frac{1}{2}\left(\mathcal{E}_M^2 - \mathcal{B}_M^2\right)}$$

$$\longrightarrow e^{-\int d^4x_E \frac{1}{4}F_{\mu\nu}F_{\mu\nu}} = e^{-\int d^4x_E \frac{1}{2}\left(\mathcal{E}_E^2 + \mathcal{B}_E^2\right)}$$

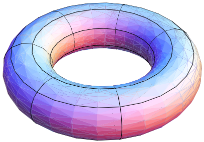
In this way, the $U(1)$ gauge links are given by a phase

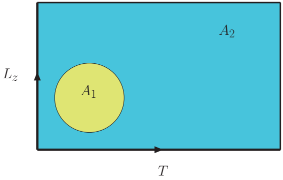
$$U_\mu(x) = e^{iaqA_\mu(x)}$$

Consequences:

$$M(\mathcal{E}_M) = M_0 - 2\pi\alpha\mathcal{E}_M^2 + \ldots \longrightarrow M(\mathcal{E}_E) = M_0 + 2\pi\alpha\mathcal{E}_E^2 + \ldots$$

On a compact torus, not all values of the field strength are allowed: G. ‘t Hooft NPB 153 (1979)





$$0 = \Phi = \Phi_1 + \Phi_2 \qquad A_1 = TL_z - A_2$$

$$\exp\{-iq\mathcal{E}A_2\} = \exp\{iq\mathcal{E}A_1\}$$

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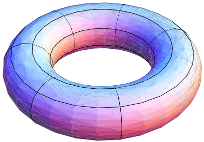
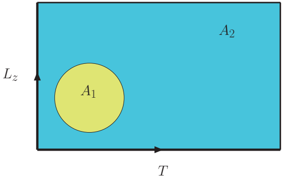




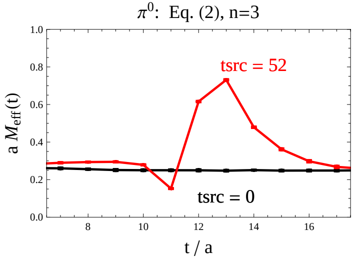
$$0 = \Phi = \Phi_1 + \Phi_2 \qquad\qquad A_1 = TL_z - A_2$$

$$\exp\{-iq\mathcal{E}A_2\} = \exp\{iq\mathcal{E}A_1\}$$

$$\exp\{-iq\mathcal{E}A_2\} = \exp\{iq\mathcal{E}(TL_z - A_2)\} \qquad \longrightarrow \exp\{iq\mathcal{E}TL_z\} = 1$$

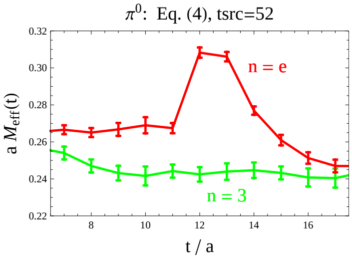
$$q\mathcal{E} = \frac{2\pi}{TL_z}n \qquad\qquad \text{for } n = 1, 2, \ldots$$

Non-Quantized



- $n = 3,\ t_{src} = 0$
- $n = 3,\ t_{src} = 52$

Quantized



- $n = 3,\ t_{src} = 52$
- $n = e,\ t_{src} = 52$

$$aM_{eff}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$$

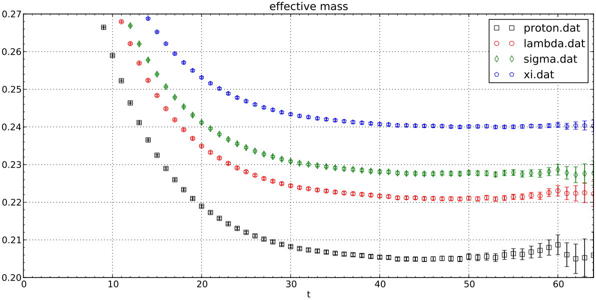
Hadron Correlation Functions

In free ($\mathcal{E}M$) field, hadron 2-point correlation functions

$$C(t) = \sum_n Z_n e^{-E_n t} \qquad \lim_{t\to\infty} C(t) = Z_0 e^{-E_0 t}$$

form effective mass

$$m_{eff}(t) = \frac{1}{d} \ln\left(\frac{C(t)}{C(t+d)}\right)$$



In a background field, what do we expect the correlation functions to look like?

$$J=0, Q=0; \qquad C(t,\mathcal{E}) = \sum_n Z_n(\mathcal{E}) e^{-E_n(\mathcal{E})t}$$

$$J=1/2, Q=0; \qquad C(t,\mathcal{E}) = \sum_n Z_n(\mathcal{E},\mu) e^{-E_n(\mathcal{E},\mu)t}$$

$$J=0, Q=1; \qquad C(t,\mathcal{E},\mu) = \sum_n Z_n(\mathcal{E}) G(E_n,\mathcal{E},t)$$

$$J=1/2, Q=1; \qquad C(t,\mathcal{E},\mu) = \sum_n Z_n(\mathcal{E},\mu) G(E_n,\mathcal{E},\mu,t)$$

Consider spin-less, relativistic particle of unit charge coupled to an electric field

$$\mathcal{L} = D_\mu \pi^\dagger D_\mu \pi + m_{eff}^2 \pi^\dagger \pi, \quad D_\mu = \partial_\mu + iA_\mu, \quad A_\mu = (0, 0, -\mathcal{E} t, 0)$$

integrating by parts and changing variables

$$D^{-1} = p_\tau^2 + \mathcal{E}^2 \tau^2 + E_{k_\perp}^2 \equiv 2\left(\mathcal{H} + \frac{1}{2} E_{k_\perp}^2\right),$$

$$\tau = t - \frac{k_z}{\mathcal{E}}, \qquad E_{k_\perp}^2 = E_k^2 - k_z^2$$

solution B.C. Tiburzi Nucl.Phys. A 814 (2008)

$$D(\tau', \tau) = \frac{1}{2} \int_0^\infty ds \langle \tau', s | \tau, 0 \rangle e^{-s E_{k_\perp}^2 / 2}$$

$$\langle \tau', s | \tau, 0 \rangle = \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E} s}} \exp\left\{ -\frac{\mathcal{E}}{2 \sinh \mathcal{E} s} \left[(\tau'^2 + \tau^2) \cosh \mathcal{E} s - 2\tau' \tau \right] \right\}$$

Take $\tau = 0$, $\vec{k} = 0$:

$$C(\tau, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau, \mathcal{E})$$

$$G(\tau, \mathcal{E}) = \frac{1}{2} \int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E} s}} \exp\left\{-\frac{1}{2}\left(\mathcal{E}\tau^2 \coth \mathcal{E} s + s\, m_{eff}^2\right)\right\}$$

in the weak field limit

$$C(\tau, \mathcal{E}) = Z(\mathcal{E}) \exp\left\{-M(\mathcal{E})\tau - \frac{\mathcal{E}^2}{M(\mathcal{E})^4}\left(\frac{1}{6}(M(\mathcal{E})\tau)^3 + \frac{1}{4}(M(\mathcal{E})\tau)^2 + \frac{1}{4}(M(\mathcal{E})\tau)\right)\right\}$$

$$M(\mathcal{E}) = M_0 + 2\pi\alpha\mathcal{E}^2 + \mathcal{O}(\mathcal{E}^4)$$

computing hadron deformations in background $\mathcal{E}M$ fields amounts to spectroscopy neutron in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

$$S = \int d^4x\, \overline{\psi}(x) \left[\not{\partial} + E(\mathcal{E}) - \frac{\mu(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x) \,,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,,$$

$$\sigma_{\mu\nu} F_{\mu\nu} = 2\vec{K} \cdot \mathcal{E} \,,$$ for background $\mathcal{E}$-field and $\vec{K} = i\vec{\gamma}\gamma_4$

$$\mu(\mathcal{E}) = \mu + \mu'' \mathcal{E}^2 + \dots$$ anomalous magnetic coupling

with $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$, construct

$$G_\pm(t, \mathcal{E}) \equiv \mathrm{tr}[\mathcal{P}_\pm G(t, \mathcal{E})] = Z(\mathcal{E}) \left(1 \pm \frac{\mathcal{E}\mu}{2M^2} \right) \exp\left[-t\, E_{eff}(\mathcal{E}) \right] \,,$$

$$\mathcal{P}_\pm = \frac{1}{2}[1 \pm K_3] \qquad E_{eff} = E(\mathcal{E}) - \frac{\mu(\mathcal{E})^2 \mathcal{E}^2}{8M^3}$$

$$= M + \frac{1}{2}\mathcal{E}^2 \left(4\pi\alpha_E - \frac{\mu^2}{4M^3} \right) + \dots$$

neutron in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

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$$\sigma_{\mu\nu} F_{\mu\nu} = 2\vec{K} \cdot \mathcal{E}\,, \qquad \text{for background } \mathcal{E}\text{-field and } \vec{K} = i\vec{\gamma}\gamma_4$$

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proton in background electric field: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

$$S = \int d^4x\, \overline{\psi}(x) \left[\not{D} + E(\mathcal{E}) - \frac{\tilde{\mu}(\mathcal{E})}{4M} \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x)\,,$$

$$D_\mu = \partial_\mu + iQA_\mu \qquad\qquad \mu = Q + \tilde{\mu}(0)$$

boost projected correlation functions

$$G_\pm(t,\mathcal{E}) = Z(\mathcal{E}) \left(1 \pm \frac{\tilde{\mu}\mathcal{E}}{2M^2} \right) D\left(t, E_{eff}(\mathcal{E})^2 \mp Q\mathcal{E}, \mathcal{E} \right)$$

$$D(t, E^2, \mathcal{E}) = \int_0^\infty ds \sqrt{\frac{Q\mathcal{E}}{2\pi \sinh(Q\mathcal{E}s)}} \exp\left[-\frac{Q\mathcal{E}t^2}{2} \coth(Q\mathcal{E}s) - \frac{E^2 s}{2} \right]$$

Results I am going to present are from

- mesons: W. Detmold, B.C. Tiburzi, AWL PRD 79 (2009)
- proton and neutron: W. Detmold, B.C. Tiburzi, AWL PRD 81 (2010)

To date, we have set $q_{sea} = 0$ (Quenched $\mathcal{E}M$)

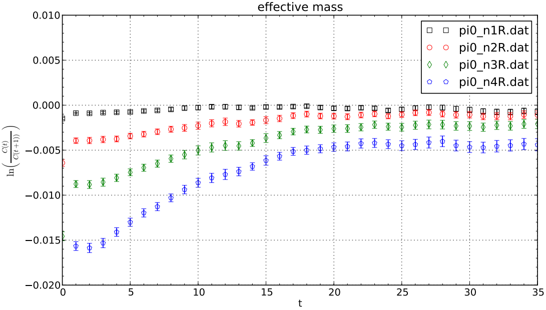
$$m_\pi \sim 390 \;\; \texttt{MeV} \qquad L = 2.5 \;\; \texttt{fm}$$

TABLE I: Propagators generated to date with our 2008-09 and 2009-10 USQCD allocations.

V	a_s [fm]	a_s/a_t	$a_t m_u^0$	$a_t m_s^0$	m_π [MeV]	m_K [MeV]	Field Strength	$N_{src} \times N_{cfg}$	total # of props(u,d,s)
$20^3 \times 128$	0.123	3.5	−0.0840	−0.0743	390	546	0	15×200	6,000
							±1	15×200	9,000
							±2	10×200	6,000
							±3	10×200	6,000
							±4	10×200	6,000
$24^3 \times 128$	0.123	3.5	−0.0840	−0.0743	390	546	0	10×195	3,900
							±1	10×195	5,850
							±2	10×195	5,850
							±3	10×195	5,850
							±4	10×195	5,850
$32^3 \times 256$	0.123	3.5	−0.0860	−0.0743	225	467	0	7×106	2,226

Numerical Results

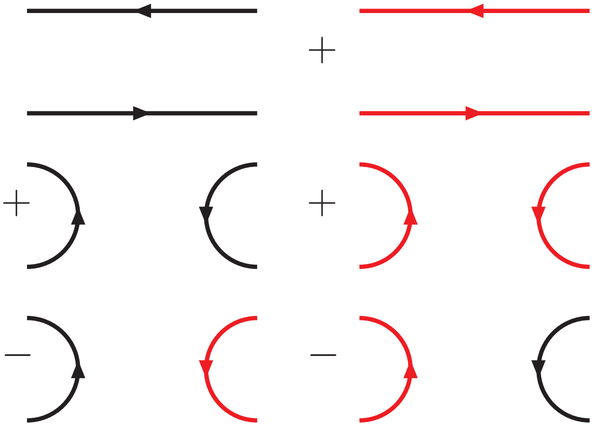
π^0



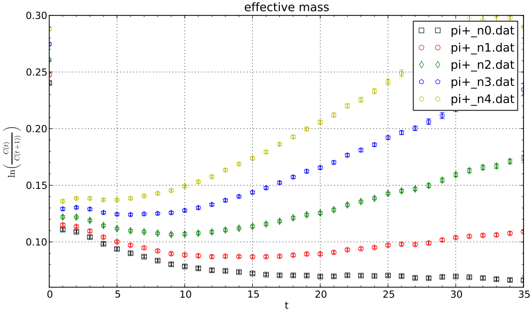
$$G(t) = \frac{C_n(t)}{C(t)}$$

Numerical Results

π^0



π^+

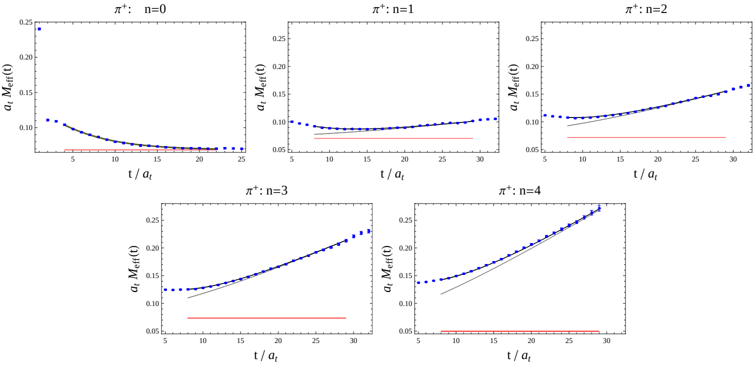


$$C(\tau, \mathcal{E}) = \sum_n Z_n(\mathcal{E}) G(\tau, \mathcal{E})$$

$$G(\tau, \mathcal{E}) = \frac{1}{2} \int_0^\infty ds \sqrt{\frac{\mathcal{E}}{2\pi \sinh \mathcal{E} s}} \exp\left\{ -\frac{1}{2} \left(\mathcal{E} \tau^2 \coth \mathcal{E} s + s\, m_{eff}^2 \right) \right\}$$

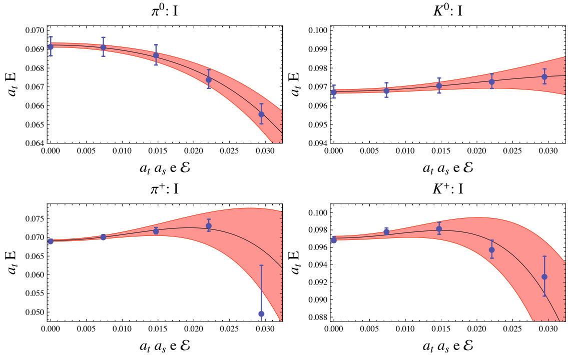
Numerical Results

π^+



	n				
	0	1	2	3	4
$m(\mathcal{E})$	0.0691(4)	0.0702(6)	0.0718(8)	0.0733(16)	0.0497(129)

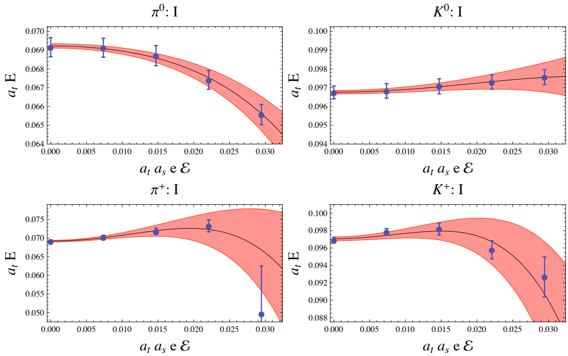
Numerical Results



$$m(\mathcal{E}) = m_0 + \alpha_E^{latt}\mathcal{E}^2 + \bar{\alpha}_{EEE}^{latt}\mathcal{E}^4$$

$a_s/a_t = 3.5, L = 20, T = 128$	$\lvert n\rvert$ = 1	2	3	4
$\epsilon_\pi = \left(\frac{e\mathcal{E}}{m_\pi^2}\right)^2 = \left(\frac{2\pi n}{m_\pi L\, m_\pi T}\right)^2$	0.02	0.09	0.20	0.35

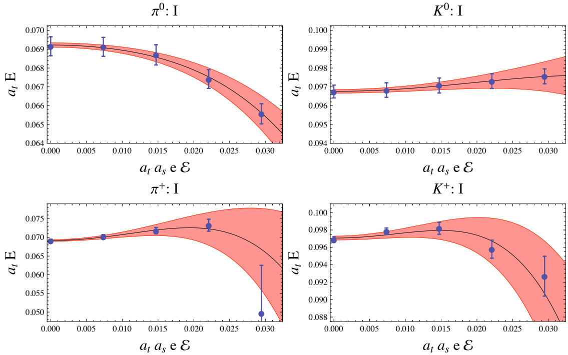
Numerical Results



$$m(\mathcal{E}) = m_0 + \alpha_E^{latt}\mathcal{E}^2 + \bar{\alpha}_{EEE}^{latt}\mathcal{E}^4$$

	π^0	π^+	K^0	K^+
α_E^{latt}	-2.6(5)(9)	18(4)(6)	1.5(4)(7)	8(3)(1)
$\bar{\alpha}_E^{latt}$	1.8(5)	24(10)	0.6(5)	17(5)

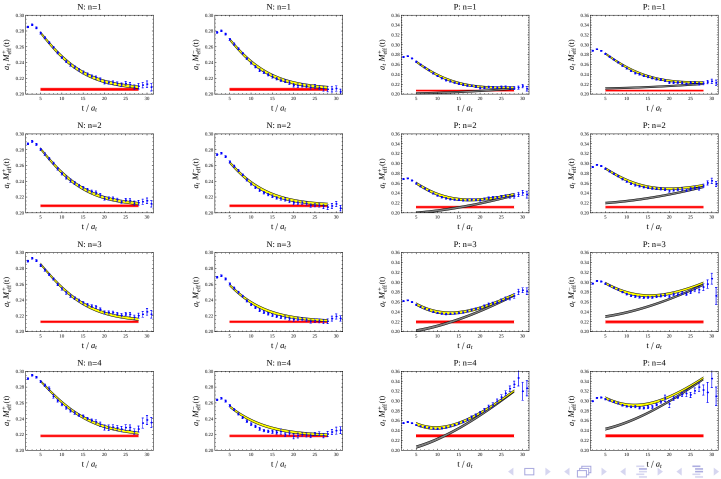
Numerical Results



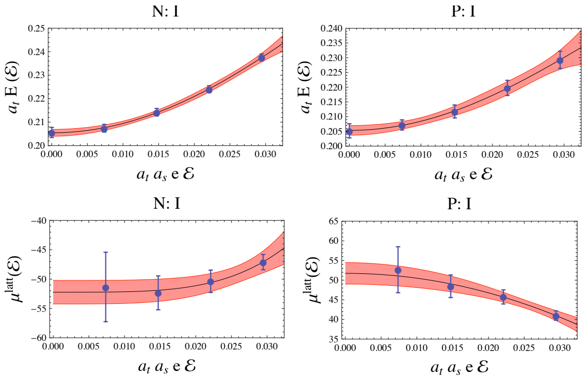
$m_\pi \simeq 390$ MeV	π^0	π^+	K^0	K^+
$\alpha_E[10^{-4}\,\mathrm{fm}^{-3}]$	-0.20(4)(7)	1.4(3)(5)	0.11(3)(5)	0.62(23)(08)

$\alpha_E^{\pi^\pm} = 2.4 \pm 0.5$ Burgi (1996) and Gasser et. al. (2006)

Neutron and Proton effective mass

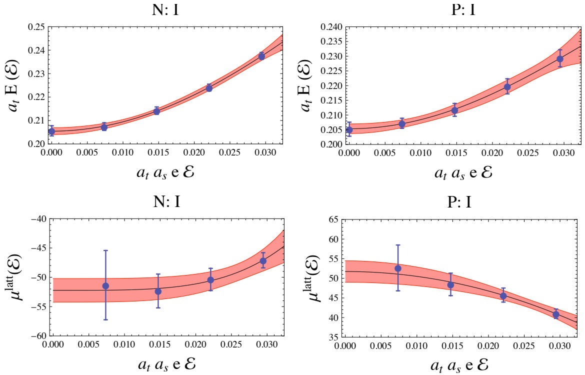


Numerical Results



N	α_E^{latt}	$\tilde{\mu}^{latt}$	μ^{latt}
neutron	40(9)(2)	-52(2)(1)	-52(2)(1)
proton	32(13)(1)	52(3)(1)	83.9(3)(1)

Numerical Results



N	$\alpha_E[10^{-4}\,\mathtt{fm}^{-3}]$	$\tilde{\mu}[\mu_N]$	$\mu[\mu_N]$
neutron	3.1(7)(2)	-1.63(6)(3)	-1.63(6)(3)
proton	2.4(1.0)(0.1)	1.63(6)(3)	2.63(9)(3)

$\mu_V(m_\pi \simeq 390\ \mathtt{MeV}) = 4.3(2)(1)(1)[\mu_N]$ $\mu_V^{phys} = 4.7[\mu_N]$

In the last two years, we have established a program to compute polarizabilites of hadrons, as well as magnetic moments of spin-$1/2$ baryons utilizing background electric fields.

Several systematics we need to address

- sea quark electric charges
- As polarizabilites are singular in the chiral limit, they are also sensitive to finite-volume effects

	m_π[MeV]			
L[fm]	450	390	300	225
2.5	○	✓	○	
3.0	○	✓	○	
4.0		⊗	⊗	⊗

Future:

- utilize background magnetic fields
- explore non-constant fields to extract nucleon spin-polarizabilites
- explore methods to include sea-quark electromagnetic charges