Low-energy pion-photon reactions and chiral symmetry

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- Tests of chiral perturbation theory via low-energy $\pi^-\gamma$ reactions
- COMPASS@CERN: Primakoff effect to extract $\pi^-\gamma$ cross sections
- Pion Compton scattering in ChPT: electric/magnetic polarizabilities
- Radiative corrections to $\pi^-\gamma \to \pi^-\gamma$, isospin-breaking correction
- Neutral and charged pion-pair production: $\pi^-\gamma \to \pi^-\pi^0\pi^0$ or $\pi^+\pi^-\pi^-$
- Total cross sections and 2π invariant mass spectra at one-loop order
- Radiative corrections to $\pi^-\gamma \to \pi^-\pi^0\pi^0$ (simpler case)

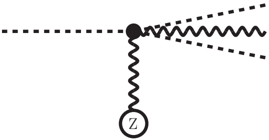
Publications: N. Kaiser, J. Friedrich, EPJA36, 181 ('08); NPA812, 186 ('08),
N. Kaiser, NPA848, 198 ('10); EPJA46, 373 ('10); EPJA47 ('11) in print

- Pions $\pi^{\pm 0}$: Goldstone bosons of spontaneous chiral symmetry breaking in QCD, $SU(2)_L \times SU(2)_R \to SU(2)_V$
- Their low-energy dynamics: systematically (and accurately) calculable in Chiral Perturbation Theory (= loop-expansion with effective Lagrangian)
- 2-loop prediction for $I = 0$ $\pi\pi$-scattering length: $a_0 m_\pi = 0.220 \pm 0.005$ confirmed by NA48/2@CERN: $K^+ \to \pi^+\pi^- e^+ \nu_e$ ($\pi^+\pi^-$ mass distribut.)
- Implications: quark condensate $\langle 0|\bar{q}q|0\rangle$ is large, linear term dominates quark mass expansion of m_π^2: $m_\pi^2 f_\pi^2 = -\langle 0|\bar{q}q|0\rangle m_q + \mathcal{O}(m_q^2 \ln m_q)$
- DIRAC@CERN: Pionium lifetime $\tau_{pred} = (2.9 \pm 0.1) \cdot 10^{-15}$ sec

$$\Gamma((\pi^+\pi^-)_{atom} \to \pi^0\pi^0) = \frac{2}{9}\alpha^3 p_{cm} m_\pi^2 (a_0 - a_2)^2 + \dots$$

- Cusp effect in $2\pi^0$ mass spectrum of $K^+ \to \pi^+\pi^0\pi^0$ at $\pi^+\pi^-$ threshold (high statistics wins): $(a_0 - a_2)m_\pi = 0.257 \pm 0.006$ $(0.265 \pm 0.005)_{ChPT}$ [include isospin-breaking correc.: J. Gasser, B. Kubis, A. Rusetsky et al.]
- Electromagnetic processes with pions allow for further tests of ChPT
- Pion polarizability difference (2-loops): $\alpha_\pi - \beta_\pi = (5.7 \pm 1.0) \cdot 10^{-4}\,\text{fm}^3$, experimental determinations from Serpukhov and Mainz in conflict with it

Primakoff effect:



- Scattering of high energy pions in nuclear Coulomb field (high Z) allows to extract cross sections for $\pi^-\gamma$ reactions (equivalent-photon method)

$$\frac{d\sigma}{ds\,dQ^2} = \frac{Z^2\alpha}{\pi(s-m_\pi^2)}\,\frac{Q^2-Q_{min}^2}{Q^4}\,\sigma_{\pi^-\gamma}(s)\,, \qquad Q_{min} = \frac{s-m_\pi^2}{2E_{beam}}$$

- $s = (\pi^-\gamma \text{ invariant mass})^2$, $Q \to 0$ momentum transfer by virtual photon
- expt. difficulty: isolate Coulomb peak from strong interaction background
- COMPASS@CERN:
  π-Compton scattering $\pi^-\gamma \to \pi^-\gamma$: electric and magnetic polarizabilities
  π^0-production $\pi^-\gamma \to \pi^-\pi^0$: test QCD chiral anomaly, $F_{\gamma 3\pi} = e/(4\pi^2 f_\pi^3)$
- pion-pair product. $\pi^-\gamma \to 3\pi$: $\sqrt{s} > 1\text{GeV}$ meson spectroscopy, exotics, high statistics allows to continue event rates even down to threshold

- Pion Compton-scattering: $\pi^-(p_1) + \gamma(k_1, \epsilon_1) \to \pi^-(p_2) + \gamma(k_2, \epsilon_2)$, T-matrix in center-of-mass frame in Coulomb gauge: $\epsilon^0_{1,2} = 0$:

$$T_{\pi\gamma} = 8\pi\alpha\Big\{ -\vec{\epsilon}_1 \cdot \vec{\epsilon}_2\, A(s,t) + \vec{\epsilon}_1 \cdot \vec{k}_2\, \vec{\epsilon}_2 \cdot \vec{k}_1\, \frac{2}{t}\Big[A(s,t) + B(s,t)\Big]\Big\}$$

independent Mandelstam variables: $s = (p_1 + k_1)^2,\ t = (k_1 - k_2)^2$

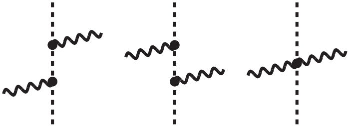
- Corresponding differential cross section:

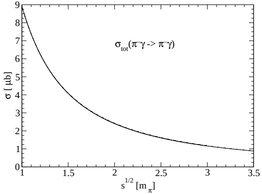
$$\frac{d\sigma}{d\Omega_{cm}} = \frac{\alpha^2}{2s}\Big\{\big|A(s,t)\big|^2 + \big|A(s,t) + (1+z)B(s,t)\big|^2\Big\}$$

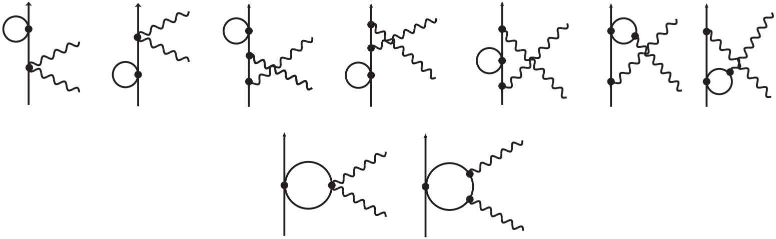
where $t = (s - m_\pi^2)^2(z-1)/2s$ with $z = \cos\theta_{cm}$, scattering angle

- Tree diagrams (s-channel pole diagram vanishes, $\epsilon_1 \cdot (2p_1 + k_1) = 0$):

$$A(s,t)^{(tree)} = 1, \qquad B(s,t)^{(tree)} = \frac{s - m_\pi^2}{m_\pi^2 - s - t}$$







- Pion-loop diagrams (photon scattering off the pion's "pion cloud"):

$$A(s,t)^{(loop)} = \frac{1}{(4\pi f_\pi)^2}\left\{ -\frac{t}{2} - 2m_\pi^2 \ln^2 \frac{\sqrt{4m_\pi^2 - t} + \sqrt{-t}}{2m_\pi} \right\} \sim t^2 > 0$$

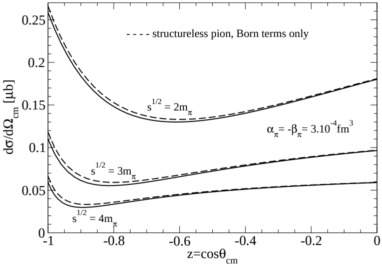
with $f_\pi = 92.4\,$MeV, expression corresponds to isospin limit: $m_{\pi^0} = m_\pi$

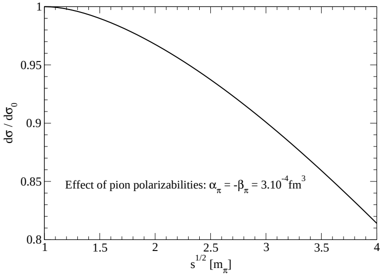
- Electric/magnetic polarizabilities = low-energy const. with $\alpha_\pi + \beta_\pi = 0$

$$A(s,t)^{(pola)} = -\frac{\beta_\pi m_\pi t}{2\alpha} < 0\,, \qquad \alpha_\pi - \beta_\pi = \frac{\alpha}{24\pi^2 f_\pi^2 m_\pi}(\bar\ell_6 - \bar\ell_5)$$

Pion Compton-scattering in ChPT

- Combination $\bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3$ is determined via radiative pion decay $\pi^+ \to e^+ \nu_e \gamma$, PIBETA@PSI: axial-to-vector coupl. ratio $F_A/F_V \simeq 0.44$
- Current-algebra relation: $\langle 0|A^\mu V^\nu|\pi\rangle \simeq f_\pi \langle \pi|V^\mu V^\nu|\pi\rangle$ plus corrections
- One-loop "prediction": $\alpha_\pi = -\beta_\pi \simeq 3.0 \cdot 10^{-4}\,\mathrm{fm}^3$
- $\sigma_{tot}(s)$ absolutely insensitive to pion's low-energy structure
- Small effect on backward angular distributions of $d\sigma/d\Omega_{cm}$





- Pion-loop compensates partly reduction of $d\sigma/d\Omega_{cm}$ by polarizabilities
- Effect of pion polarizabilities on π-Compton cross section: less than 20%
- M. Ivanov: 2-loop corrections to $d\sigma/d\Omega_{cm}$ are very small

Pion polarizabilities in ChPT

- Gasser et al., NPB745, 84 (2006): Pion polarizabilities to 2 loops
- Analytical expression in terms of low-energy constants $\bar{\ell}_j$:

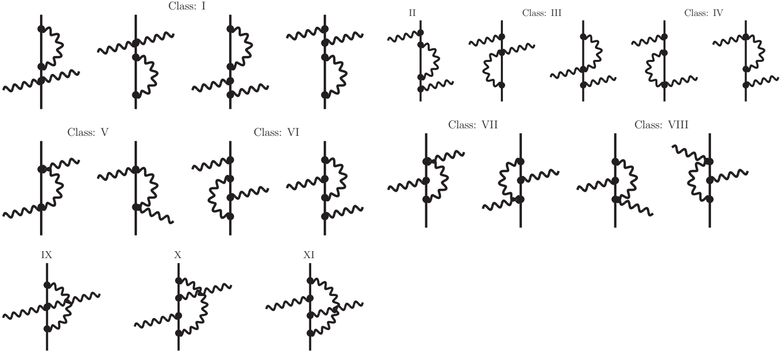
$$\alpha_\pi - \beta_\pi = \frac{\alpha(\bar{\ell}_6 - \bar{\ell}_5)}{24\pi^2 f_\pi^2 m_\pi} + \frac{\alpha m_\pi}{(4\pi f_\pi)^4}\Big\{c^r + \frac{8}{3}\Big(\bar{\ell}_2 - \bar{\ell}_1 + \bar{\ell}_5 - \bar{\ell}_6 + \frac{65}{12}\Big)\ln\frac{m_\pi}{m_\rho} + \frac{4}{9}(\bar{\ell}_1 + \bar{\ell}_2) - \frac{\bar{\ell}_3}{3} + \frac{4\bar{\ell}_4}{3}(\bar{\ell}_6 - \bar{\ell}_5) - \frac{187}{81} + \Big(\frac{53\pi^2}{48} - \frac{41}{324}\Big)\Big\}$$

- Contribution of nonfactorizable acnode diagram evaluated analytically (and corrected in comparison to Bürgi (1996)), improved values of $\bar{\ell}_j$
- 2-loop prediction including realistic estimate of theoretical errors:

$$\alpha_\pi - \beta_\pi = (5.7 \pm 1.0)\cdot 10^{-4}\,\mathrm{fm}^3\,, \qquad \alpha_\pi + \beta_\pi = (0.16 \pm 0.1)\cdot 10^{-4}\,\mathrm{fm}^3$$

- Good reasons to believe that chiral prediction is stable against higher order corrections: ChPT at 2-loop order works very well for $\gamma\gamma \to \pi^0\pi^0$
- Existing expt. determinations $\alpha_\pi - \beta_\pi = (15.6 \pm 7.8)\cdot 10^{-4}\,\mathrm{fm}^3$ from Serpukhov (via Primakoff) and $\alpha_\pi - \beta_\pi = (11.6 \pm 3.4)\cdot 10^{-4}\,\mathrm{fm}^3$ from Mainz (via $\gamma p \to \gamma\pi^+ n$) violate chiral low-energy theorem by a factor 2!
- $\alpha_\pi + \beta_\pi = \frac{1}{2\pi^2}\int \frac{d\omega}{\omega^2}\sigma_{\rm abs}^{\pi\gamma}(\omega)$ agrees with results from dispersion sum rules

- Pion-structure effects small: necessary to include radiative corr. of $\mathcal{O}(\alpha)$
- Start with structureless pion: extensive exercise in one-loop scalar QED
- Advantage of Coulomb gauge: all s-channel pole diagrams vanish



- Dimensional regularization to treat both ultraviolet divergencies ($d < 4$) and infrared divergencies ($d > 4$), composite constant:

$$\xi = \frac{1}{d-4} + \frac{1}{2}(\gamma_E - \ln 4\pi) + \ln\frac{m_\pi}{\mu}$$

Alternative: introduce regulator photon mass m_γ, $\xi_{IR} = \ln(m_\pi/m_\gamma)$

- UV divergencies ξ_{UV} cancel: scalar QED is renormalizable, Crossing: $A(s,u)$ and $(u-m_\pi^2)(s-m_\pi^2)^{-1}B(s,u)$ sym. under $s \leftrightarrow u$, $u = 2m_\pi^2 - s - t$
- Infrared-finite after inclusion of soft photon bremsstrahlung: $d\sigma/d\Omega_{\rm cm}\cdot\delta_{\rm soft}$

$$\delta_{\rm soft} = \alpha\,\mu^{4-d}\int_{|\vec{l}\,|<\lambda}\frac{d^{d-1}l}{(2\pi)^{d-2}\,l_0}\left\{\frac{2m_\pi^2-t}{p_1\cdot l\,p_2\cdot l}-\frac{m_\pi^2}{(p_1\cdot l)^2}-\frac{m_\pi^2}{(p_2\cdot l)^2}\right\}$$

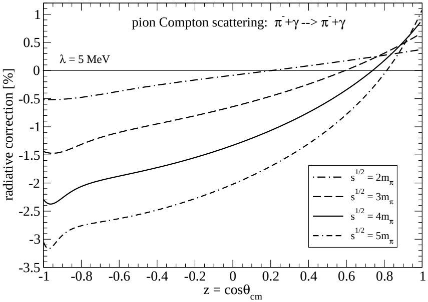
- Evaluate it in dim. regularization: ξ_{IR} from photon loops gets canceled, radiative correction depends on a small energy resolution scale λ:

$$\delta_{\rm real} = \frac{\alpha}{\pi}\left\{\left[2+\frac{4\hat{t}-8}{\sqrt{\hat{t}^2-4\hat{t}}}\ln\frac{\sqrt{4-\hat{t}}+\sqrt{-\hat{t}}}{2}\right]\ln\frac{m_\pi}{2\lambda}+\frac{\hat{s}+1}{\hat{s}-1}\ln\hat{s}\right.$$
$$\left.+\int_0^{1/2}dx\,\frac{(\hat{s}+1)(\hat{t}-2)}{\sqrt{W}[1-\hat{t}x(1-x)]}\ln\frac{\hat{s}+1+\sqrt{W}}{\hat{s}+1-\sqrt{W}}\right\}$$

where $\hat{s}=s/m_\pi^2$, $\hat{t}=t/m_\pi^2$ and $W=(\hat{s}-1)^2+4\hat{s}\hat{t}x(1-x)$

- Terms beyond $\ln(m_\pi/2\lambda)$ specific for evaluation in center-of-mass frame
- Idealized experiment with undetected soft photons filling in momentum space a small sphere of radius λ in the center-of-mass frame
- Further experiment-specific soft/hard γ-radiation can be accounted for

Results:



- QED radiative corrections are maximal in backward directions $z \simeq -1$
- Same kinematical signature as pion polarizability difference $\alpha_\pi - \beta_\pi$
- Suppressed by factor of ~ 10
- Limit of zero photon momentum $k_1 = k_2 = 0$, pure Thomson amplitude $T^{(0)}_{\pi^-\gamma} = -8\pi\alpha\,\vec{\epsilon}_1 \cdot \vec{\epsilon}_2$ survives, correspond. nonrenormalization factor:

$$1 + \frac{\alpha}{4\pi}\Big[(4\xi_{IR} - 4\xi_{UV})^{(\mathrm{I})} + (8\xi_{UV} - 8)^{(\mathrm{V})} + (1 - 2\xi_{UV})^{(\mathrm{VII})} + (1 - 2\xi_{UV})^{(\mathrm{VIII})} + (4 - 4\xi_{IR} - 2\xi_{UV})^{(\mathrm{IX})} + (\xi_{UV} + 1)^{(\mathrm{X})} + (\xi_{UV} + 1)^{(\mathrm{XI})}\Big] = 1$$

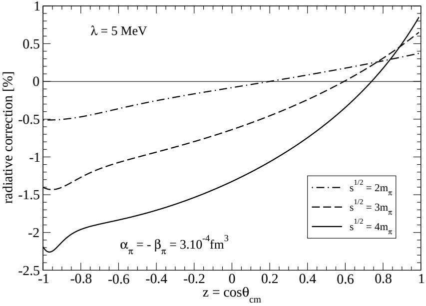
Radiative corrections including pion structure

- Include leading pion-structure in form of polarizability difference $\alpha_\pi - \beta_\pi$
- Reinterpret $\gamma\gamma$ contact vertex as representing the pion polarizabilities:

$$\sim F_{\mu\nu}F^{\mu\nu}\,, \qquad 8\pi i \beta_\pi m_\pi \Big(k_1\cdot k_2\,\epsilon_1\cdot\epsilon_2 - \epsilon_1\cdot k_2\,\epsilon_2\cdot k_1\Big)$$

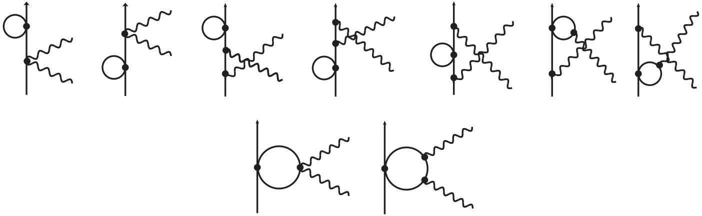
- Reference cross section: point-like $d\sigma^{(pt)}/d\Omega_{cm}$ + polarizability improved

$$\frac{d\sigma^{(\text{pola})}}{d\Omega_{\text{cm}}} = \frac{\alpha\beta_\pi m_\pi^3 (s-m_\pi^2)^2(1-z)^2}{2s^2[s(1+z)+m_\pi^2(1-z)]}$$



- Relative size and angular depend. not affected by leading pion-structure

- Isospin-breaking induced by charged/neutral pion mass difference (elm)



$$A(s,t)^{(isobr)} = \frac{m_\pi^2 - m_{\pi^0}^2}{(2\pi f_\pi)^2}\left\{ -\frac{1}{2} - \frac{2m_\pi^2}{t}\ln^2\frac{\sqrt{4m_\pi^2 - t} + \sqrt{-t}}{2m_\pi}\right\} \sim t$$

entirely from depend. of chiral $\pi\pi$ interaction on $m_{\pi^0}^2$, no elm counterterm

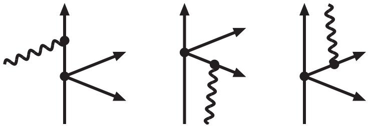
- Small contribution to pion polarizability difference

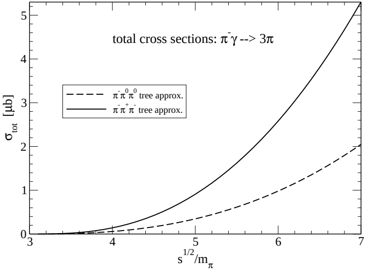
$$\delta(\alpha_\pi - \beta_\pi) = \frac{\alpha(m_\pi^2 - m_{\pi^0}^2)}{24\pi^2 f_\pi^2 m_\pi^3} \simeq 1.3 \cdot 10^{-5}\,\mathrm{fm}^3$$

- Affects backward cross section at level of few permille at most
- One order of magnitude smaller than "genuine" radiative corrections

Tree level cross sections for $\pi^-\gamma \to 3\pi$

- Coulomb gauge $\epsilon \cdot p_1 = \epsilon \cdot k = 0$, photon does not couple to incoming π^-
- parametrize $U = \sqrt{1 - \vec{\pi}^{\,2}/f_\pi^2} + i\vec{\tau}\cdot\vec{\pi}/f_\pi$, no $\gamma 4\pi$ vertex at leading order





- Example: total cross section for $\pi^-(p_1) + \gamma(k,\epsilon) \to \pi^-\pi^0\pi^0$

$$\sigma_{tot}(s) = \frac{\alpha}{32\pi^2 f_\pi^4 (s - m_\pi^2)^3} \int_{2m_\pi\sqrt{s}}^{s-3m_\pi^2} dw \sqrt{\frac{s - w - 3m_\pi^2}{s - w + m_\pi^2}}\,(s-w)^2$$

$$\times \left[w \ln \frac{w + \sqrt{w^2 - 4m_\pi^2 s}}{2m_\pi\sqrt{s}} - \sqrt{w^2 - 4m_\pi^2 s} \right]$$

- $(s-w)/f_\pi^2$ factor: chiral $\pi\pi$-interaction, rest mainly 3-body phase space
- How large are next-to-leading order corrections from chiral loops + cts?

Neutral pion-pair production

- 3-body process: $\pi^-(p_1) + \gamma(k, \epsilon) \to \pi^-(p_2) + \pi^0(q_1) + \pi^0(q_2)$
- general form of T-matrix (in Coulomb gauge)

$$T_{3\pi} = \frac{2e}{f_\pi^2}\Big[\vec{\epsilon}\cdot\vec{q}_1 A_1 + \vec{\epsilon}\cdot\vec{q}_2 A_2\Big]\,, \qquad A_2 = A_1\Big|(s_1 \leftrightarrow s_2, t_1 \leftrightarrow t_2)$$

- amplitudes A_1 and A_2 depend on five (independ.) Mandelstam variables:

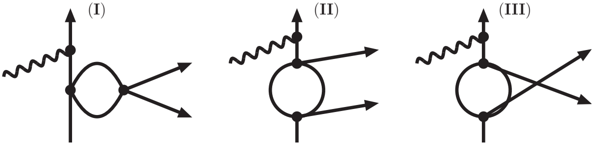
$$s = (p_1+k)^2,\;\; s_1 = (p_2+q_1)^2,\;\; s_2 = (p_2+q_2)^2,\;\; t_1 = (q_1-k)^2,\;\; t_2 = (q_2-k)^2$$

- convenient for permutation of identical neutral pions ($s_1 \leftrightarrow s_2$, $t_1 \leftrightarrow t_2$)
- tree-level amplitudes:

$$A_1^{(\text{tree})} = A_2^{(\text{tree})} = \frac{2m_\pi^2 + s - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2}$$

- written with physical parameters (f_π^2, m_π^2) instead of leading-order ones, requires extra renormalization contribution which accounts for difference

- Pion-loop corrections (example I)



$$A_1^{(I)} = \frac{1}{(4\pi f_\pi)^2}\frac{2m_\pi^2+s-s_1-s_2}{3m_\pi^2-s-t_1-t_2}\left\{\left(\xi+\ln\frac{m_\pi}{\mu}\right)(s_1+s_2+t_1+t_2-11m_\pi^2) + (s_1+s_2+t_1+t_2-7m_\pi^2)\left[J(3m_\pi^2+s-s_1-s_2)-\frac{1}{2}\right]\right\}$$

- Ultraviolet divergence in dimensional regularization (ChPT convention)

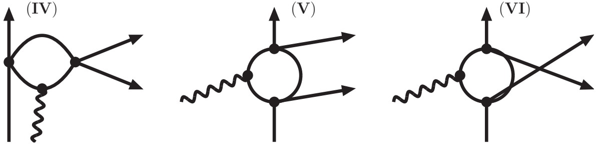
$$\xi = \mu^{d-4}\left\{\frac{1}{d-4}+\frac{1}{2}(\gamma_E-1-\ln 4\pi)\right\}$$

- Loop function (from loop with two pion-propagators)

$$J(s)=\sqrt{\frac{s-4m_\pi^2}{s}}\left[\ln\frac{\sqrt{|s-4m_\pi^2|}+\sqrt{|s|}}{2m_\pi}-\frac{i\pi}{2}\theta(s-4m_\pi^2)\right],\quad s<0 \text{ or } s>4m_\pi^2$$

Neutral pion-pair production

- Pion-loop corrections (example IV)



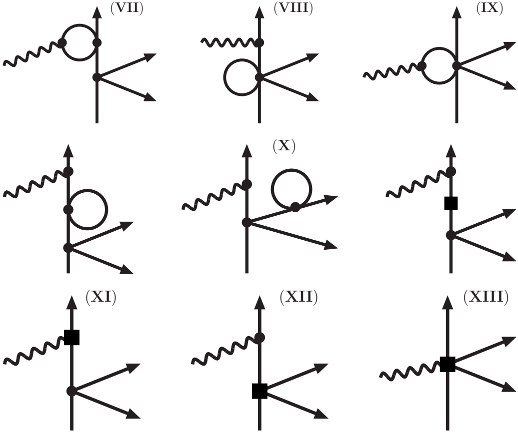
$$A_1^{(\mathrm{IV})} = \frac{2m_\pi^2 + s - s_1 - s_2}{(4\pi f_\pi)^2}\Bigg\{\xi + \ln\frac{m_\pi}{\mu} - \frac{1}{2} + J(3m_\pi^2 + s - s_1 - s_2)$$
$$+\frac{m_\pi^2 - s}{2m_\pi^2 - t_1 - t_2} + \frac{2(s - m_\pi^2)}{(2m_\pi^2 - t_1 - t_2)^2}\Big\{(s_1 + s_2 - s - m_\pi^2 - t_1 - t_2)$$
$$\times\Big[J(m_\pi^2 + s - s_1 - s_2 + t_1 + t_2) - J(3m_\pi^2 + s - s_1 - s_2)\Big]$$
$$+2m_\pi^2\Big[G(m_\pi^2 + s - s_1 - s_2 + t_1 + t_2) - G(3m_\pi^2 + s - s_1 - s_2)\Big]\Big\}\Bigg\}$$

- Loop function (from loop with three pion-propagators)

$$G(s) = \left[\ln\frac{\sqrt{|s - 4m_\pi^2|} + \sqrt{|s|}}{2m_\pi} - \frac{i\pi}{2}\theta(s - 4m_\pi^2)\right]^2, \quad s < 0 \text{ or } s > 4m_\pi^2$$

Neutral pion-pair production

- Chiral loop and counterterm corrections (completed)



- Chiral 6π-vertex: challenging combinatorics involved, $6! = 720$
- Pion wavefunction renormalization factor, chiral counterterms $\sim \ell_1, \ell_2, \ell_4$
- First crucial check: ultraviolet divergence ξ drops out in total sum for $A_{1,2}$

Neutral pion-pair production

- Introduce low-energy constants that subsume chiral logarithm $\ln(m_\pi/\mu)$

$$\ell_j^r = \frac{\gamma_j}{32\pi^2}\left(\bar{\ell}_j + 2\ln\frac{m_\pi}{\mu}\right), \quad \gamma_1 = \frac{1}{3},\ \gamma_2 = \frac{2}{3},\ \gamma_3 = -\frac{1}{2},\ \gamma_4 = 2$$

- Complete counterterm contribution:

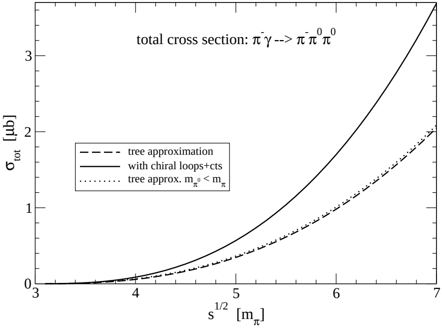
$$A_1^{(\mathrm{ct})} = \frac{1}{(4\pi f_\pi)^2}\frac{1}{3m_\pi^2 - s - t_1 - t_2}\Big\{\frac{\bar{\ell}_1}{3}(s_1+s_2-s-m_\pi^2)^2 + \frac{\bar{\ell}_2}{3}\Big[s^2+s_1^2+s_2^2$$
$$+t_2^2 - 2ss_1 + (s-2s_1+2s_2-t_1)t_2 + m_\pi^2(s-6s_2+t_1-2t_2+6m_\pi^2)\Big]$$
$$-\frac{\bar{\ell}_3}{2}m_\pi^4 + 2\bar{\ell}_4 m_\pi^2(s+2m_\pi^2-s_1-s_2)\Big\}$$

- Finite loop corrections with $\xi + \ln(m_\pi/\mu)$ terms deleted altogether
- Counterterms with low-energy constants $\ell_{5,6,7}$ do not contribute: ℓ_5-term requires 2 external photons, $\langle r_\pi^2\rangle = (\bar{\ell}_6 - 1)/(4\pi f_\pi)^2$ pion mean square charge radius, ℓ_7-term breaks isospin symmetry
- Values of low-energy constants: $\bar{\ell}_1 = -0.4 \pm 0.6$, $\bar{\ell}_2 = 4.3 \pm 0.1$, $\bar{\ell}_3 = 2.9 \pm 2.4$, $\bar{\ell}_4 = 4.4 \pm 0.2$, determined with improved empirical input

Neutral pion-pair production

- Total cross section for $\pi^-\gamma \to 3\pi$

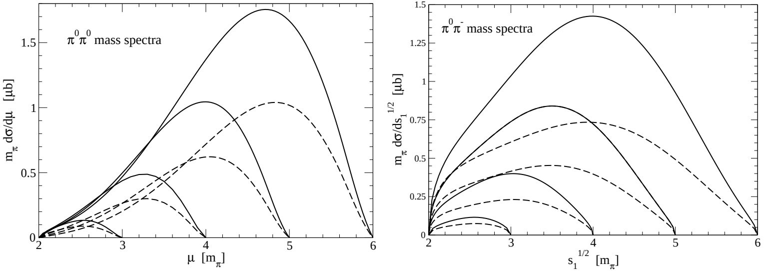
$$\sigma_{\rm tot}(s) = \frac{\alpha}{32\pi^3 f_\pi^4 (s - m_\pi^2)} \iint_{z^2<1} d\omega_1 d\omega_2 \int_{-1}^{1} dx \int_0^\pi d\phi \left|\hat{k} \times (\vec{q}_1 A_1 + \vec{q}_2 A_2)\right|^2$$



- enhancement of $\sigma_{\rm tot}(s)$ by factor 1.5 - 1.8 through chiral corrections
- suggestive explanation: $\pi^+\pi^- \to \pi^0\pi^0$ final state interaction $(1+0.20)^2$

$$\frac{1}{3}(a_0 - a_2) = \frac{3m_\pi}{32\pi f_\pi^2}\left[1 + \frac{m_\pi^2}{36\pi^2 f_\pi^2}\left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} + \frac{9\bar{\ell}_4}{2} + \frac{33}{8}\right)\right]$$

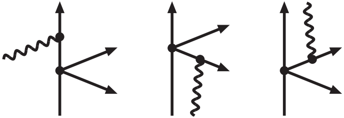
- Uncertainty induced by errorbars of $\bar{\ell}_j$: about $\pm 5\%$ for $\sigma_{\rm tot}(s)$, mainly $\bar{\ell}_1$
- More exclusive observables: two-pion mass spectra
- $\pi^0\pi^0$ invariant mass^2: $\mu^2 = s - s_1 - s_2 + 3m_\pi^2$, $\pi^0\pi^-$ invariant mass^2: s_1, range of invariant masses: $2m_\pi < \mu, \sqrt{s_1} < \sqrt{s} - m_\pi$



- Mass spectra reproduce enhancement by chiral correct. seen in $\sigma_{\rm tot}(s)$
- No further specific dynamical details visible in two-pion mass spectra

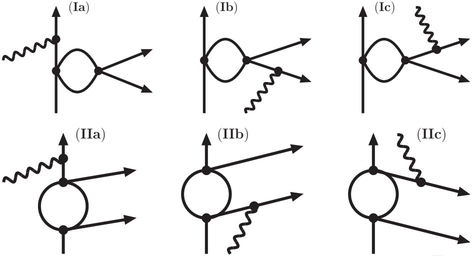
Charged pion-pair production

- 3-body process: $\pi^-(p_1) + \gamma(k,\epsilon) \to \pi^+(p_2) + \pi^-(q_1) + \pi^-(q_2)$
- Photon couples to all charged pions: $\to$ many more diagrams

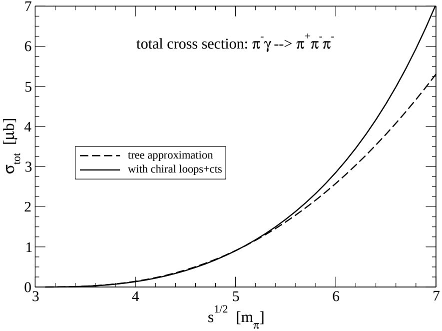


$$A_1^{(\text{tree})} = \frac{s + m_\pi^2 - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_2}{t_1 - m_\pi^2} - 1$$

$$A_2^{(\text{tree})} = \frac{s + m_\pi^2 - s_1 - s_2}{3m_\pi^2 - s - t_1 - t_2} + \frac{s - s_1 - s_2 + t_1}{t_2 - m_\pi^2} - 1$$



- Total cross section



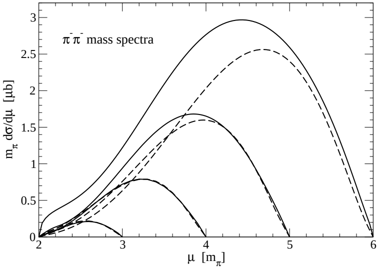
- $\sigma_{\rm tot}(s)$ for $\sqrt{s} < 6m_\pi$ almost unchanged in comparison to tree approx.
- suggestive explanation: $\pi^-\pi^- \to \pi^-\pi^-$ final state interaction $(1-0.02)^2$

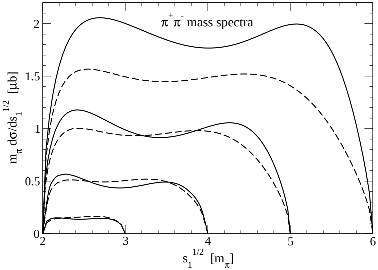
$$a_2 = -\frac{m_\pi}{16\pi f_\pi^2}\left[1 - \frac{m_\pi^2}{12\pi^2 f_\pi^2}\left(\bar{\ell}_1 + 2\bar{\ell}_2 - \frac{3\bar{\ell}_3}{8} - \frac{3\bar{\ell}_4}{2} + \frac{3}{8}\right)\right]$$

- Uncertainty induced by errorbars of $\bar{\ell}_j$: about $\pm 5\%$ for $\sigma_{\rm tot}(s)$
- Preliminary analysis of COMPASS data seems to agree with tree approx.

Charged pion-pair production

- More exclusive observables: two-pion mass spectra

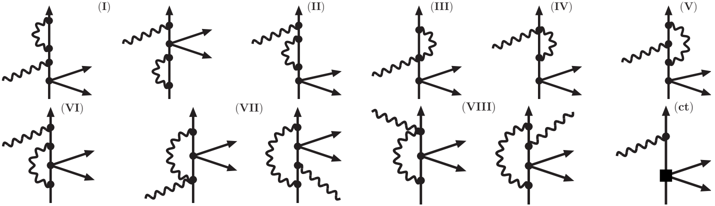




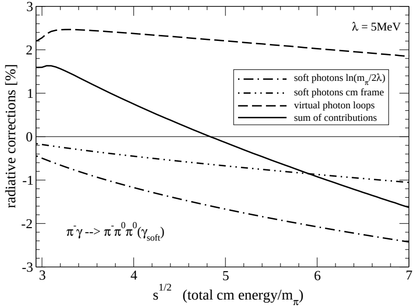
- Dip in $\pi^+\pi^-$ mass spectr. at intermediate $\sqrt{s_1}$ produced by chiral correc.
- Squared T-matrix $|\hat{k} \times (\vec{q}_1 A_1 + \vec{q}_2 A_2)|^2$ with its full dependence on pion energies and angles includes still more dynamical information
- It is expected that high statistics COMPASS data can reveal such details
- Role of $\rho(770)$ resonance ($\Gamma_\rho = 150\,\text{MeV}$) needs to be investigated, ρ-exchange model should respect chiral symmetry and gauge invariance

Radiative corrections to neutral pion-pair production

- Chiral $\pi^+\pi^- \to \pi^0\pi^0$ transition amplitude factors out of all photon loops



- Radiative corr. to total cross section vary between about $+2\%$ and -2%



- Radiative corrections to $\pi^-\gamma \to \pi^+\pi^-\pi^-$ could be much more sizeable, Coulomb singularity from γ-exchange between charged pions: $\alpha\pi/v_{\rm rel}$