Parametric excitation of a 1D gas in INTEGRABLE and NON-INTEGRABLE cases

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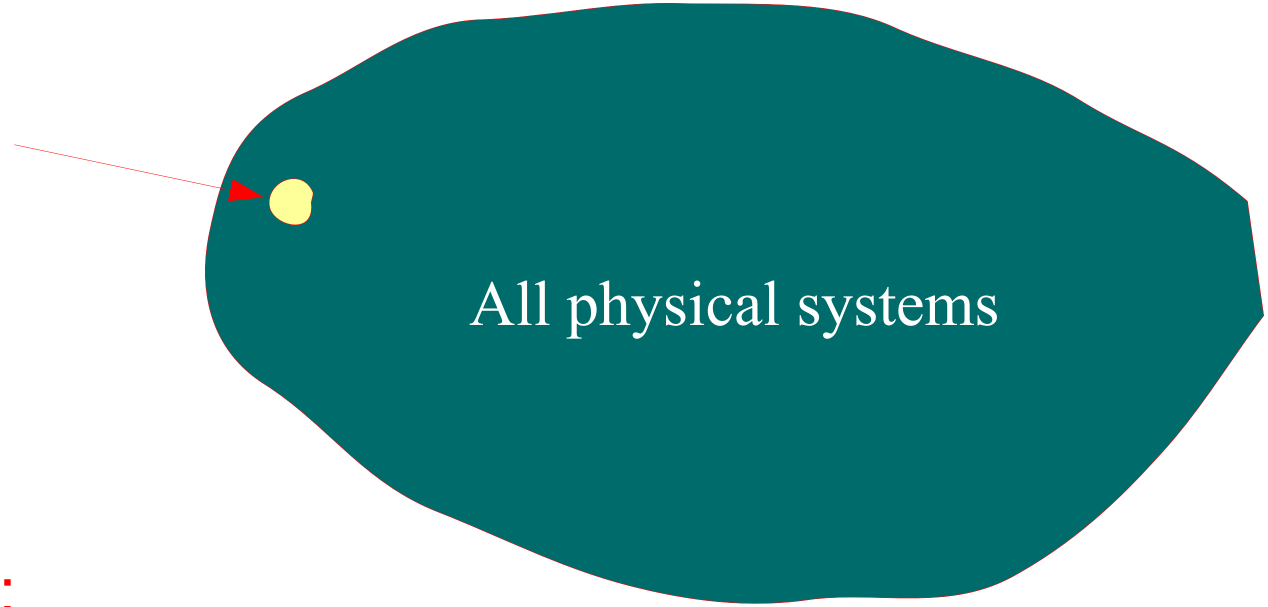
Outline

- Intro
  - Quantum integrability: why interesting?
  - Phenomenological signatures of integrability
- Our models
  - non-integrable: mobile impurity in a Fermi gas
  - integrable: mobile impurity of the same mass
  - integrable: Lieb-Liniger gas
- Static properties
  - spectrum
  - wavefunctions
- Dynamics
  - linear response theory
  - dynamic structure factor
- Conclusions and perspectives

Quantum integrability: why interesting?



Why bother?

Theoretical perspective:

Interesting because one can treat strongly correlated systems exactly!

- spectrum
- thermodynamics

$$L k_j = 2\pi n_j - 2\sum_{i=1}^{N} \arctan[(k_j - k_i)/c]$$

$$E = \frac{1}{2}\sum_{j=1}^{N} k_j^2, \quad K = \sum_{j=1}^{N} k_j$$

Mathematics meets physics

- METHODS and THEORIES for studying correlation functions (Algebraic Bethe Ansatz)

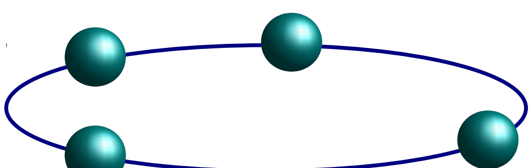
Quantum integrability: why interesting?

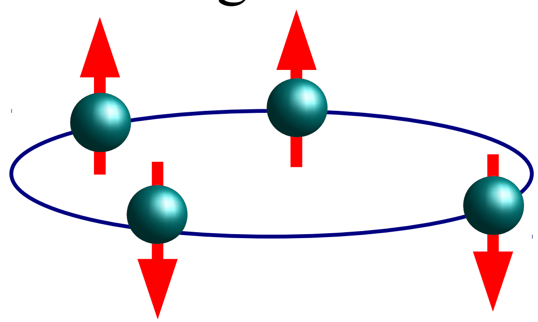
Ultracold gases:
- very clean and isolated systems
- control over experimental parameters

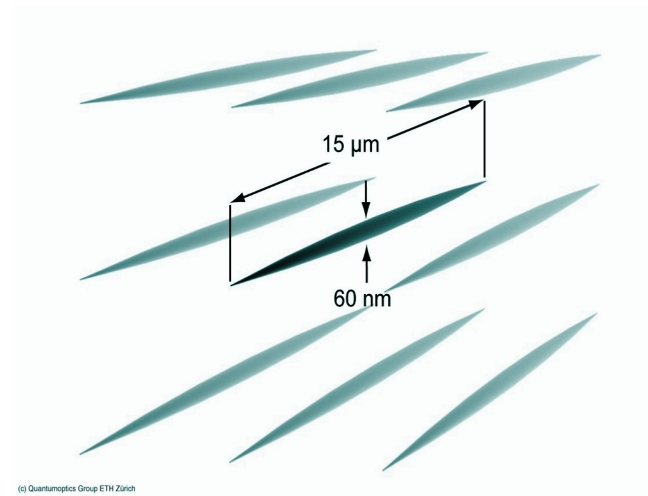
Optical potential or large magnetic field gradients (atom chips) $\Rightarrow$ $\hbar\omega \gg \epsilon$

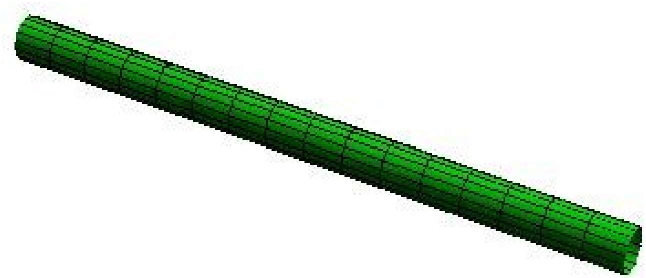
One-dimensional gas with 1D coupling constant $g_{1D} = \frac{\hbar^2}{\mu a_{1D}} \approx \frac{\hbar^2 a}{\mu l_0^2}$

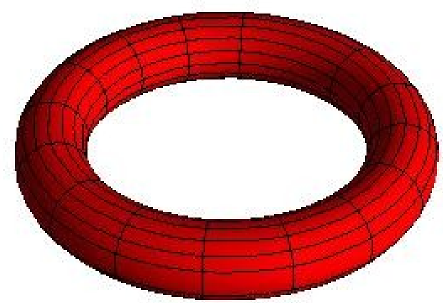
Lieb-Liniger and Yang-Gaudin integrable models











Quantum integrability: why interesting?

Experimentalist's viewpoint:

Ok, suppose we can create integrable systems but why?

Think about the following:

- What is the difference between integrability and non-integrability in terms of observables?

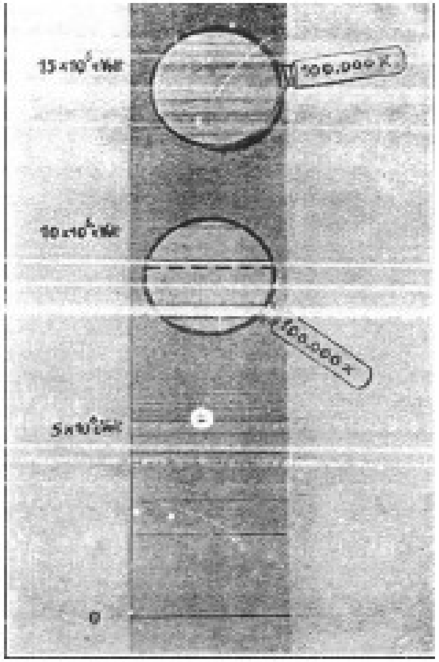
- What measurements should we perform?

Phenomenological signatures of quantum integrability

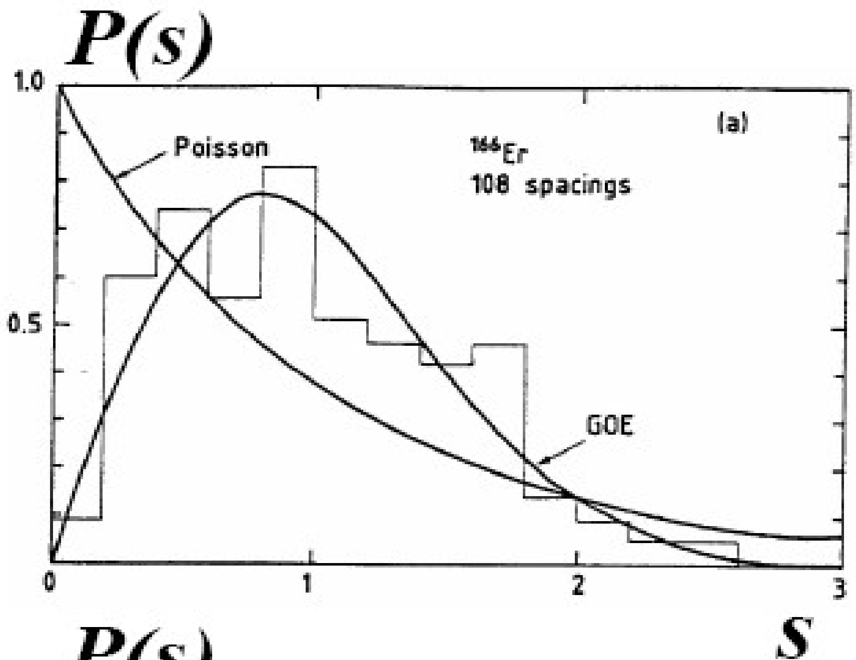
- Nearest Neighbor Spacing (NNS) distribution
- Localization of eigenstates
- Transport and thermalization dynamics

- This talk: response to external perturbation

NNS distribution

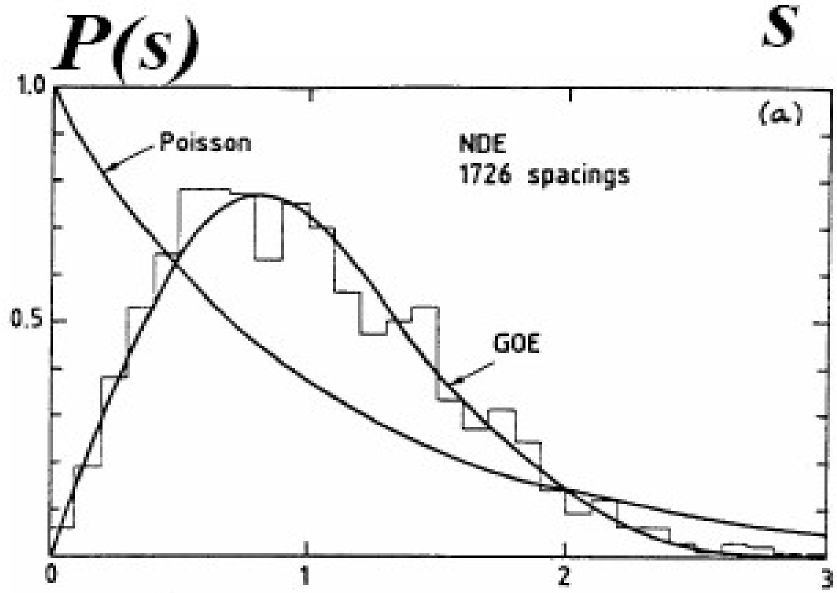


N. Bohr, Nature 137 (1936) 344.



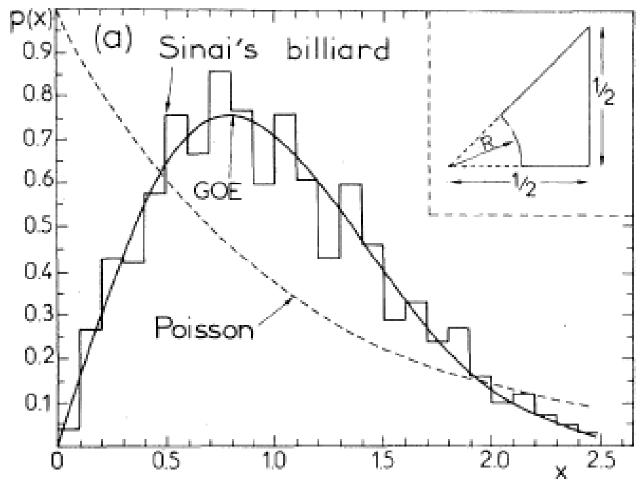
Particular nucleus

^{166}Er



Spectra of several nuclei combined (after spacing) rescaling by the mean level

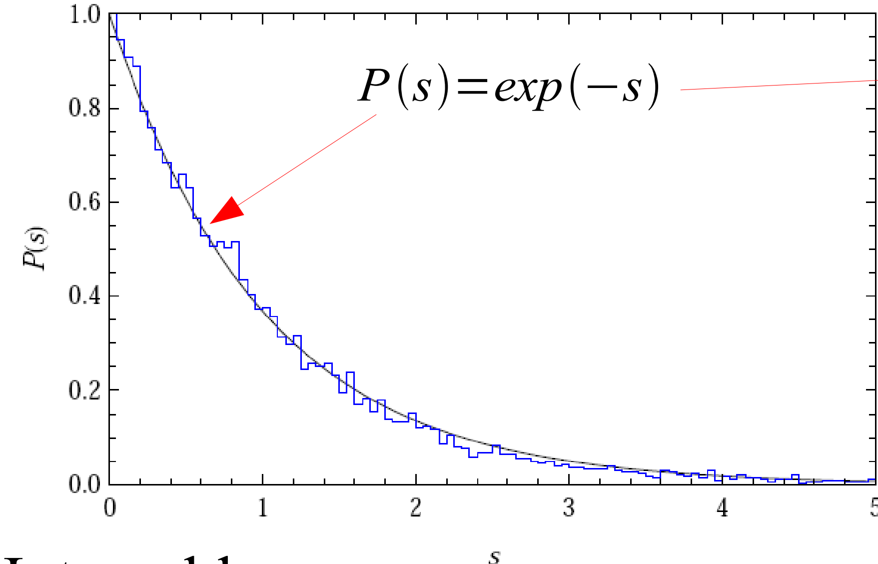
NNS distribution

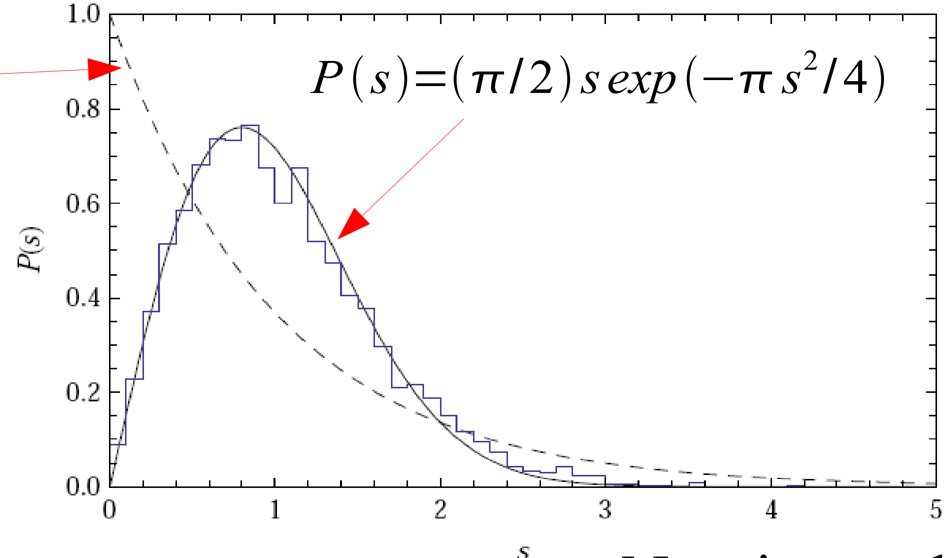


Bohigas, Giannoni, Schmit (1984) conjectured:
Spectra of time-reversal-invariant systems whose classical analogs are K systems show the same fluctuation properties as predicted by GOE (Gaussian Orthogonal Ensemble of random matrices)

universality of the laws of level fluctuations in quantal spectra already found in nuclei and to a lesser extent in atoms. Then, they should also be found in other quantal systems, such as molecules, hadrons, etc.

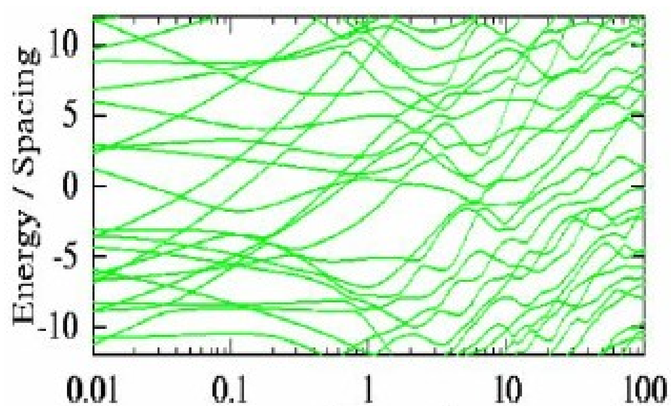
NNS distribution





Integrable
Levels are not correlated
(Berry & Tabor 1977)
Poisson distribution

Non-integrable
Levels repel e.a. (non-crossing rule)
(v. Neumann & Wigner 1929)
Wigner-Dyson statistics



Localization of eigenstates

Classical integrability: N degrees of freedom ⇒ N integrals of motion

$$H(q,p) \rightarrow H(\phi, J)$$

J, ϕ - action-angle variables

J_i - generalized momenta

ϕ_i - generalized coordinates

correspondence principle ⇐

quantum mechanical eigenstates are localized in J-space

⇕

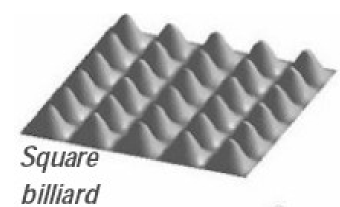
eigenstates localized in different places do not repel each other

⇕

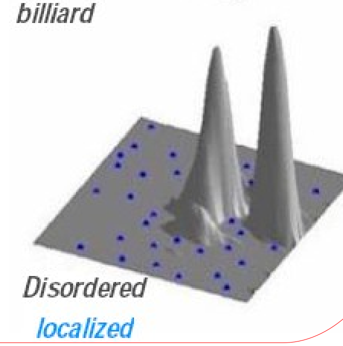
Poisson spectral statistics

Examples:

- eigenstates of an ideal gas in a box are localized in momentum space

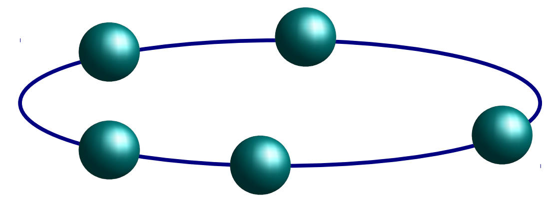


- Anderson transition in disordered systems – localization in coordinate space



Lieb-Liniger model

Integrable model of N bosons on a ring

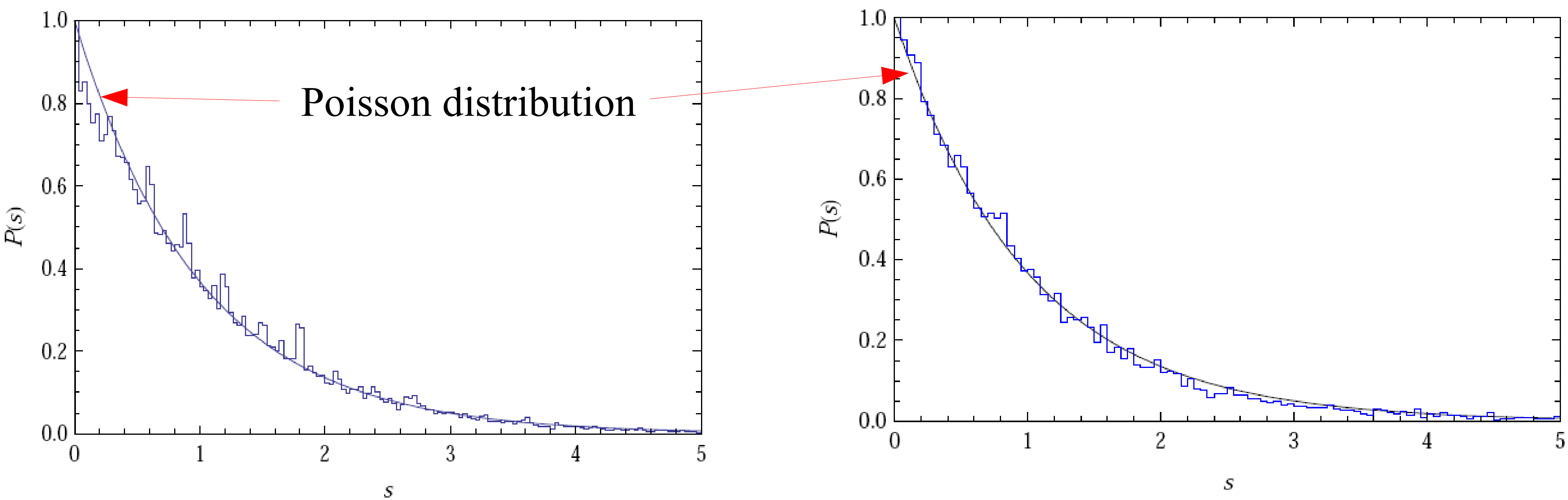


$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + c\sum_{i<j}\delta(x_i - x_j)$$

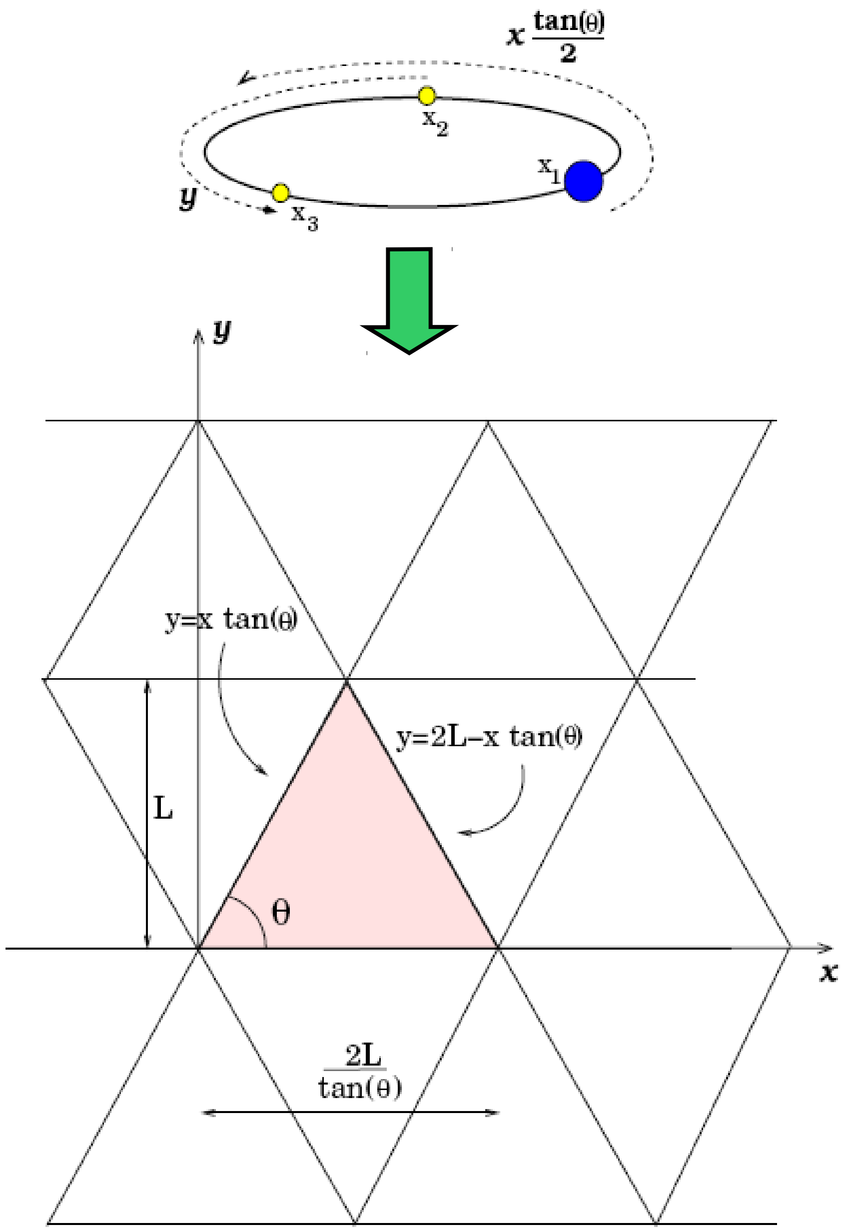
State of the system is determined by a set of N (half-)integers $\{n_j\}$

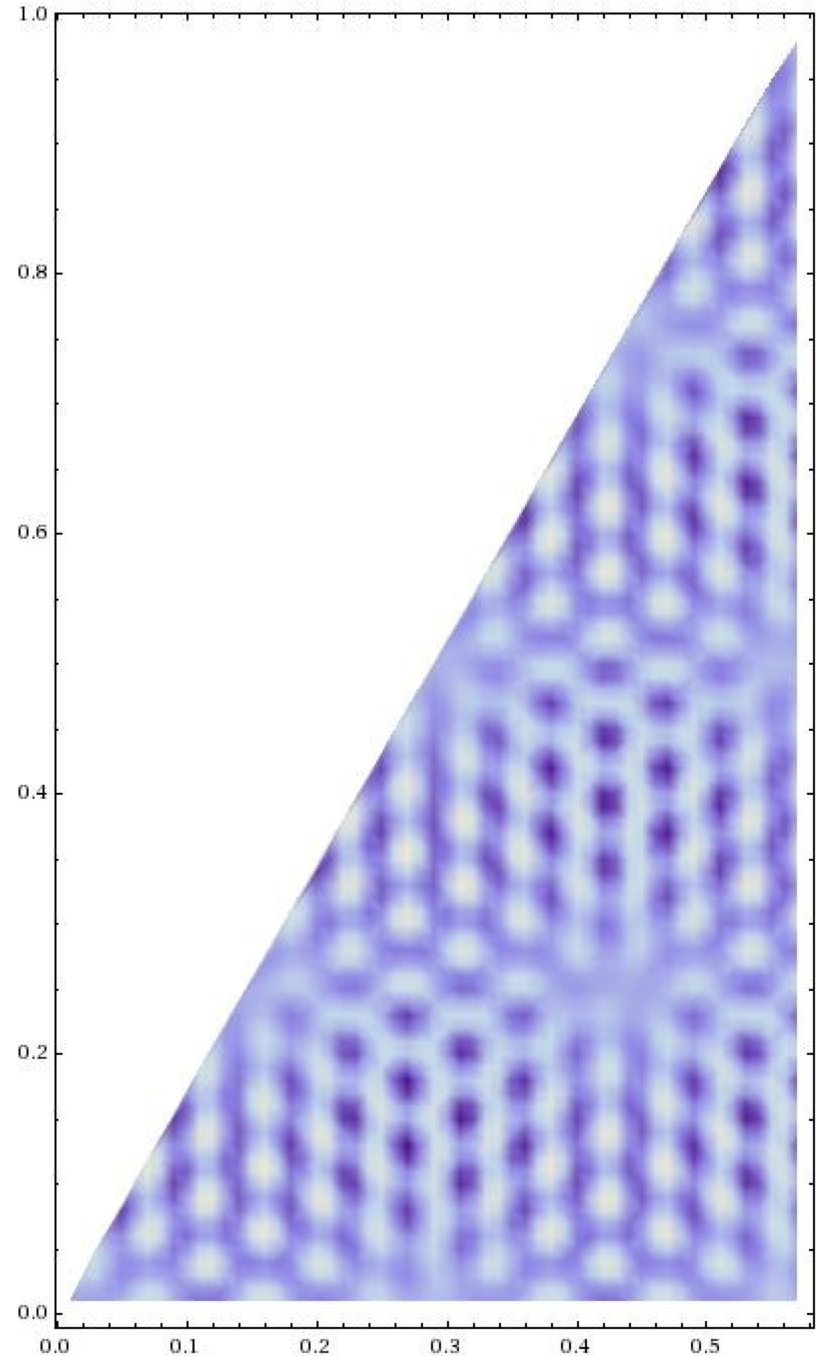
$$Lk_j = 2\pi n_j - 2\sum_{i=1}^{N}\arctan[(k_j - k_i)/c] \Rightarrow E = \frac{1}{2}\sum_{j=1}^{N}k_j^2, \quad K = \sum_{j=1}^{N}k_j$$

$N = 3$ $\qquad$ $N = 4$



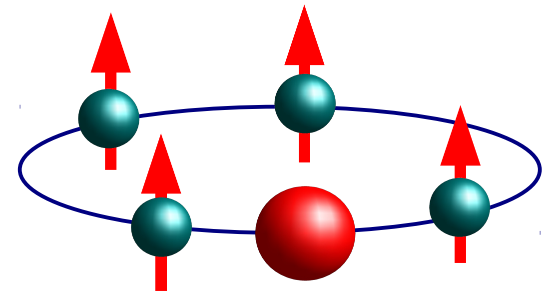
Wavefunction for N=3





Mobile impurity in a Fermi gas

Single mobile impurity immersed in a gas of N-1 fermions on a ring

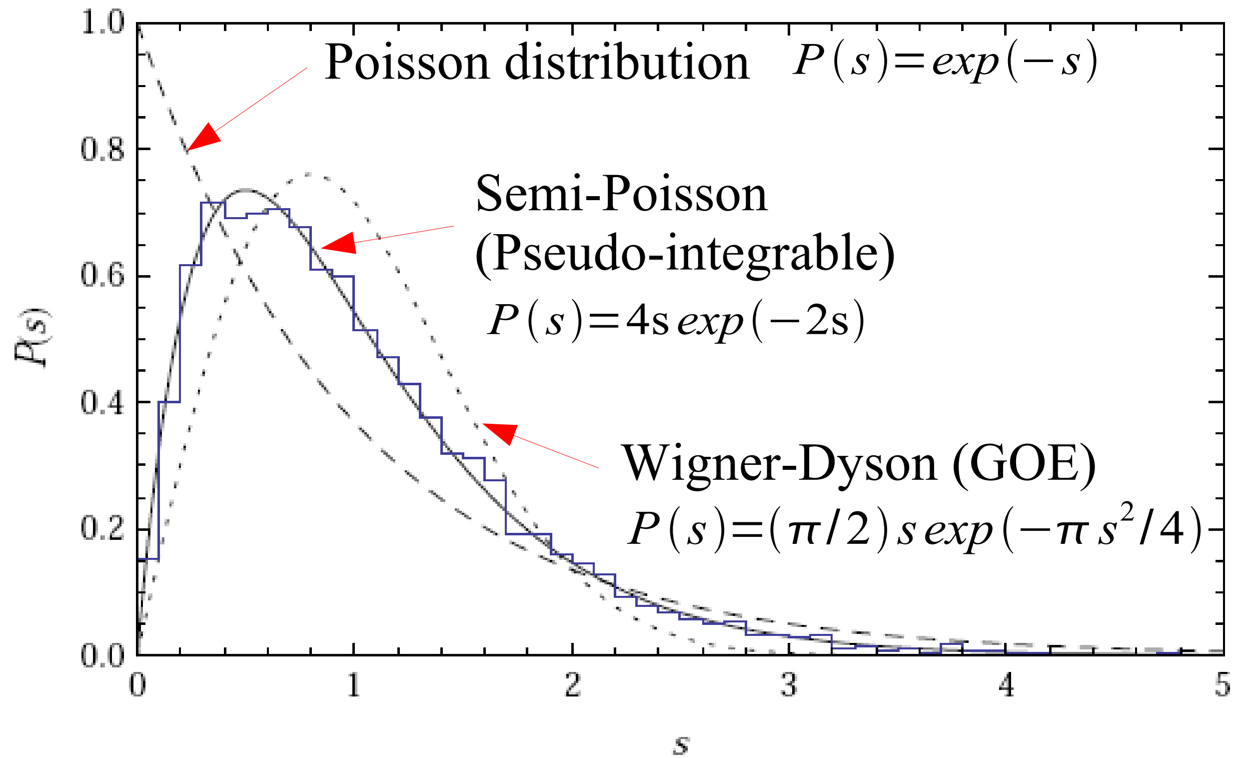


$$H=-\frac{m}{2M}\frac{\partial^2}{\partial x_1^2}+\sum_{i=2}^{N}\left[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2}+c\,\delta(x_i-x_1)\right]$$

The model is not integrable for $M \neq m$ and we solve it numerically for M/m=87/40 (Rb-K mixture) and N=3,4

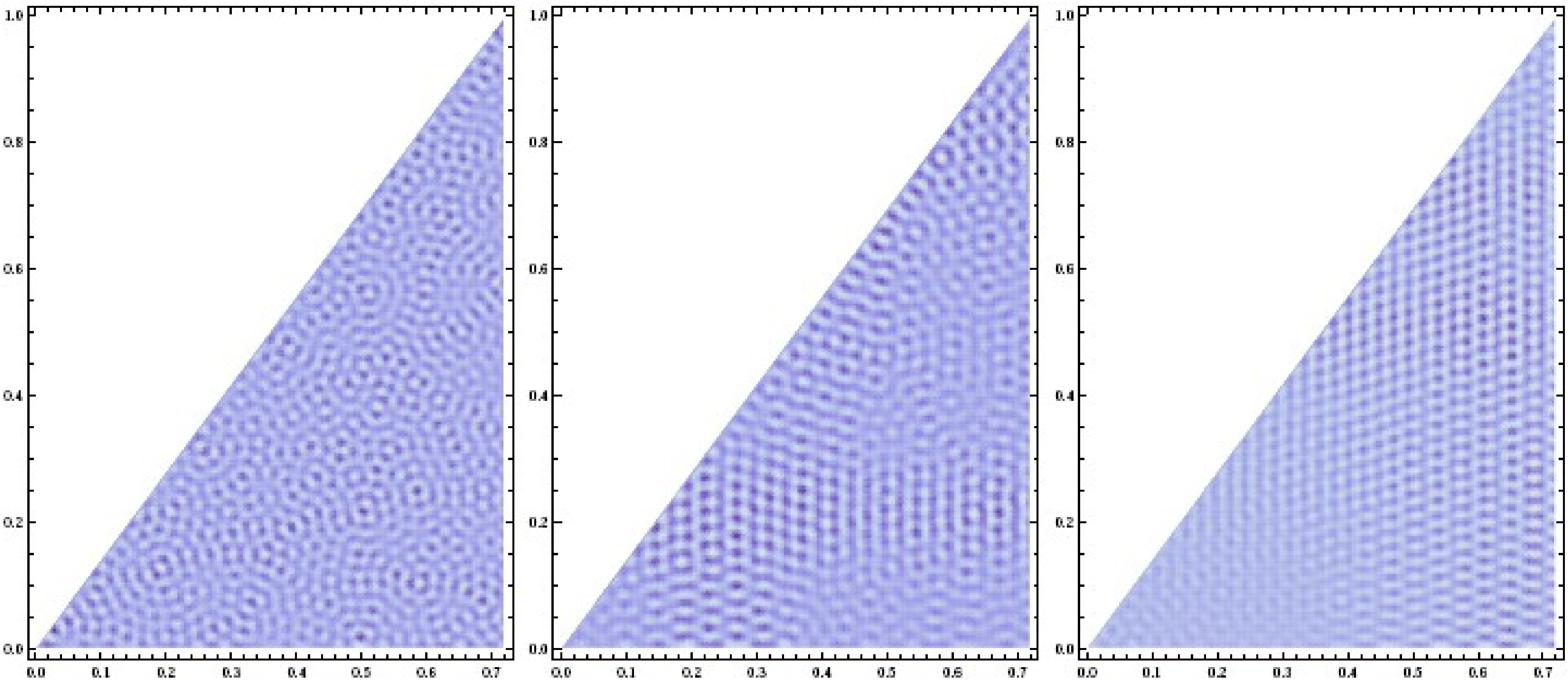
$N=3$

NNS distribution for 7000 states



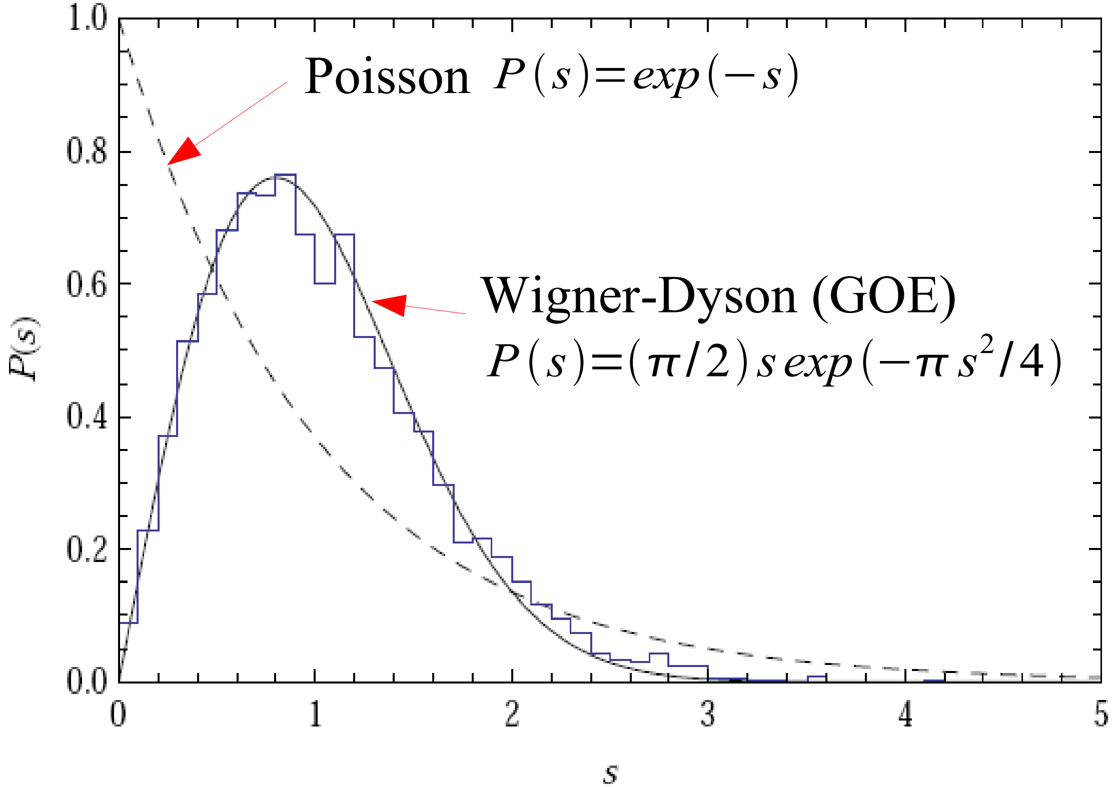
Wavefunctions N=3

State number 1835 1296 1676

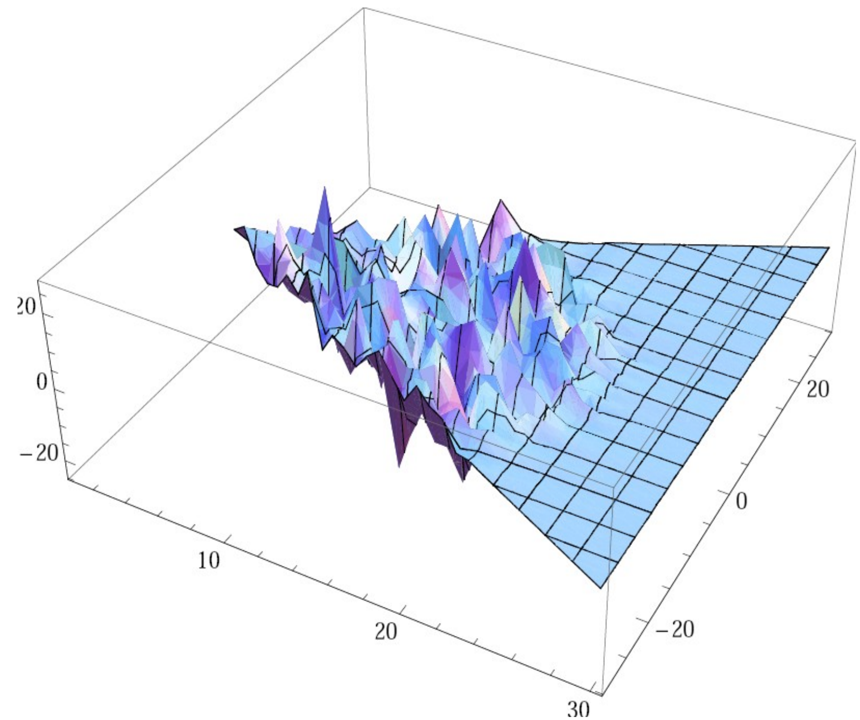


N=4

NNS
distribution
for 3879 states



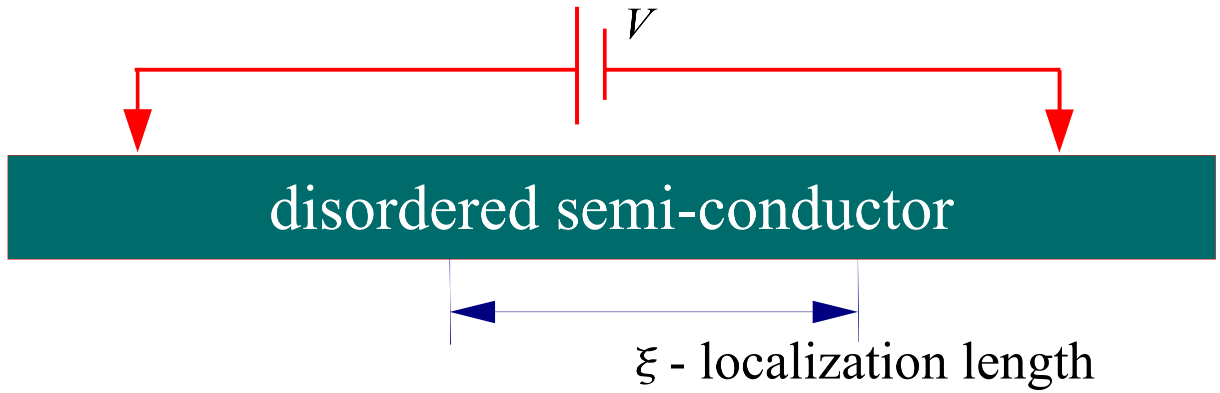
Eigenfunction for $x_1=x_2$
(actually, its Fourier transform)



Transport and thermalization of integrable systems

If initial state (not necessarily eigenstate) is localized in J-space, it will stay localized during time evolution

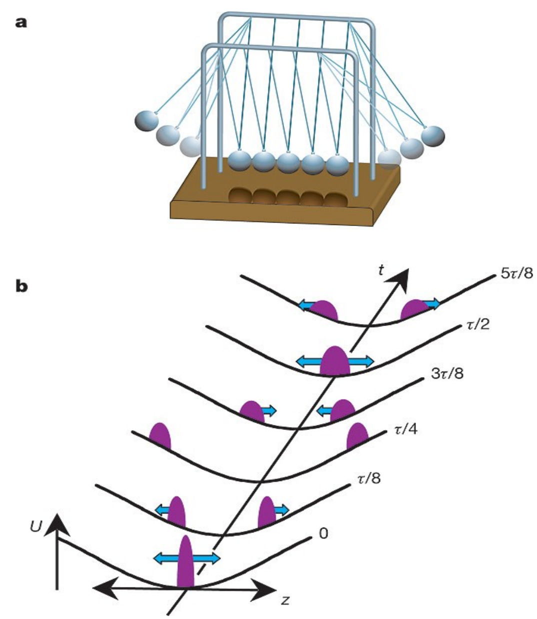
- Anderson insulator



- ``A Quantum Newton's cradle'' (Kinoshita, Wenger, & Weiss 2006)

Localization in momentum space!

Fast thermalization for weaker transversal confinement, when $\hbar\omega \sim \epsilon$ and the system is not integrable



Modulation of *c*. Linear response

Integrable

$$H=-\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+c\sum_{i<j}\delta(x_i-x_j)$$

Non-integrable

$$H=-\frac{m}{2M}\frac{\partial^2}{\partial x_1^2}+\sum_{i=2}^{N}[-\frac{1}{2}\frac{\partial^2}{\partial x_i^2}+c\,\delta(x_i-x_1)]$$

Add weak modulation of the coupling constant $c(t)=c+2\delta g\cos(\omega t),\ \delta g\ll c$

$$F=\delta g\sum_{i<j}\delta(x_i-x_j) \Rightarrow H=H_0+2F\cos\omega t \Leftarrow F=\delta g\sum_{j=2}^{N}\delta(x_j-x_1)$$

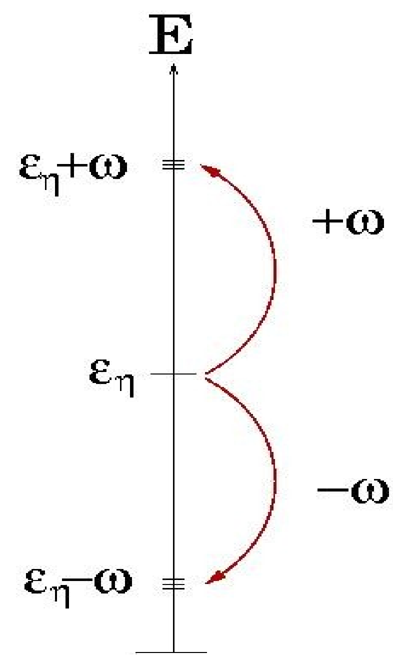
Dynamic structure factor

- Initial state : $|\Psi_0\rangle = |\psi_\eta\rangle$.
- The probability to remain in the state η decreases with the rate :

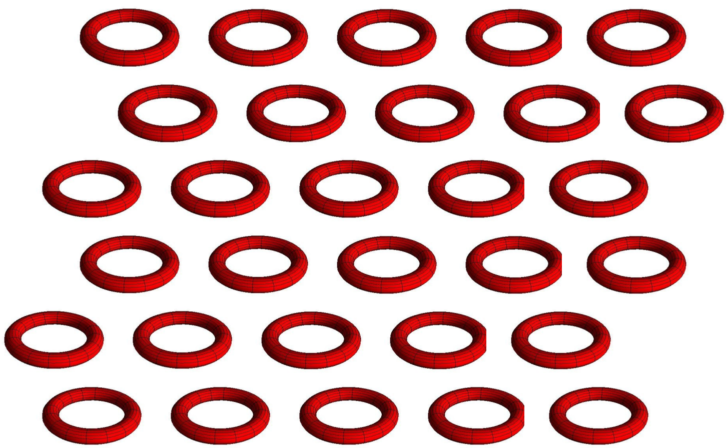
$$\Omega = 2\pi\,[S(\epsilon_\eta,\omega)+S(\epsilon_\eta,-\omega)]$$

where S is the dynamic structure factor :

$$S(\epsilon_\eta,\omega)=\sum_\nu \delta(\omega-(\epsilon_\nu-\epsilon_\eta))|\langle\psi_\nu|F|\psi_\eta\rangle|^2$$



Linear response



Diffusion of the state population:

$$\frac{\partial f(\epsilon)}{\partial t} = \frac{\partial}{\partial \epsilon}\left[D(\epsilon,\omega)\frac{\partial f(\epsilon)}{\partial \epsilon}\right],$$

where f is the probability density and

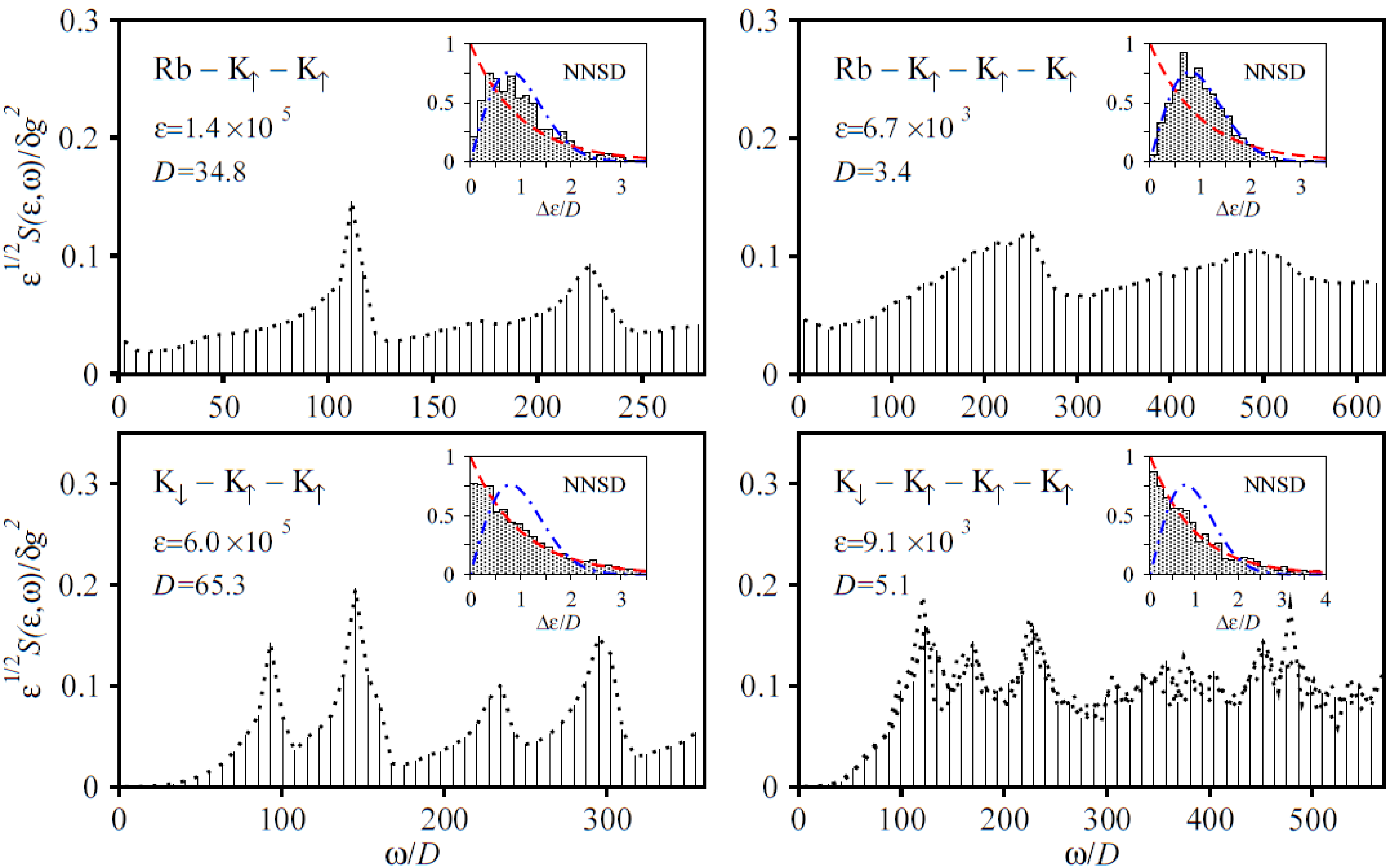
$$D(\epsilon,\omega) = 2\pi S(\epsilon,\omega)\omega^2$$

is the diffusion constant. Energy transfer from the external field to the system

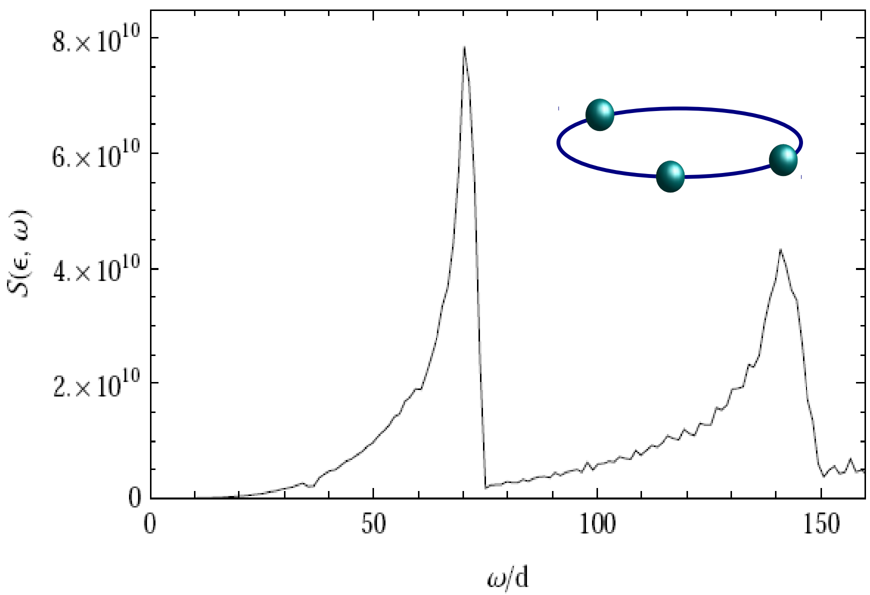
$$\frac{\partial E}{\partial t} = 2\pi\omega^2 \int \epsilon \frac{\partial}{\partial \epsilon}\left[S(\epsilon,\omega)\frac{\partial f(\epsilon)}{\partial \epsilon}\right] d\epsilon$$

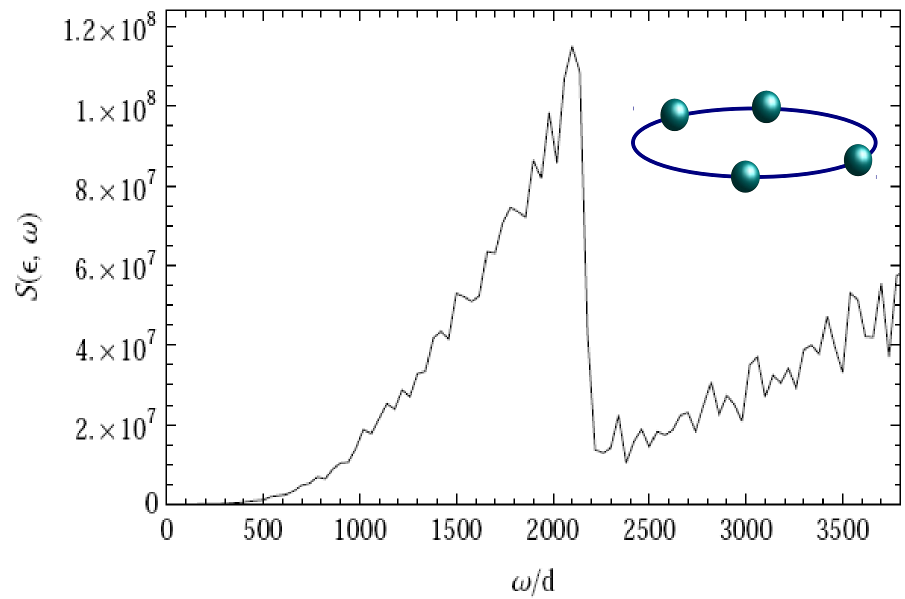
Asymptotic behavior of $S(\epsilon,\omega)$ at small ω gives the dissipative part of the response of the system to a slow variation of its Hamiltonian and, therefore, measures the degree at which this variation can be assumed adiabatic!

Dynamic structure factor

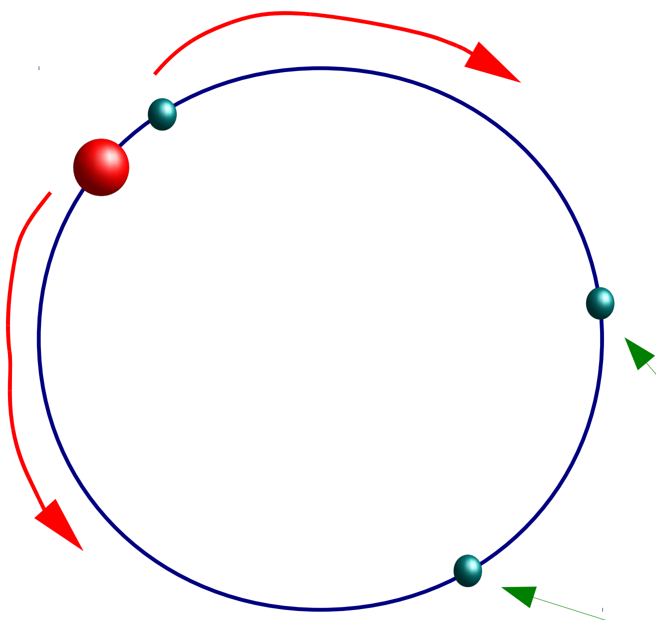


Dynamic structure factor. Lieb-Liniger





Large ω behavior. Binary approximation



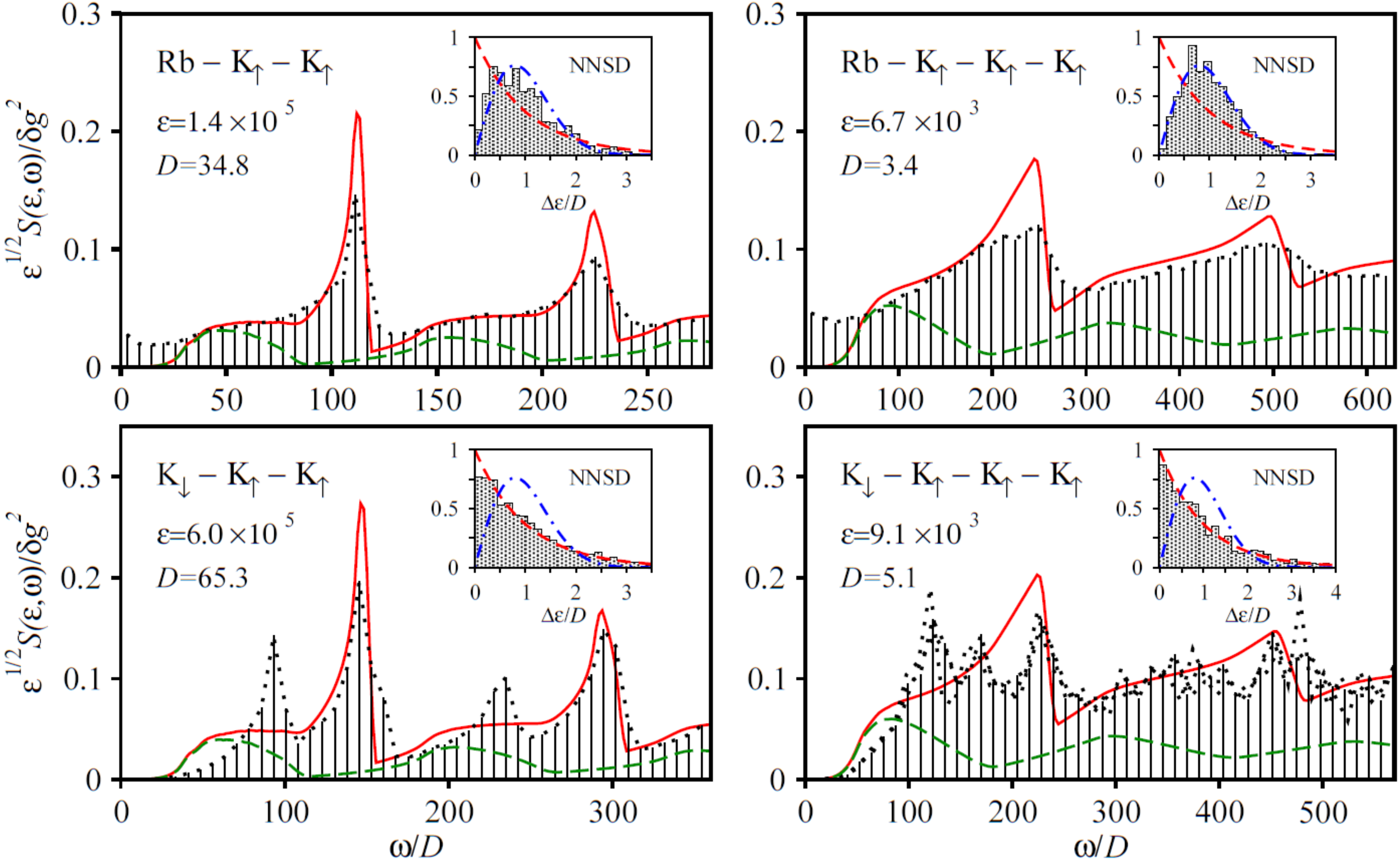
$$S(\epsilon_\nu,\omega)=\sum_\eta \delta(\omega-\epsilon_\eta+\epsilon_\nu)|\langle\psi_\eta|F|\psi_\nu\rangle|^2$$

$$\tilde{S}(E,\omega)=(Number\ of\ interacting\ pairs)\times$$

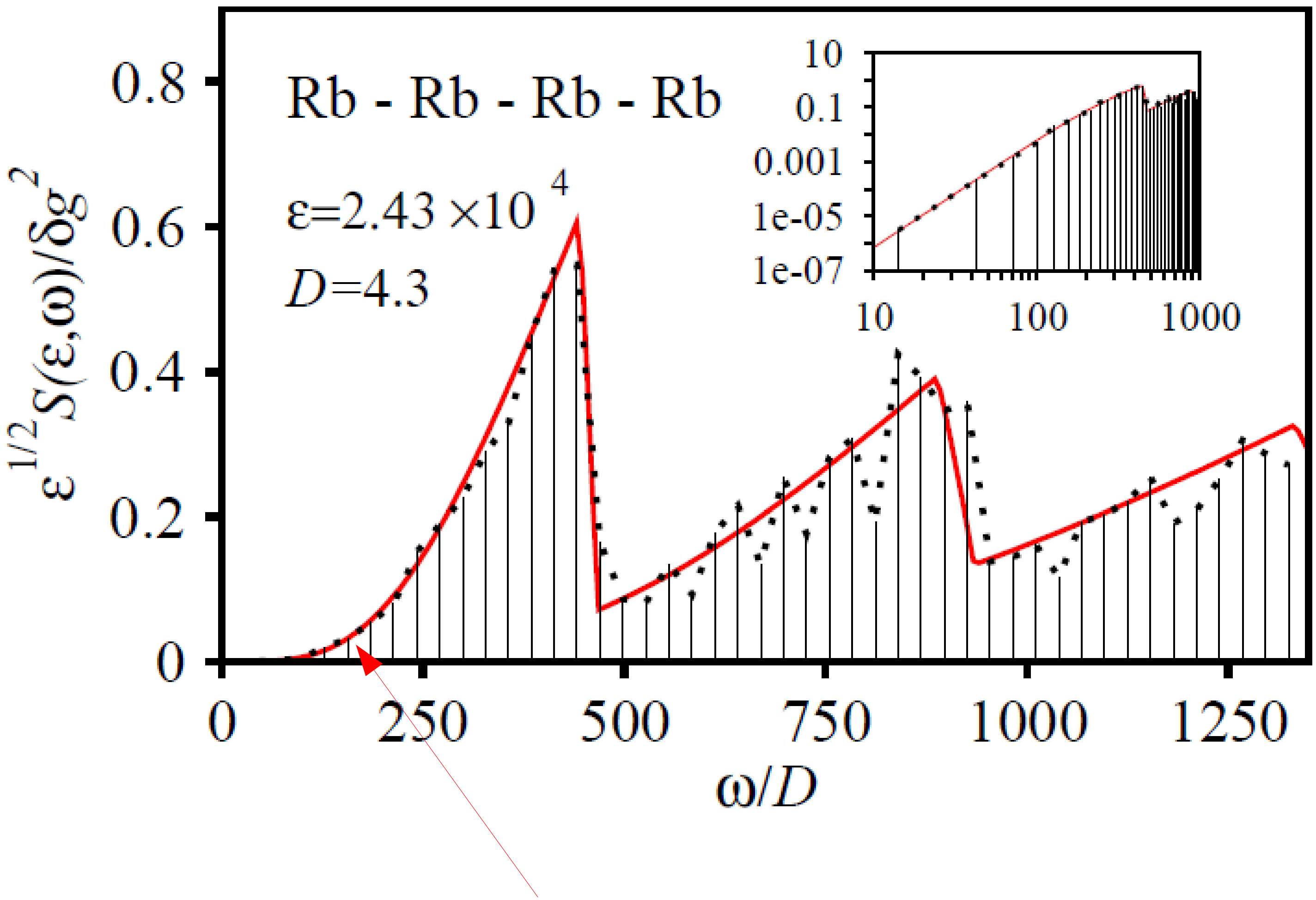
$$\times\sum_{\nu,\eta}\rho_{N-2}(E-\epsilon_\nu^{2body})\,\delta(\omega-\epsilon_\eta^{2body}+\epsilon_\nu^{2body})|\langle\psi_\eta^{2body}|F|\psi_\nu^{2body}\rangle|^2$$

Density of states for
the other N-2 atoms

Large ω behavior. Two-body physics

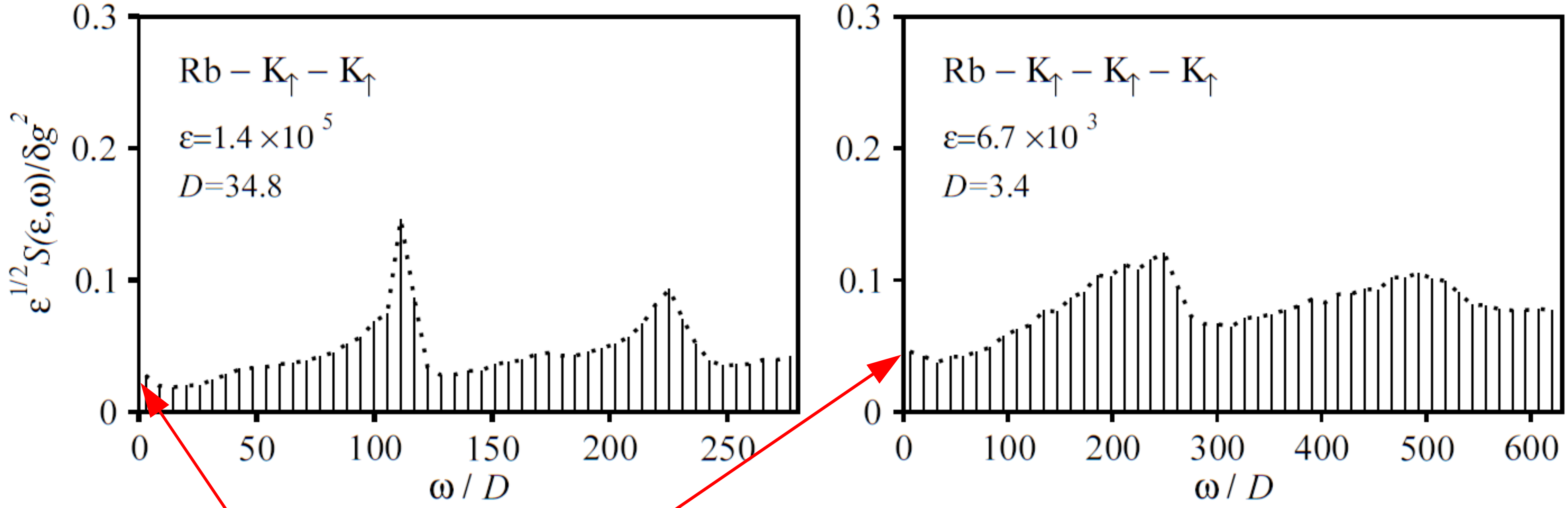


Binary approximation. Lieb-Liniger



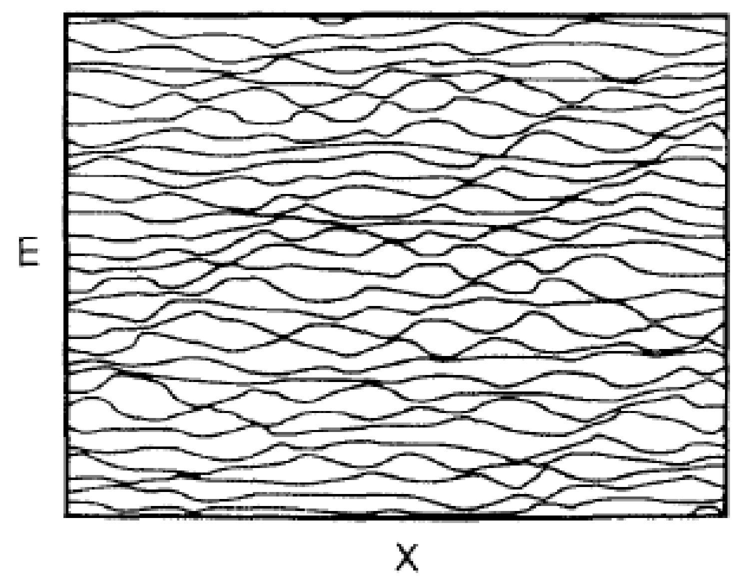
Binary approximation works very well also for small ω !

Small frequency behavior: non-integrable case

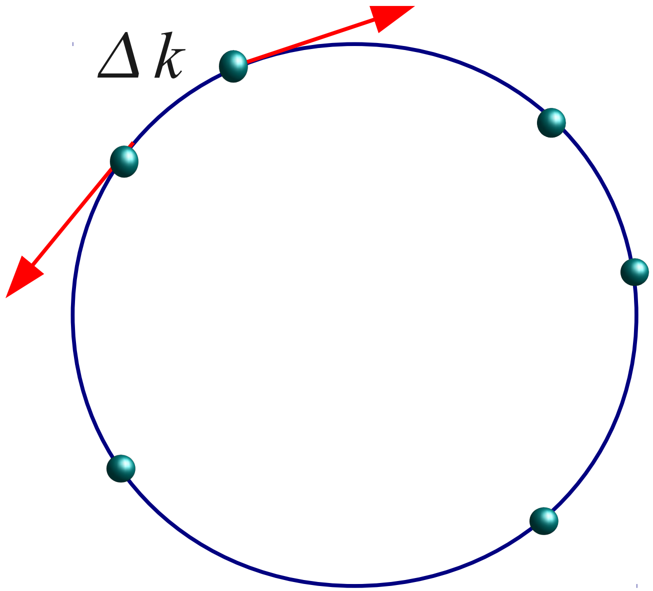


$$S(\epsilon,\omega)\rightarrow const\,,\ \omega\sim d$$
$$S(\epsilon,\omega)\propto\omega\,,\ \omega\ll d$$

consistent with the approach of Wilkinson (1989) who considered statistics of Landau-Zener tunnelings in the RMT approach



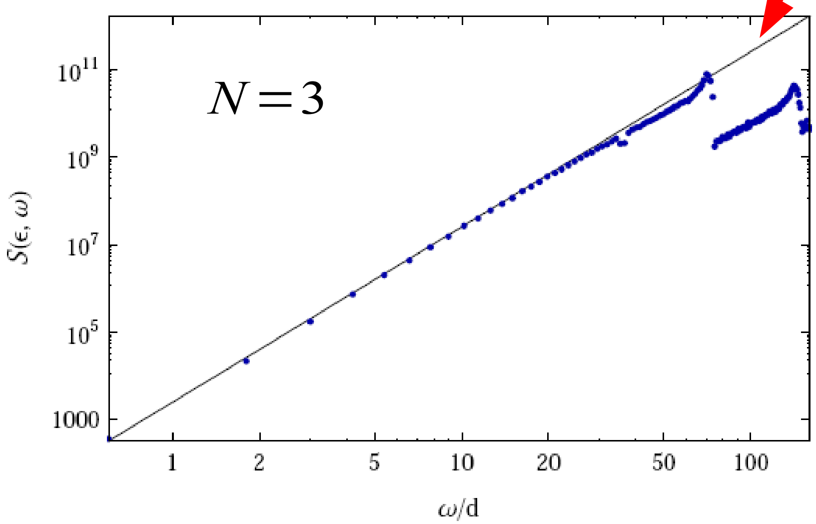
Small frequency behavior: integrable case

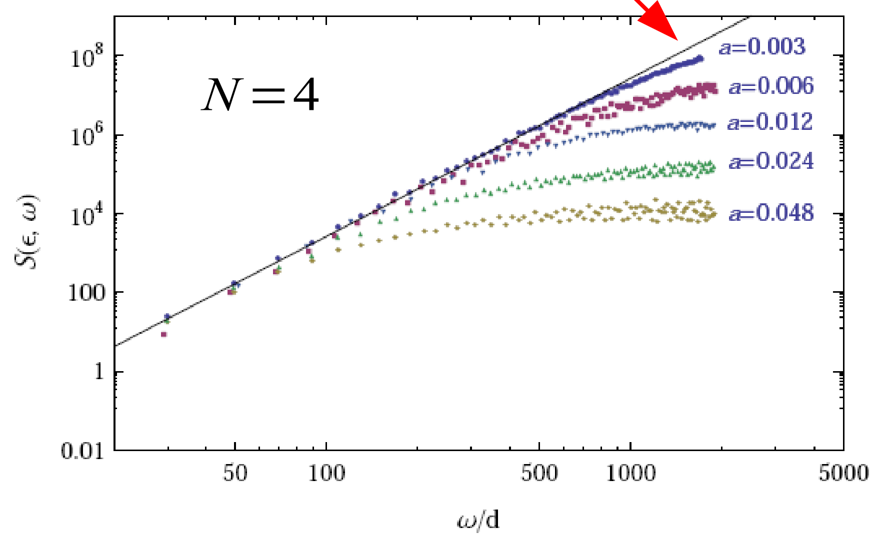


$$\delta E = \omega \approx \frac{\Delta k(\Delta k' - \Delta k)}{\mu} \approx \frac{\Delta k}{\mu}\frac{2\pi \times integer}{L}$$

$$\langle \psi_{\Delta k'} | F | \psi_{\Delta k} \rangle \propto \Delta k \Delta k' \propto \omega^2$$

$$S(\epsilon, \omega) \approx \frac{\zeta(5)}{2^7 \pi^{11/2}} \frac{N(N-2)\Gamma(N/2+1/2)}{\Gamma(N/2)} \frac{A^2 L^3 \omega^4}{\sqrt{\epsilon}}$$





Heating rate & adiabaticity

- Heating rate

$$\frac{\partial f(\epsilon)}{\partial t} = \frac{\partial}{\partial \epsilon}\left[D(\epsilon,\omega)\frac{\partial f(\epsilon)}{\partial \epsilon}\right],$$

$$\boxed{D(\epsilon,\omega) = 2\pi S(\epsilon,\omega)\omega^2 \propto \omega^6} \quad \text{cf.} \quad \boxed{D_{Kubo} \propto \omega^2}$$

This can be observed by measuring the heating rate versus frequency

- Adiabaticity

  - Initial state : $|\Psi_0\rangle = |\psi_\eta\rangle$.
  - The probability to remain in the state η decreases with the rate :

$$\Omega = 2\pi\,[S(\epsilon_\eta,\omega) + S(\epsilon_\eta,-\omega)]$$

Integrable case. According to our model:
$S(\epsilon,\omega) = 0,\ \omega < \sim (2\pi)^2/L^2$

Non-integrable system reacts to frequencies smaller than mean level spacing, which grows exponentially with the system size

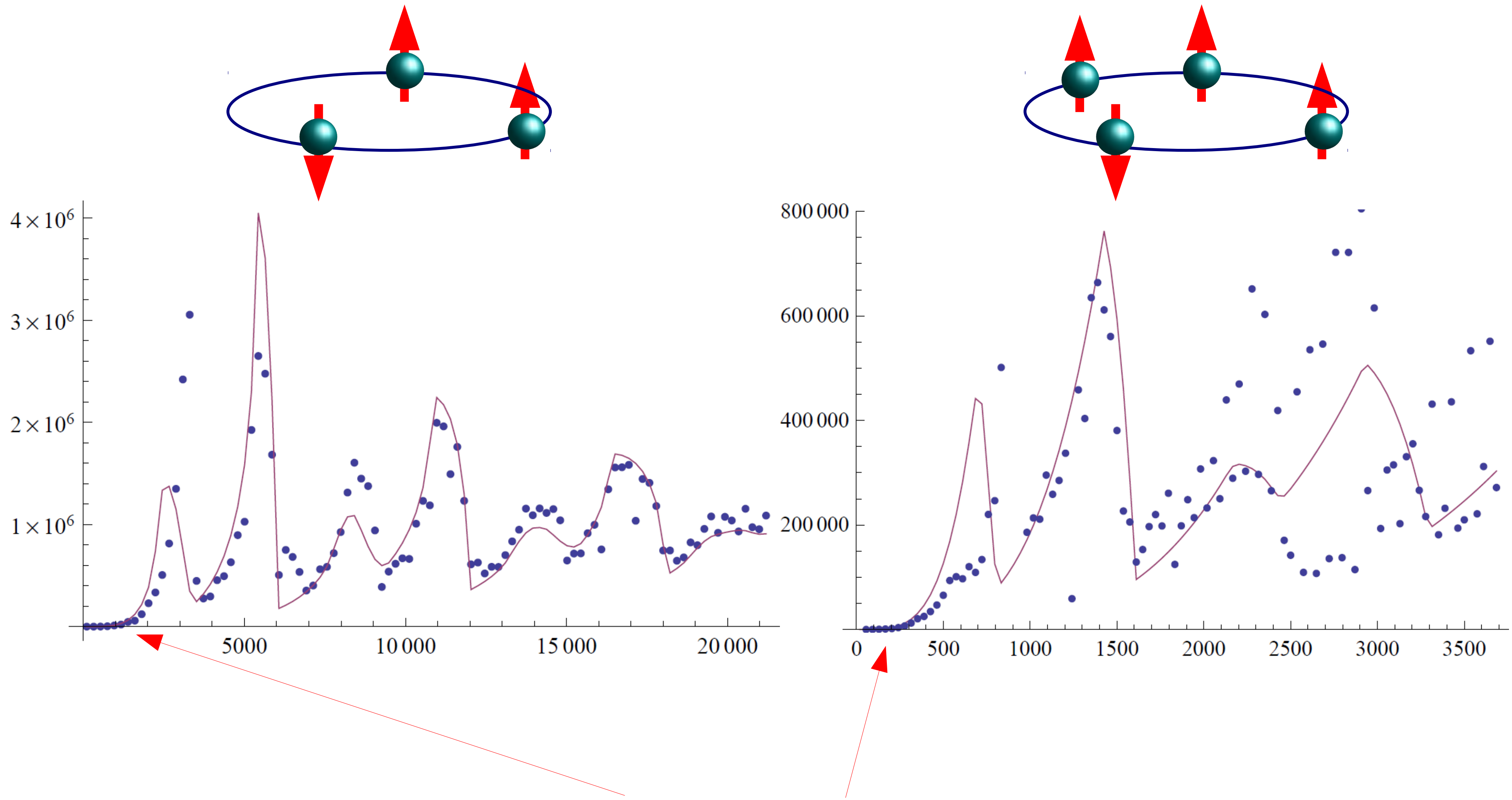
Conclusions & Perspectives

- Response of integrable and non-integrable systems to external perturbation can be dramatically different
- Dynamic structure factor for our integrable model is strongly suppressed at low frequencies meaning that
- Energy neighbors are not coupled by the perturbation
- Pseudo-gap important for adiabatic manipulations

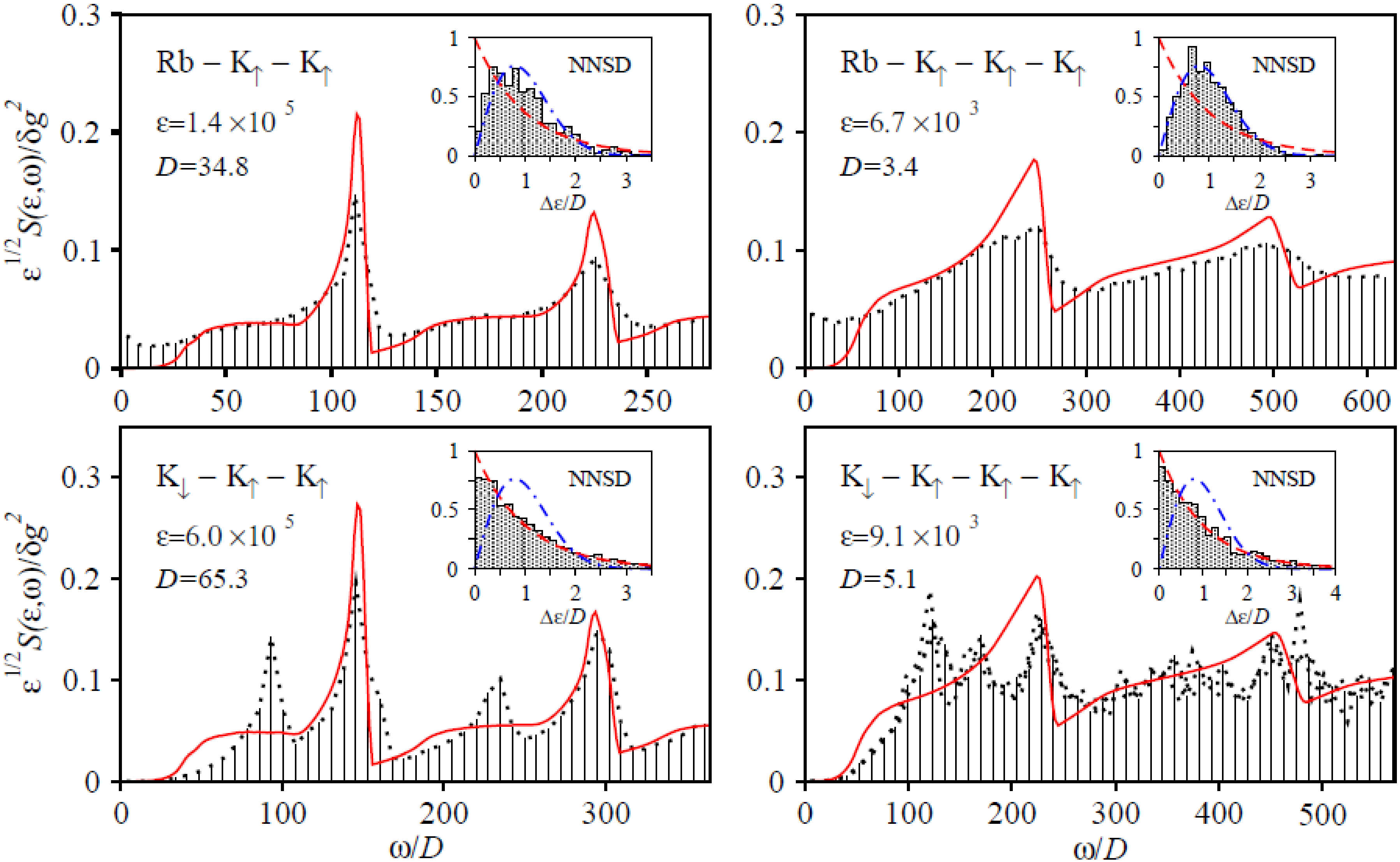
Perspectives:

- N>4
- Degenerate regime

Yang-Gaudin model



Suppression at small ω like in the bosonic Lieb-Liniger case!



Rb – K↑ – K↑
ε=1.4 ×10 5
D=34.8
NNSD
Δε/D
Rb – K↑ – K↑ – K↑
ε=6.7 ×10 3
D=3.4
NNSD
Δε/D
K↓ – K↑ – K↑
ε=6.0 ×10 5
D=65.3
NNSD
Δε/D
K↓ – K↑ – K↑ – K↑
ε=9.1 ×10 3
D=5.1
NNSD
Δε/D
ε 1/2 S(ε,ω)/δg 2
ω/D