Hydrodynamics — A tool for Strongly Coupled Systems

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Hydrodynamics

Hydrodynamics in a Nutshell

- Hydrodynamics = Conservation of Energy and Momentum

$$\partial_\mu T^{\mu\nu} = 0$$

- $T^{\mu\nu}$ = Energy Momentum Tensor. For fluids, can only depend on

$$\epsilon(P, T), u^\mu, g^{\mu\nu}, \text{gradients thereof}$$

- In equilibrium and local rest frame ($u^\mu = (1, 0, 0, 0)$),

$$T^{\mu\nu}_{\text{eq,LRF}} = \text{diag}(\epsilon, P, P, P)$$

Hydrodynamics in a Nutshell

- Only possible form

$$T^{\mu\nu} = (\epsilon + P)\, u^{\mu} u^{\nu} - P g^{\mu\nu} + \text{gradients}$$

- Gradients:

$$\text{gradients} = \eta \nabla^{<\mu} u^{\nu>} + \zeta\, (g^{\mu\nu} - u^{\mu} u^{\nu})$$

- η... Shear viscosity coefficient
- ζ... Bulk viscosity coefficients (usually neglected)

Hydrodynamics in a Nutshell

- Neglect η, ζ and take non-relativistic limit:

$$u_\nu \partial_\mu T^{\mu\nu} = 0 \to \partial_t \rho + \partial_i \left(\rho v^i \right) = 0$$

Equation of Continuity

- Neglect η, ζ and take non-relativistic limit:

$$\left(g^i_\nu - u_\nu u^i \right) \partial_\mu T^{\mu\nu} = 0 \to \rho \left(\partial_t v^i + v^j \partial_j v^i \right) = -\partial^i P$$

Euler Equation

- Include η, ζ:

$$\left(g^i_\nu - u_\nu u^i \right) \partial_\mu T^{\mu\nu} = 0 \to \mathrm{Navier-Stokes\ Equation}$$

Non-relativistic Ideal Fluid Dynamics

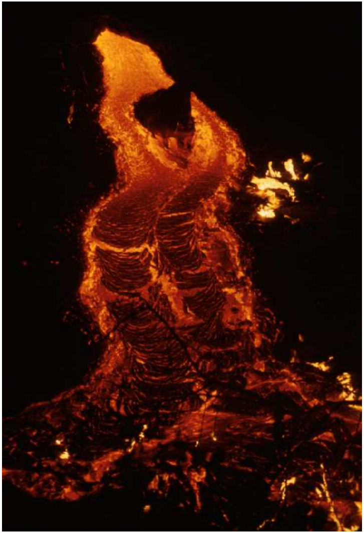
$$\partial_t v^i + v^m \partial_m v^i = -\frac{1}{\rho}\partial_j \delta^{ij} p$$

[L. Euler, 1755]

- Non-linear
- Non-dissipative: “Ideal Fluid Dynamics”

Non-linear & Non-dissipative: Turbulence

Non-linear & Dissipative: Laminar



Non-linear & Dissipative: Laminar

Viscosity dampens turbulent instability!

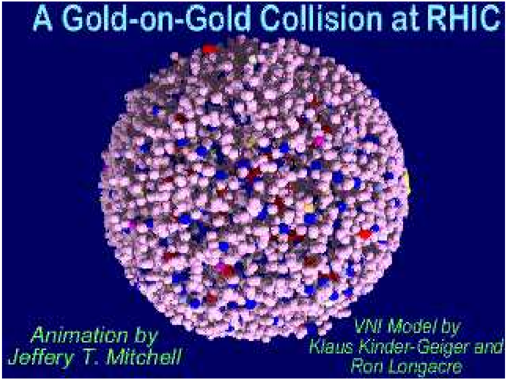
Strongly Coupled Systems

- Consider dimensionless ratio η/s where s is entropy density.
- Generally
$$\frac{\eta}{s} \propto \lambda$$
where λ is mean free path
- Strong interactions mean small λ. Small η/s means good fluid behavior

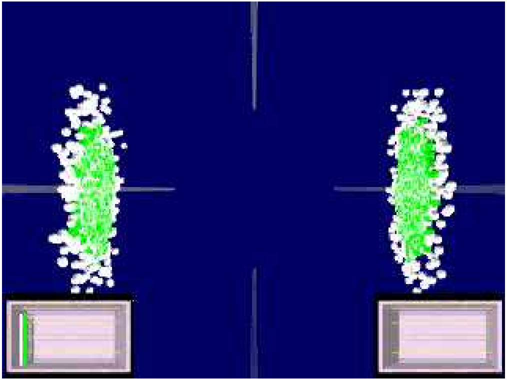
Strongly Coupled Systems Are Good Fluids!

Heavy-Ion Colliders: Relativistic Heavy-Ion Collider

Au+Au Collisions at RHIC



Au+Au Collisions at RHIC



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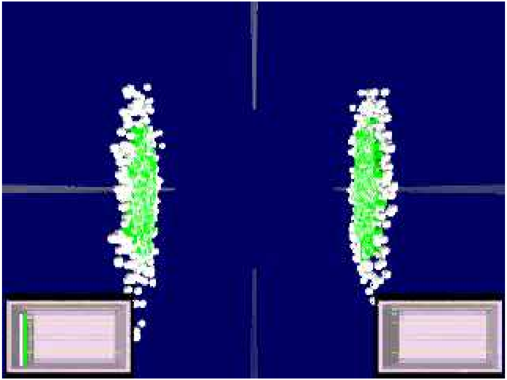

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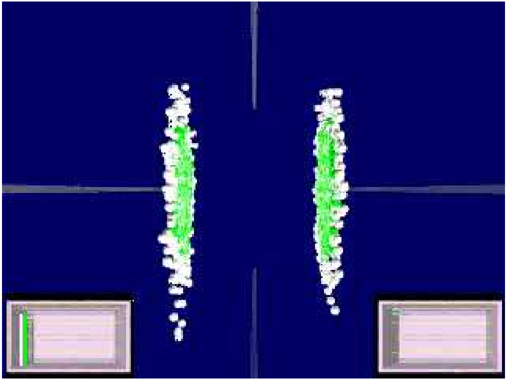

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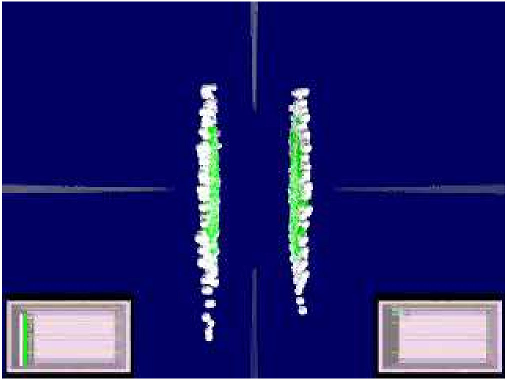

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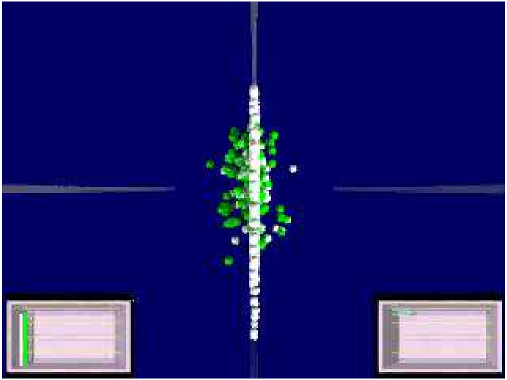

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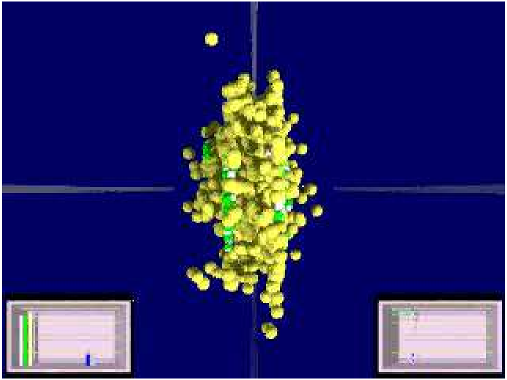

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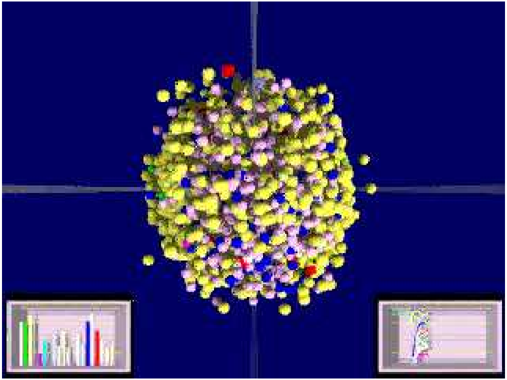
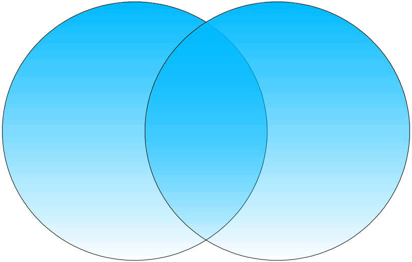


Experimental Observables



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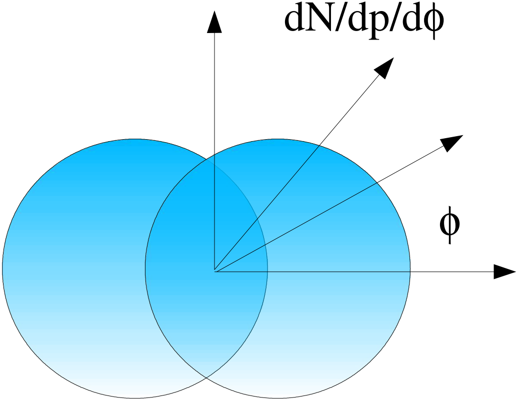


Experimental Observables

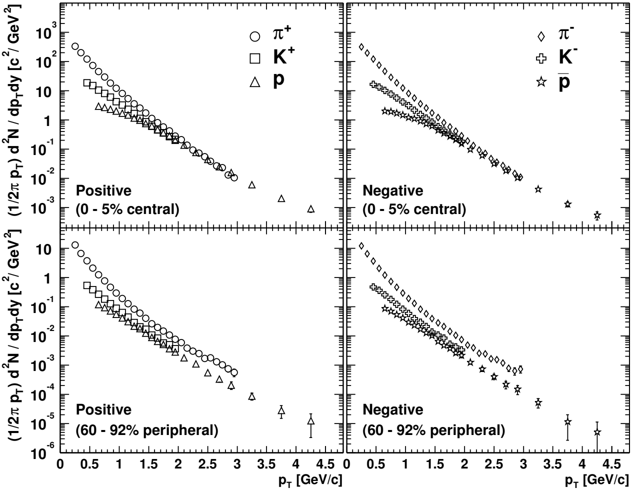
- For ultrarelativistic heavy-ion collisions,

$$\frac{dN}{dp_\perp d\phi dy} = \langle \frac{dN}{dp_\perp d\phi dy} \rangle_\phi (1 + 2 v_2(p_\perp) \cos(2\phi) + \ldots)$$

- Radial flow: $\langle \frac{dN}{dp_\perp dy} \rangle_\phi$
- Elliptic flow: $v_2(p_\perp)$

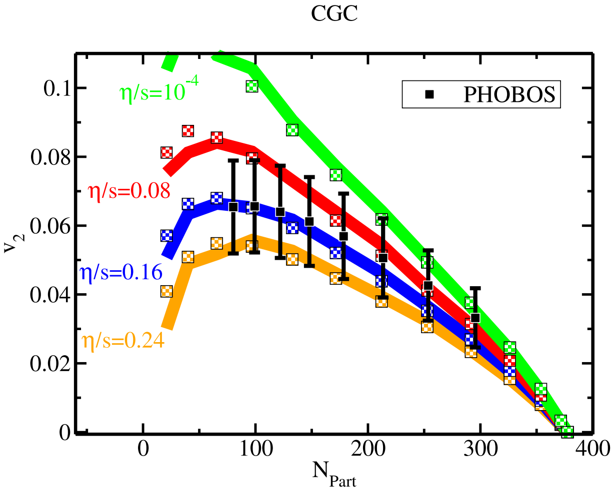
Experimental Data

Hadron Spectra at RHIC $\sqrt{s} = 200$ GeV



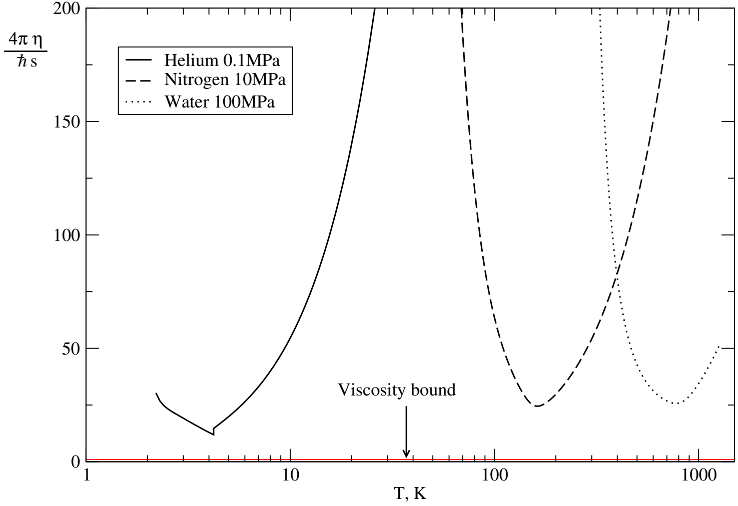
[PHENIX, 2004]

Strongly Coupled Systems — Example RHIC



[Luzum & Romatschke, 2008]

Viscosity over Entropy Density: Various Results



[Kovtun, Son, Starinets 2005]

Strongly Coupled Systems — Example RHIC

- (One) Current Problem: “Freeze-Out”
- Freeze-Out: converting fluid to particle

$$T^{\mu\nu}_{\text{fluid}} = T^{\mu\nu}_{\text{particle}} = \int \frac{d^3p}{(2\pi)^3} \sum_i \delta(p^2 - m_i^2) p^\mu p^\nu f(x,p)$$

- But: where to make transition? How treat multi-particle freeze-out?

Strongly Coupled Systems — Example Cold Atoms

Experimental setup:

- 50-50 mixture of two lowest hyperfine states of ^6Li
- magnetically tuned to a broad Feshbach resonance
- cooled by evaporation in an optical trap
- initial energy per particle E_0 measured from trapped cloud profile

Attempted fluid description — Ideal Fluids

Ideal Fluids

- Continuity Equation

$$\partial_t \rho + \partial_i \left(\rho v^i \right) = 0$$

- Euler Equation in Trapping potential $V(x)$

$$\rho \left(\partial_t \vec{v} + \vec{v} \cdot \vec{\partial} \vec{v} \right) = -\vec{\partial} P - \rho \vec{\partial} V$$

- Relation of mass and number density: $\rho = m \times n$.

Attempted fluid description — Ideal Fluids

Initial State: hydrostatics

- $$\vec{\partial} P = mn\vec{\partial} V$$
- Thermodynamics:

  $$\epsilon + P = T\,s + \mu\,n\,, \quad d\epsilon = T\,ds + \mu\,dn\,.$$

  implies $dP = nd\mu$

  $$\mu = \mu_0 + mV(x)$$
- Typically $V(x)$ harmonic trap potential

  $$V(x) = \frac{1}{2}\omega_i^2 x_i^2$$

  so

  $$\mu = \mu_0 \left(1 - \frac{x_i^2}{R_i^2}\right)\,, \quad R_i^2 = \frac{2\mu_0}{m\omega_i^2}$$

Equation of State

- 4 hydrodynamic equations (Continuity+Euler), 5 variables: $n, \vec{v}, P$. Need equation of state to close system
- EoS in the unitarity limit:

$$P(\mu, T) = \mu^{5/2} m^{3/2} g\left(\frac{T}{\mu}\right)$$

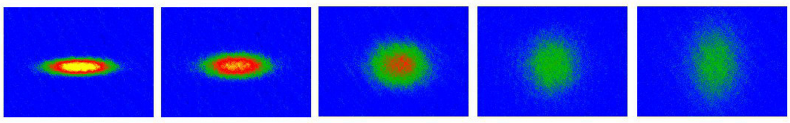
  where $g(z) \to \mathrm{const.}$ for low temperatures $T \ll T_F = \frac{(3\pi^2 n)^{2/3}}{2m}$. Then

$$n = \frac{\partial P}{\partial_\mu} \propto \mu^{3/2}$$

  Note that we don't want to enter superfluidity, which happens at

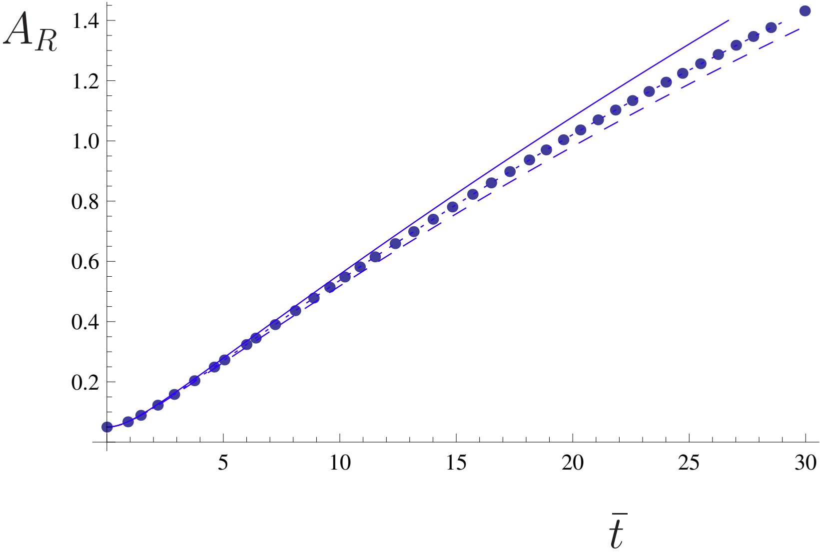
$$T_c \lesssim 0.15 T_F$$

Strongly Coupled Systems — Example Cold Atoms



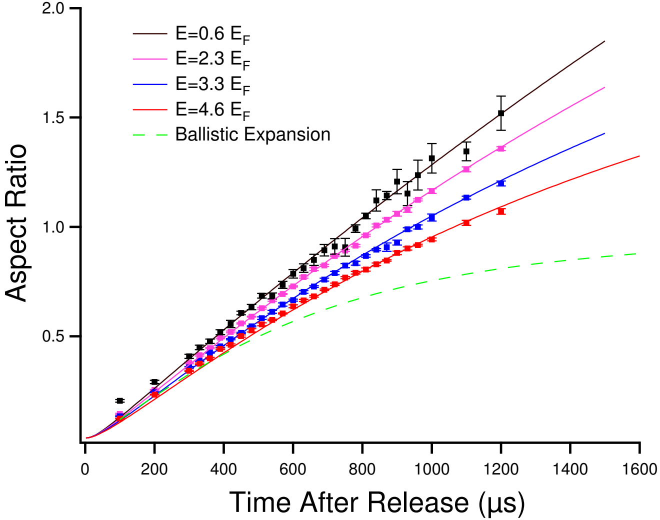
[Cao et al., Science 2011]

Strongly Coupled Systems — Example Cold Atoms



[T. Schäfer, 2010]

Strongly Coupled Systems — Example Cold Atoms



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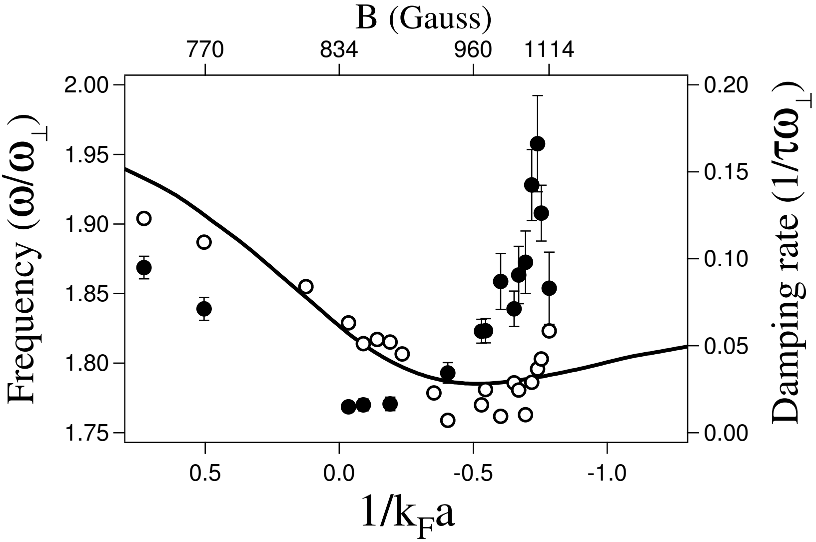
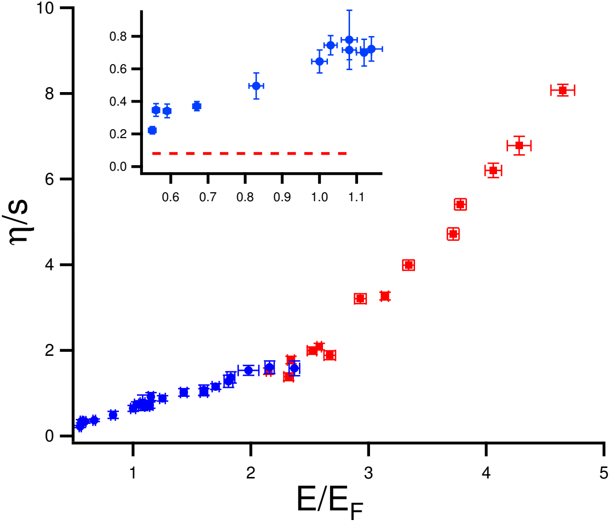
Strongly Coupled Systems — Example Cold Atoms



[Kinast, Turlapov, Thomas 2004]

Strongly Coupled Systems — Example Unitary Fermi gas



[Cao et al., Science 2011]

Same tool – same problem

- Hydrodynamics not valid in dilute corona
- Qualitatively similar to problem encountered in HIC
- Same problem also in other fields: neutrino sphere in supernovae, surface of last scattering in cosmology,...

Hydrodynamics for strongly coupled systems

- Strongly coupled systems are good fluids
- Fluid dynamics can be used to extract material properties of strongly coupled systems
- Several examples: QGP, Cold Atoms, Graphene, neutron stars(?)

Challenges

- Use different systems to improve fluid dynamics modelling (for example freeze-out)
- Look for universal features
- Learn something about turbulence?

Conclusions

- Hydrodynamics is good tool for strongly coupled systems — provided one is not interested in fast, small details
- Research on hydrodynamics continues — Lots of things to be done!