474. WE-Heraeus-Seminar

Strong Interactions: From Methods to Structures

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QCD Factorization and Heavy Quarkonium Production

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Based on work done with Kang, Nayak, Sterman, and …

Outline

- Production of heavy quarkonium
- NRQCD Factorization
- High order in α_s is not necessarily leading in $1/p_T$
- Perturbative QCD factorization
- Connect pQCD factorization to NRQCD factorization
- Summary and outlook

Production of heavy quarkonium

❑ Cross sections and mass scales:

$$\frac{d\sigma_{AB\to H(P)X}}{dy dP_T^2} \qquad \sqrt{S}, \quad P_T, \quad M_H, \qquad \textbf{Observed!}$$

$$m_Q, \quad m_Q v, \quad m_Q v^2, \ldots$$

❑ Production and distance scales:

✧ Creation of heavy quarks:

$$\Delta r \lesssim \frac{1}{2m_Q} \quad \begin{aligned} &\lesssim 0.1\text{fm (for a charm-quark pair)} \\ &\lesssim 0.025\text{fm (for a bottom-quark pair)} \end{aligned}$$

Can they be calculated using perturbative QCD?

Relevant partonic subprocesses?

✧ Binding to form a heavy quarkonium - nonperturbative:

Described by a few functions, or a few constants?

Universal – measured by experiment, or calculated on Lattice?

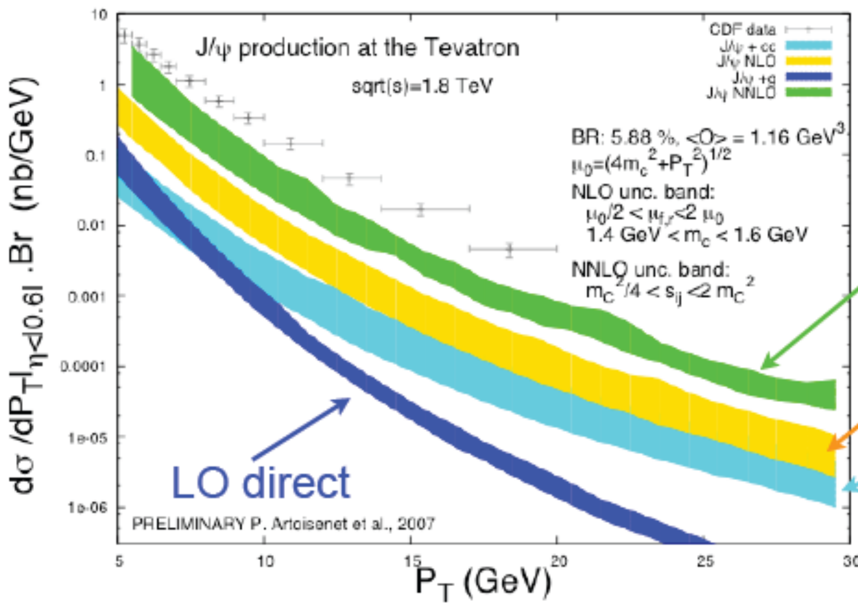
A long history

- ❑ Discovery of J/ψ – November revolution – 1974
- ❑ Color singlet model: 1976 –
  - Only the pair with right quantum numbers
  - Effectively No free parameter!
- ❑ Color evaporation model: 1977 –
  - All pairs with mass less than open meson threshold
  - One parameter per quarkonium state
- ❑ NRQCD model: 1995 –

B. Kniehl's talk

  - The most successful!
  - Infinite parameters – organized in powers of v and α_s
- ❑ QCD factorization to next-leading power: 2010 –
  - Universal fragmentation functions – evolution/resummation
  - NRQCD for input functions?

Color singlet model

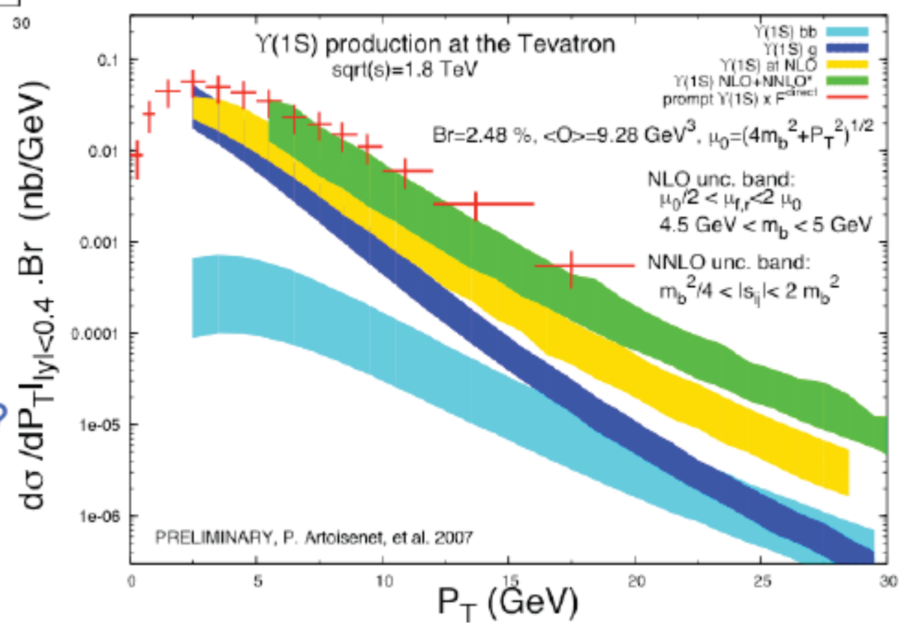


Color-singlet contribution for J/ψ and Upsilon production at Tevatron

P. Artoisenet, F. Maltoni, et.al. 2007

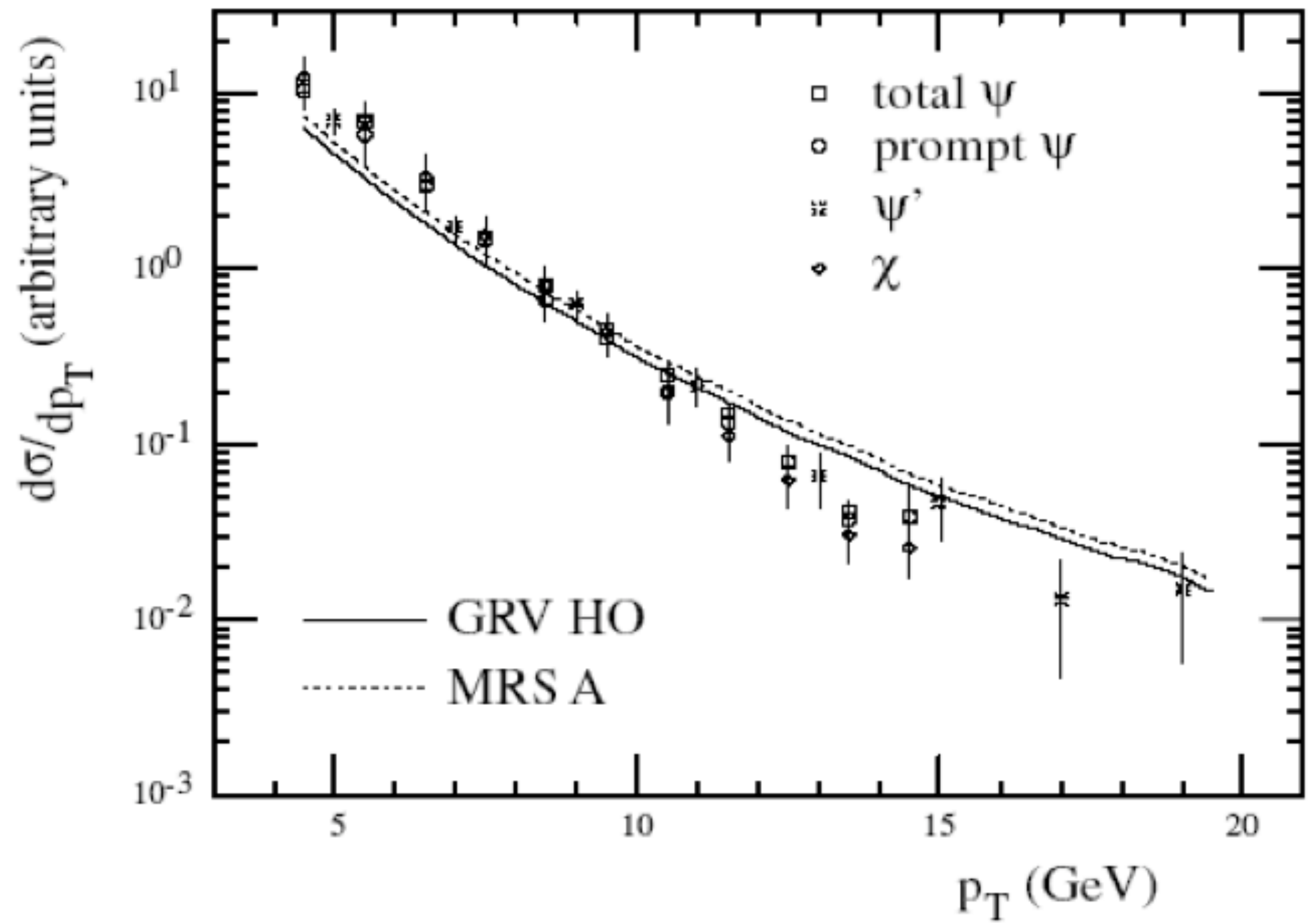
Large uncertainty band
⇒strong scale dependence

Large NLO, NNLO contribution
⇒ how perturbative series converge?



Color evaporation model

❑ **Good for total cross section, ok for P_T-distribution:**



Amundson et al, PLB 1997

NRQCD factorization

Bodwin, Braaten and Lepage 1995

- Conjectured factorization formula:

$$d\sigma_{AB\to H} = \sum_n d\hat{\sigma}_{AB\to[Q\bar{Q}(n)]}(P, m_Q)\,\langle\mathcal{O}_{[Q\bar{Q}(n)]\to H}(0)\rangle$$

- Sum over all quantum states of $Q\bar{Q}(n)$ pair:

  Color + Spin + Pair's orbital angular momentum

- Cross section to produce the $Q\bar{Q}(n)$ pair:

  Calculated in pQCD in power of α_s – complete NLO, …

- NRQCD matrix elements: $\langle\mathcal{O}_{[Q\bar{Q}(n)]\to H}(0)\rangle$

  Infinite parameters – organized in relative velocity: v^n

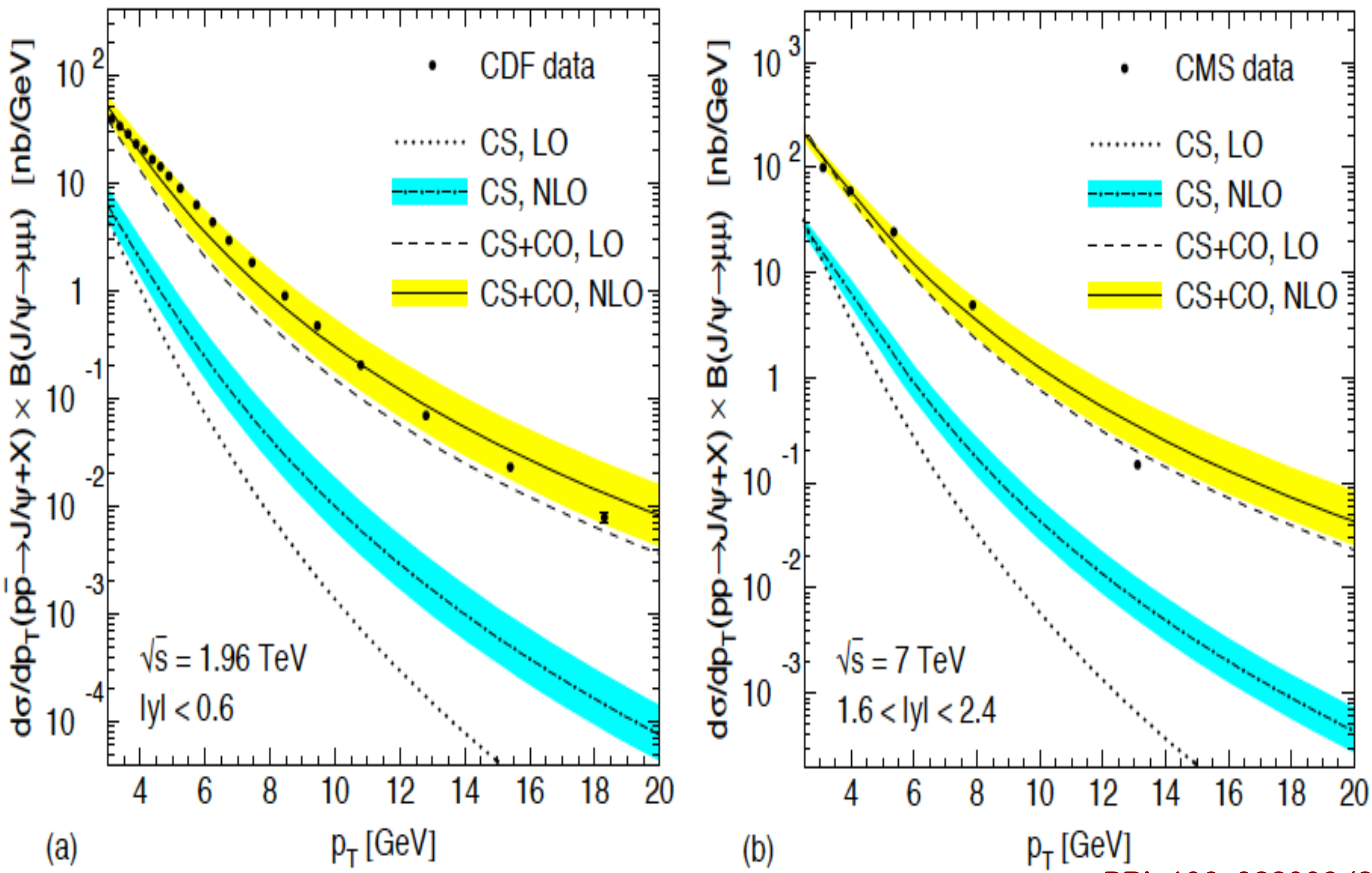
  Fitted by experimental data

B. Kniehl's talk

Both color singlet model and color evaporation model are the special cases of NRQCD model

Success of NRQCD model

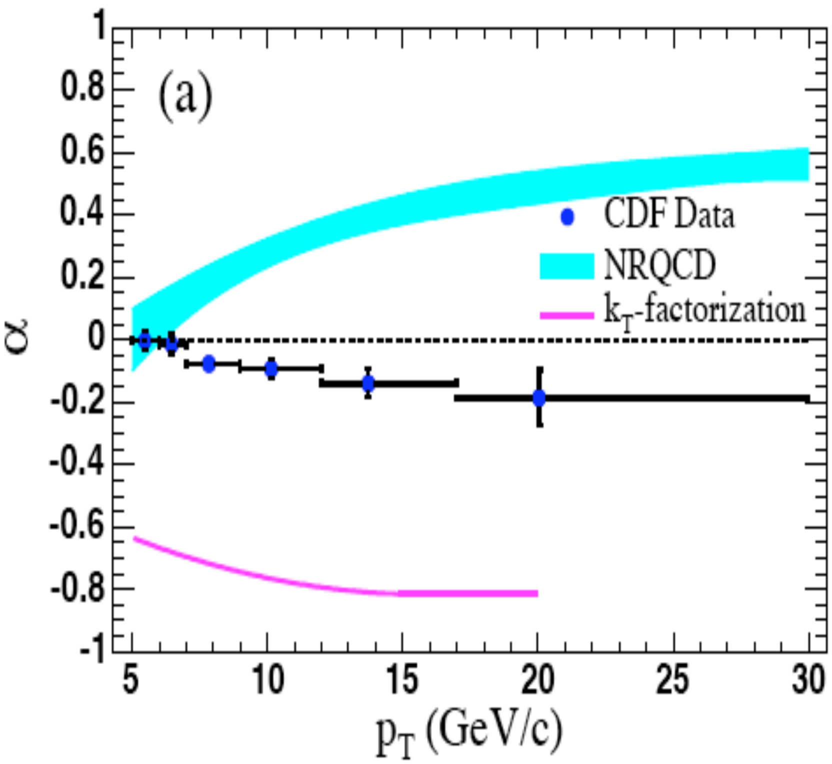
B. Kniehl's talk



PRL 106, 022003 (2011)

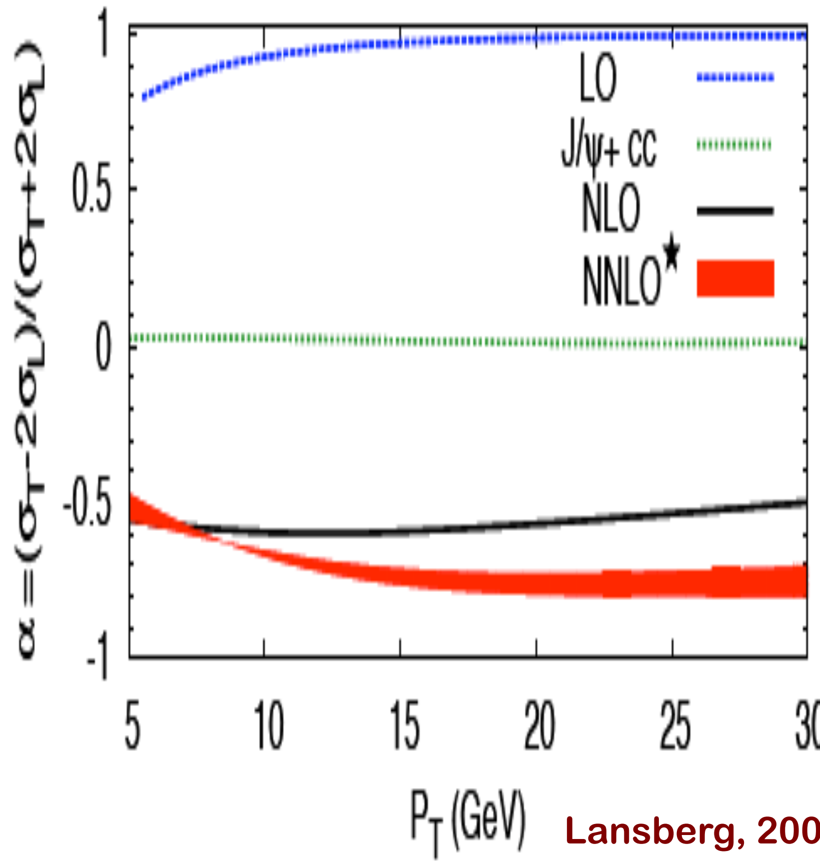
Surprises from J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

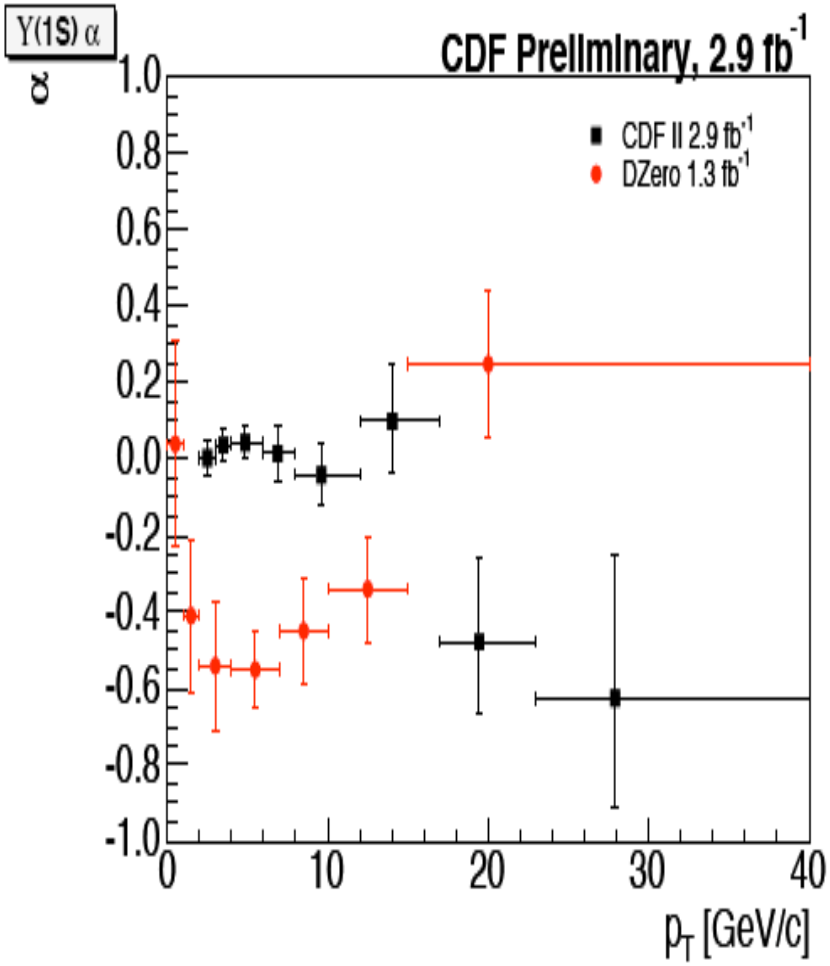
CSM

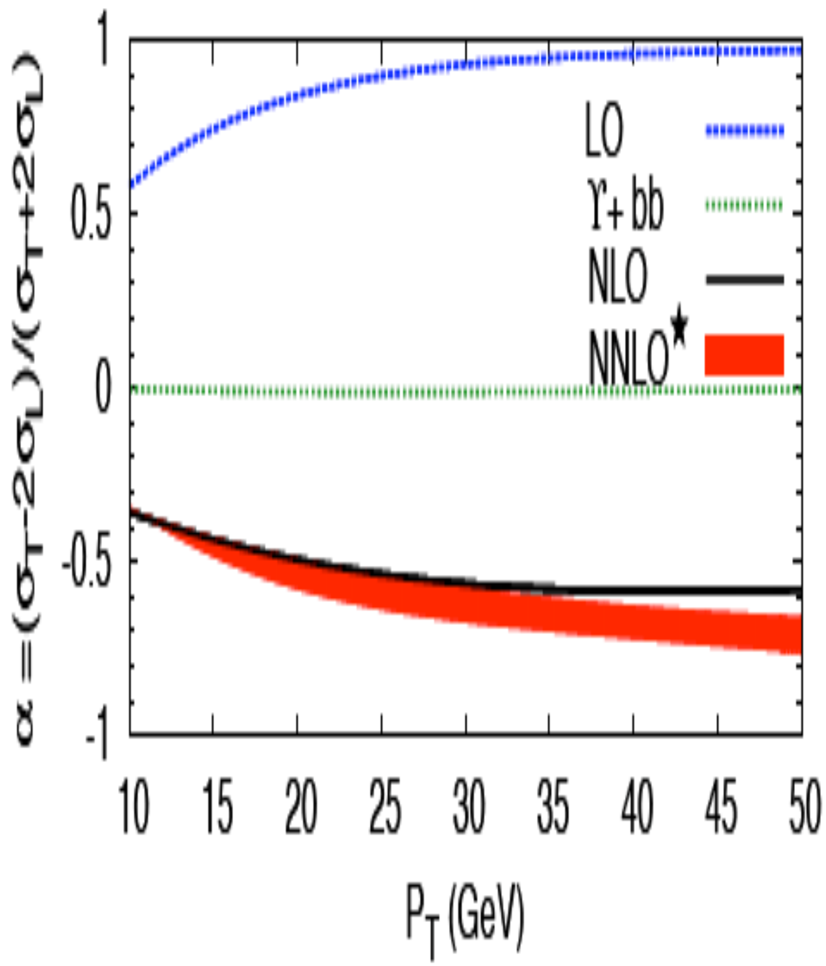


Lansberg, 2009

- ✧ **NRQCD:** Dominated by color octet – NLO is not a huge effect
- ✧ **CSM:** Huge NLO – change of polarization?

Confusions from Upsilon polarization





Resolution between CDF and D0?

P. Artoisenet, et al. 2008

Questions

- Why the high order corrections in CSM is so large?
  How many orders do we need to calculate?

- Why the high order CSM calculations predict
  longitudinally polarized J/ψ?

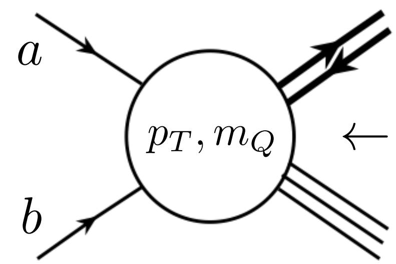
- IF NNLO CSM is so large, what is the role of octet channel in NRQCD?

- Can NRQCD factorization be proved?

- Better factorization ? …

Why high orders in CSM are so large?

- **Hard part in CSM (or NRQCD):**



$\leftarrow \quad \Delta r \sim \frac{1}{m_Q}$

Expansion in power of α_s

$$\hat{\sigma}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu, \alpha_s(\mu)) = \sum_n \hat{\sigma}^{(n)}_{ab\to Q\bar{Q}}(p_T, m_Q, \mu) \left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- **Complication with more than one hard scale:**

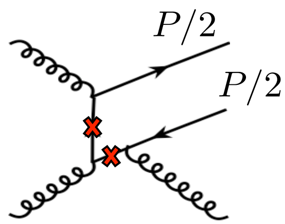
  ✧ **IF $p_T^2 \gg m_Q^2$, high order in α_s is NOT necessary smaller!**
  Different power in p_T^2

  ✧ **The size of the hard coefficients depends where the singlet pair was produced!**

α_s-expansion vs 1/p_T-expansion

- **LO in α_s but higher power in 1/p_T:**

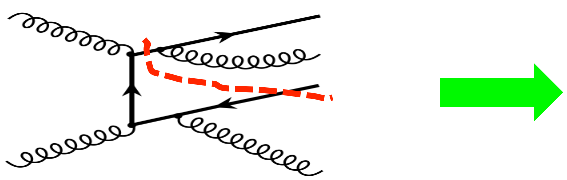
LO in α_s:

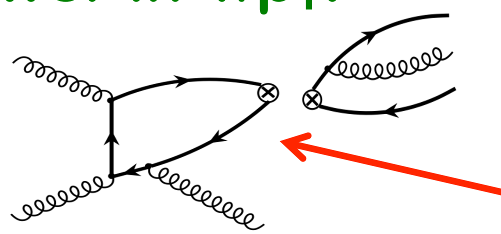


$$\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

NNLP in 1/p_T!

- **NLO in α_s but lower power in 1/p_T:**

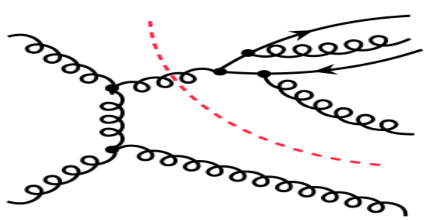




Leading power Projection "Octet pair"

$$\hat{\sigma}^{\rm NLO} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)$$

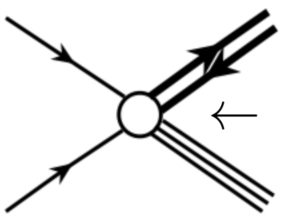
- **NNLO in α_s but leading power in 1/p_T:**



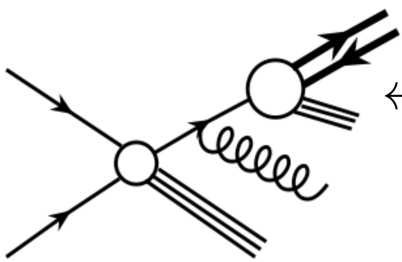
$$\hat{\sigma}^{\rm NNLO} \to \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^2(\mu)\log^m(\mu^2/\mu_0^2)$$

QCD power counting

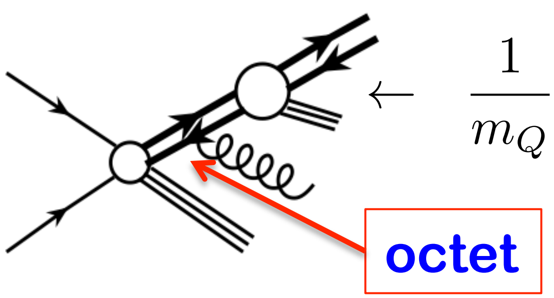
❑ Power counting for the hard part:



$$\leftarrow \Delta r \sim \frac{1}{p_T} \quad \Longrightarrow \quad \frac{1}{p_T^4}\left[\frac{m_Q^4}{p_T^4}, \ldots\right]$$



$$\leftarrow \frac{1}{m_Q} \quad \Longrightarrow \quad \frac{1}{p_T^4}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n \left[\frac{m_Q^2}{\mu_0^2}, \ldots\right] \qquad \mu_0 \sim 2m_Q$$



$$\leftarrow \frac{1}{m_Q} \quad \Longrightarrow \quad \frac{1}{p_T^6}\left[\log(\frac{p_T^2}{\mu_0^2})\right]^n \left[m_Q^2 \left(\log\frac{\mu_0^2}{m_Q^2}, \frac{m_Q^2}{\mu_0^2}, \ldots\right)\right]$$

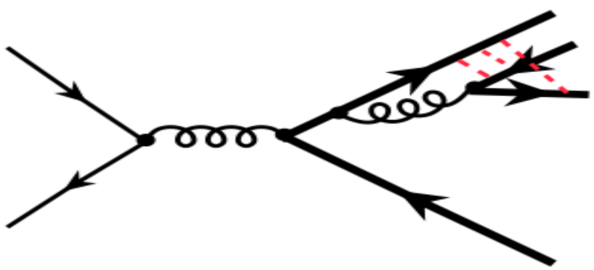
The heavy quark pair can be produced between $1/p_T$ and $1/m_Q$

Issues with NRQCD factorization

❑ **NRQCD factorization is ideal for quarkonium decay:**

- ✧ Quarkonium mass determines the available energy
- ✧ Velocity expansion is justified

❑ **NRQCD model is ill-defined for associate production:**



- ✧ More than one velocity
- ✧ Coulomb singularity

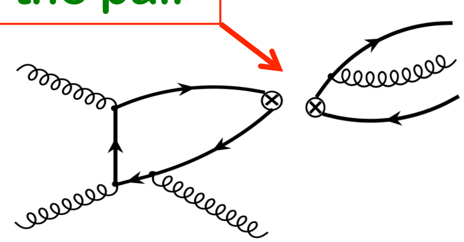
Nayak, Qiu, and Sterman, 2007

❑ **No proof of factorization for the production:**

- ✧ Heavy quark velocity can be large in high energy production

Larger phase space for the pair

$$P_T^2 \gg M_{c\bar{c}}^2 \gtrsim M_{J/\psi}^2$$



The pair radiates semi-hard gluons, and evolves in color and spin

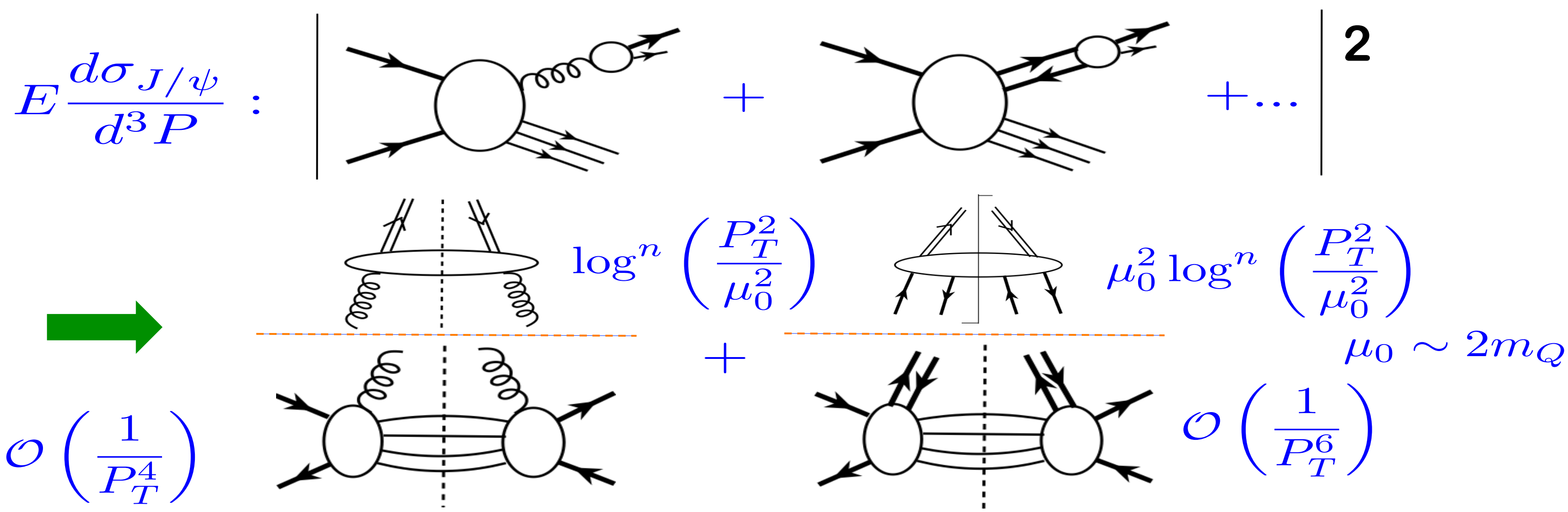
PQCD factorization

❑ Ideas:

Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010

- ✧ Expand cross section in powers of μ_0^2/p_T^2 with $\mu_0 \gtrsim 2m_Q$
- ✧ Resum logarithmic contribution into "fragmentation functions"
- ✧ Apply NRQCD to input fragmentation functions at $\mu_0 \sim 2m_Q$

❑ Factorization – all orders in α_s:



Power series in α_s without large logarithms

Perturbative coefficients

Kang, Qiu and Sterman, 2010

❑ Factorization formalism:

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}(p_T/z,\mu)\otimes D_{i\to H}(z,m_Q,\mu)$$
$$+\sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}(\kappa)]}=p_T/z,\mu)\otimes D_{[Q\bar{Q}(\kappa)]\to H}(z,m_Q,\mu)$$
$$+\mathcal{O}(m_Q^4/p_T^4)$$

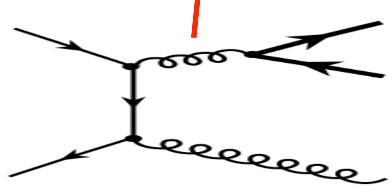
❑ Partonic hard parts – subtraction at tree-level:

$$\hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g}\otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$$\frac{\alpha_s^3(\mu)}{p_T^6}$$

$$\frac{\alpha_s^2(\mu)}{p_T^4}$$

$$\frac{\alpha_s(2m_Q)}{(2m_Q)^2}$$



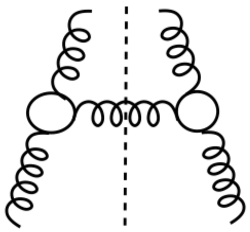
$$H^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \left(\frac{4\pi\alpha_s}{\hat{s}}\right)\left[\frac{(N_c^2-1)^2}{N_c^3}\left[1+\frac{2\hat{t}\hat{u}}{\hat{s}^2}\right]\left[\frac{\hat{s}^2}{\hat{t}\hat{u}}-1\right]-\frac{N_c^2-1}{2N_c}\left[4+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]\right]$$

Normalized to 2 → 2 amplitude square

Evolution and evolution kernels

Kang, Qiu and Sterman, 2010

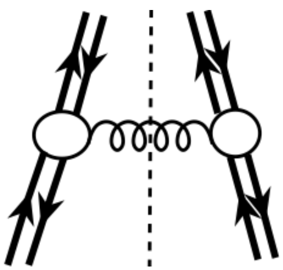
❑ Single parton:



Same as normal DGLAP
Difference from input distribution

$$D_{g\to H}(z, \mu_0, m_Q)$$

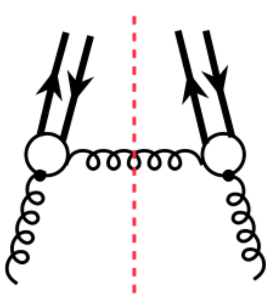
❑ Heavy quark pair:

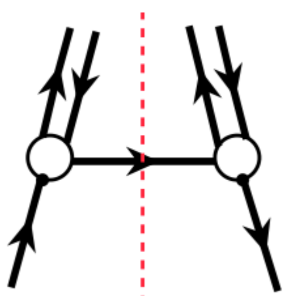


Differ from DGLAP – still logarithmic
Spin-color sensitive
Infrared safe evolution

$$D_{[Q\bar{Q}(\kappa)]\to H}(z, \mu_0, m_Q)$$

❑ Mixing:





Non-logarithmic contribution
to the evolution
Needed to remove CO divergence

$$\sigma^{(4)}_{q\bar{q}\to Q\bar{Q}gg} \qquad \text{NLO to } \hat{\sigma}_{q\bar{q}\to Q\bar{Q}}$$

Evolution equations

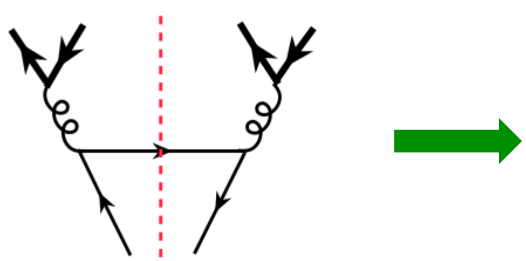
- Evolution of a heavy quark pair:

$$\frac{d}{d\ln\mu^2}D_{[Q\bar{Q}]\to H}(z,\mu^2)=\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}]\to[Q\bar{Q}]}\otimes D_{[Q\bar{Q}]\to H}(z,\mu^2)$$

- Evolution kernel:

$$K_{[Q\bar{Q}]\to[Q\bar{Q}]}(z)=4C_F\left[z(1-z)+\frac{z}{(1-z)_+}+\frac{3}{4}\delta(1-z)\right]$$

- Mixing kernel:



$$\gamma^{(1)}_{q\to g}(z)\otimes D^{(1)}_{g\to[Q\bar{Q}]}(z)+K^{(2)}_{q\to[Q\bar{Q}]}(z)$$

NRQCD for input distributions

- **Input distributions are universal, non-perturbative:**
  Should, in principle, be extracted from experimental data

- **NRQCD – single parton distributions:**

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

Nayak, Qiu and Sterman, 2005

Dominated by transverse polarization

- **NRQCD – heavy quark pair:**

$$D_{[Q\bar{Q}(\kappa)\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle\mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

Kang, Qiu and Sterman, 2010

Dominated by longitudinal polarization

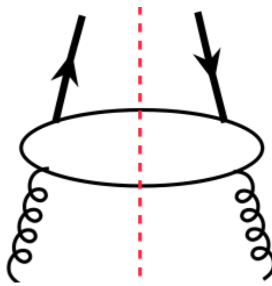
- **No proof of such factorization yet!**

  Single parton case was verified to two-loops (with gauge links)!

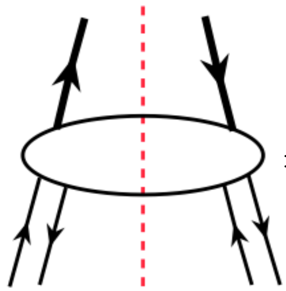
Nayak, Qiu and Sterman, 2005

Quarkonium polarization

- Fragmentation evolves perturbatively until $\mu_0 \sim 2m_Q$:



$$= D_{g\to H}(z, \mu_0, m_Q)$$



$$= D_{[Q\bar{Q}(\kappa)]\to H}(z, \mu_0, m_Q)$$

- Polarization of heavy quark pair: $[Q\bar{Q}] \to H$

$$D_{[Q\bar{Q}]\to H}(z) = \sum_n d_{[Q\bar{Q}]\to[Q\bar{Q}(n)]}(z)\langle \mathcal{O}_{[Q\bar{Q}(n)]\to H}(0)\rangle$$

$$D_{[Q\bar{Q}]\to H}(z,\mu^2) = D^L_{[Q\bar{Q}]\to H}(z,\mu^2) + 2D^T_{[Q\bar{Q}]\to H}(z,\mu^2)$$

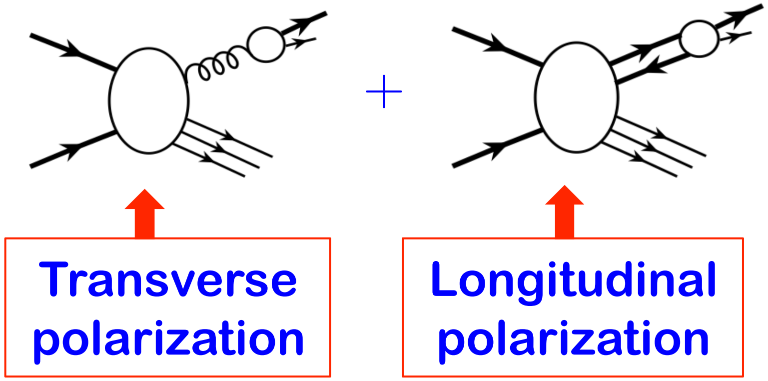
$$D^L_{[Q\bar{Q}]\to H}(z,\mu^2) = \frac{\alpha_s}{2\pi}\frac{\langle O^1({}^3S_1)\rangle}{2m}\int_{4m^2/z}^{\mu^2} dp_c^2 \frac{C_F}{N_c^2}\left[\frac{z(1-z)}{p_c^2-4m^2} - \frac{4m^2(1-z)^2}{(p_c^2-4m^2)^2}\right]$$

$$D^T_{[Q\bar{Q}]\to H}(z,\mu^2) = \frac{\alpha_s}{2\pi}\frac{\langle O^1({}^3S_1)\rangle}{2m}\int_{4m^2/z}^{\mu^2} dp_c^2 \frac{C_F}{N_c^2}\left[\frac{4m^2(1-z)^2}{(p_c^2-4m^2)^2}\right]$$

Heavy quarkonium polarization

❑ **Competition between LP and NLP:**

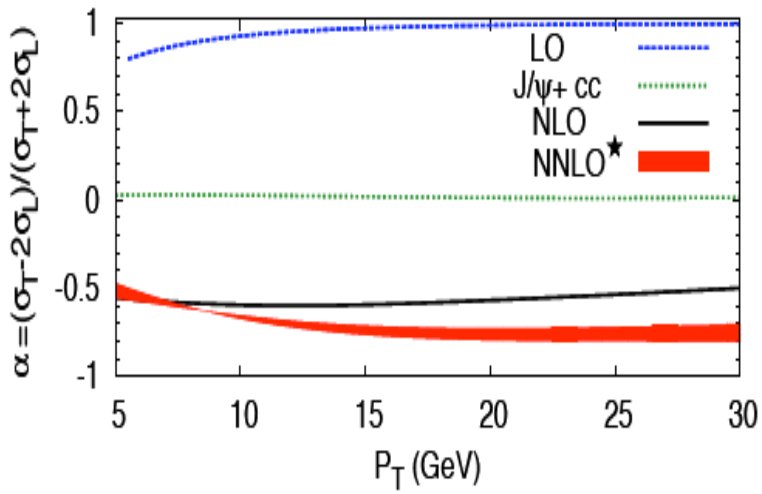
$E\frac{d\sigma_{J/\psi}}{d^3P}$:

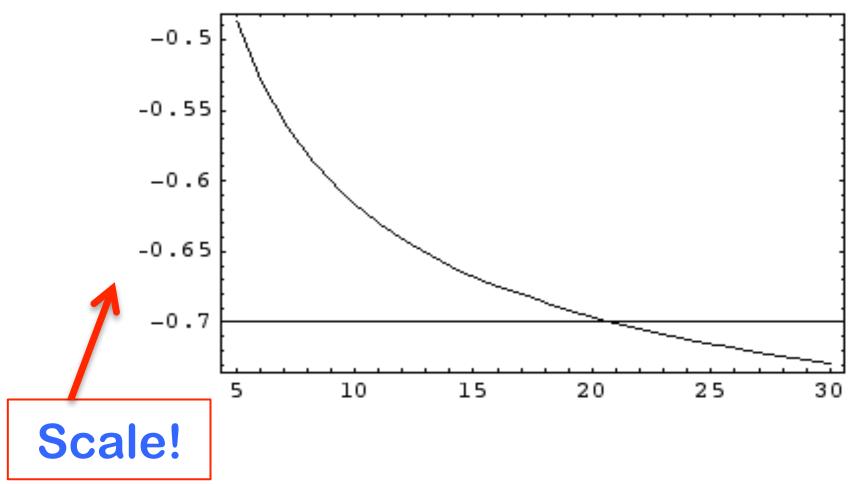


$+\ldots$

$\frac{\langle\mathcal{O}_{\rm octet}\rangle}{p_T^4}$ vs. $\frac{\langle\mathcal{O}_{\rm singlet}\rangle}{p_T^6}$

❑ **Polarization:**





FF reproduces the polarization of CSM at NLO

Summary

- Propose a new QCD factorization formula for heavy quarkonium production:
  - – Expansion in power of $1/p_T$ first

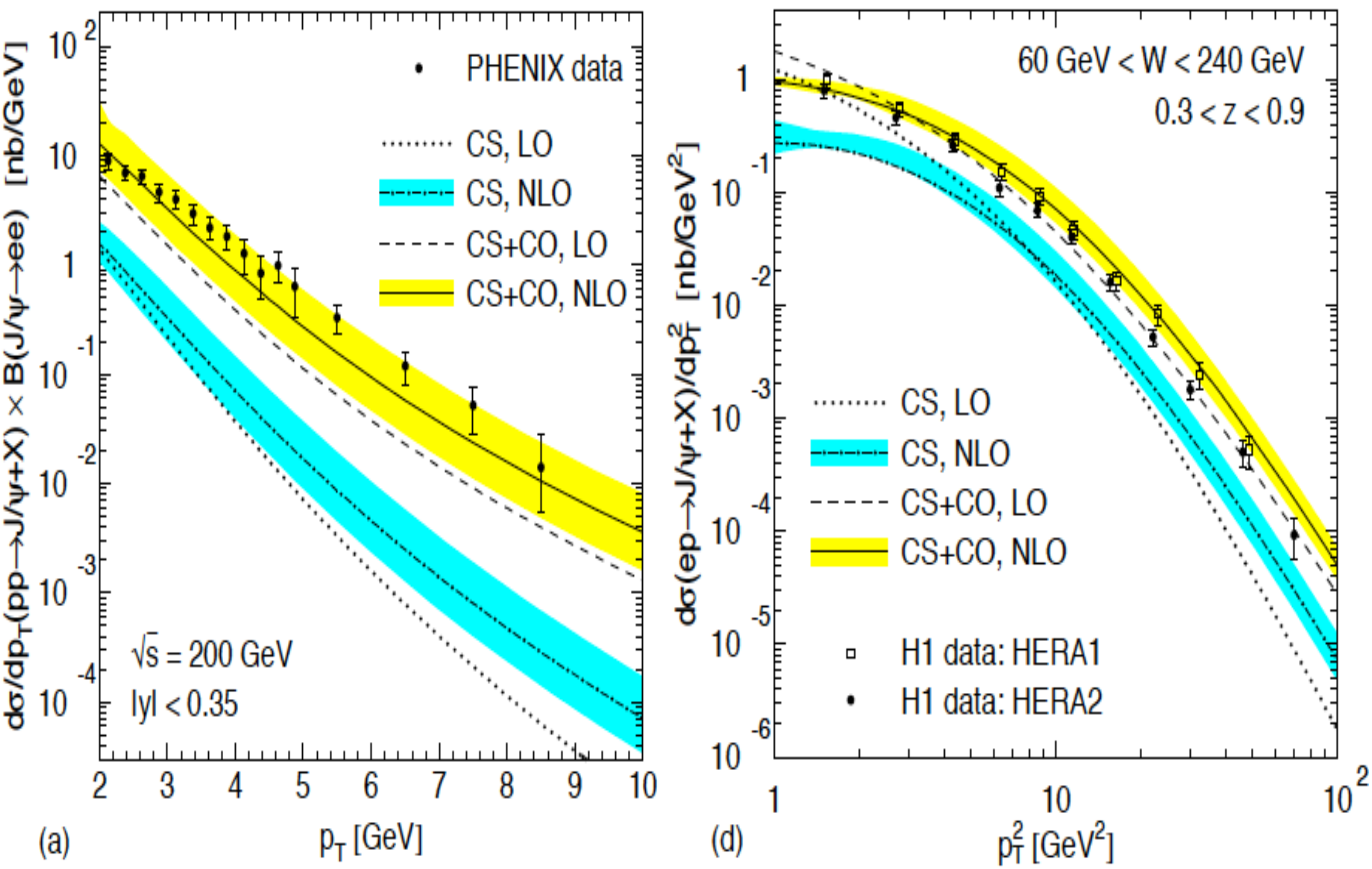
    $\sigma(p_T)$ = LP + NLP + "non-factorized"

  - – Expansion in power of α_s for each power
- Both LP and NLP in hard production dominated by octet channels (octet single parton & octet QQbar pair)
- Observed polarization of prompt quarkonium should be a result of competition between LP and NLP
- J/ψ polarization at the LHC is very interesting

Thank you!

Backup slices

Success of NRQCD model (II)



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