Reconciling J/ψ production at HERA, RHIC, Tevatron and LHC with NRQCD factorization at next-to-leading order

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Production and Decay Rates of Heavy Quarkonia

Heavy quarkonia: Bound states of heavy quark and its antiquark.

- Charmonia ($c\overline{c}$) and Bottomonia ($b\overline{b}$)
- Top decays too fast for bound state

The classic approach: Color-singlet model

- Calculate cross section for heavy quark pair in physical color-singlet (= color neutral) state. In case of J/ψ: $c\overline{c}[{}^3S_1^{[1]}]$
- Multiply by quarkonium wave function (or its derivative) at origin
- Strong disagreement with Tevatron data

Nonrelativistic QCD (NRQCD):

- 1995: Rigorous effective field theory by Bodwin, Braaten, Lepage
- Based on factorization of soft and hard scales (Scale hierarchy: $Mv^2, Mv \ll \Lambda_{\mathrm{QCD}} \ll M$)
- Could explain hadroproduction at Tevatron

J/ψ Production with NRQCD

Factorization theorem: $\sigma_{J/\psi} = \sum_n \sigma_{c\overline{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$

- n: Every possible Fock state, including color-octet states.
- $\sigma_{c\overline{c}[n]}$: Production rate of $c\overline{c}[n]$, calculated in perturbative QCD.
- $\langle O^{J/\psi}[n] \rangle$: Long distance matrix elements (LDMEs): describe $c\overline{c}[n] \to J/\psi$, universal, extracted from experiment.

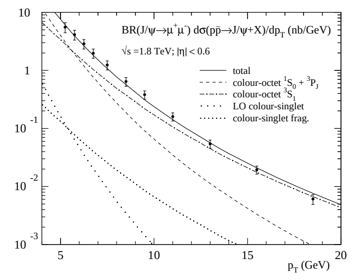
Scaling rules: MEs scale with relative velocity v ($v^2 \approx 0.2$):

scaling	v^3	v^7	v^{11}
n	${}^3S_1^{[1]}$	${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_{0/1/2}^{[8]}$	$\ldots$

- Double expansion in v and α_s.
- Leading term in v ($n = {}^3S_1^{[1]}$) equals color-singlet model.

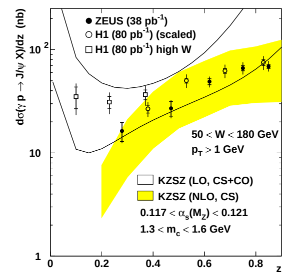
Production of J/ψ: NRQCD vs. Experiment (History)

Hadroproduction at Tevatron:



- Color octet states important
  $\Longrightarrow$ Great success for NRQCD

Photoproduction at HERA:



- MEs from fits to Tevatron data.
- Importance of color octet unclear

This work: NLO NRQCD calculation for photo- and hadroproduction
$\Longrightarrow$ Aim: Establish universality of long distance matrix elements.

Production of J/ψ: Summary of Calculations

Hadroproduction:

	$^3S_1^{[1]}$	$^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_{0/1/2}^{[8]}$
Born	Baier, Rückl (1980)	Cacciari, Krämer (1996)
NLO	Campbell et al. (2007)	Butenschön, BK (2010) Ma et al. (2010)

Photoproduction:

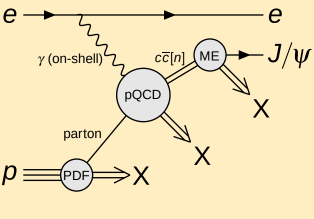
	$^3S_1^{[1]}$	$^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_{0/1/2}^{[8]}$
Born	Berger, Jones (1981)	Ko, Lee, Song (1996)
NLO	Krämer (1995)	Butenschön, BK (2009)

Open question of ME universality:

- NLO NRQCD calculation: after 14 years!
- Difficulty: virtual corrections to P states

Direct J/ψ Production

Factorization formulas: (e.g. photoproduction)



- Convolute partonic cross sections with proton PDFs:

$$\sigma_{\text{hadr}} = \sum_i \int dx\, f_{i/p}(x) \cdot \sigma_{\text{part,i}}$$

- NRQCD factorization:

$$\sigma_{\text{part,i}} = \sum_n \sigma(\gamma i \to c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

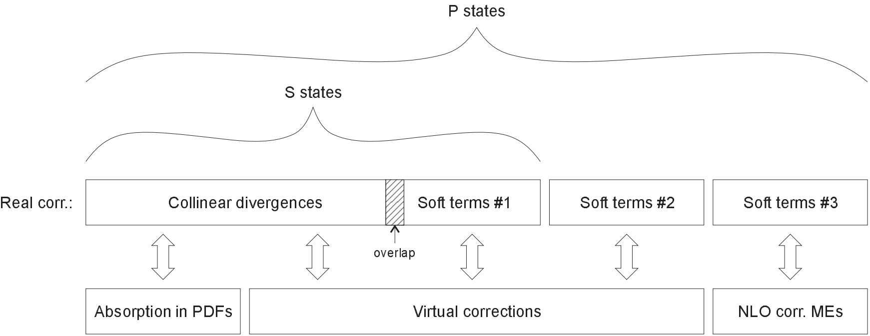
Amplitudes for $c\bar{c}[n]$ production by projector application, e.g.:

$$A_{c\bar{c}[^3S_1^{[1/8]}]} = \varepsilon_\alpha \,\mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

$$A_{c\bar{c}[^3P_J^{[8]}]} = \varepsilon_{\alpha\beta} \frac{d}{dq_\beta} \mathrm{Tr}\,[C\,\Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

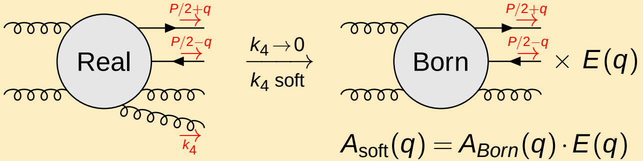
- $A_{c\bar{c}}$: Amputated pQCD amplitude for open $c\bar{c}$ production.
- q: Relative momentum between c and $\bar{c}$.

Overview of IR Singularity Structure



Structure of Soft Singularities

Soft limits of the real corrections:



$$A_{\text{soft}}(q) = A_{Born}(q) \cdot E(q)$$

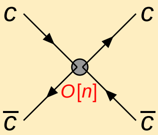
S and P states: Soft #1 + Soft #2 + Soft #3 terms:

$$A_{\text{soft,s}} = A_{\text{soft}}(0) = A_{\text{Born,s}} \cdot E(0)$$
$$A_{\text{soft,p}} = A'_{\text{soft}}(0) = A_{\text{Born,p}} \cdot E(0) + A_{\text{Born,s}} \cdot E'(0)$$

$$|A_{\text{soft,s}}|^2 = |A_{\text{Born,s}}|^2 \cdot E(0)^2$$
$$|A_{\text{soft,p}}|^2 = |A_{\text{Born,p}}|^2 \cdot E(0)^2 + 2\,\text{Re}\,A^*_{\text{Born,s}} A_{\text{Born,p}} \cdot E(0)E'(0) + |A_{\text{Born,s}}|^2 \cdot E'(0)^2$$

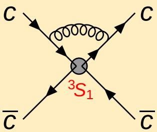
Radiative Corrections to Long Distance MEs

In NRQCD: Long distance MEs = $c\bar{c}$ scattering amplitudes:

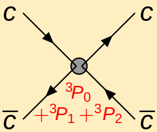
$\langle O^{J/\psi}[n]\rangle =$ 

$O[n]$ = 4-fermion operators
$(n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}, \ldots)$

Corrections to $\langle \mathbf{O}^{J/\psi}[{}^3\mathbf{S}_1^{[1/8]}]\rangle$ with NRQCD Feynman rules:



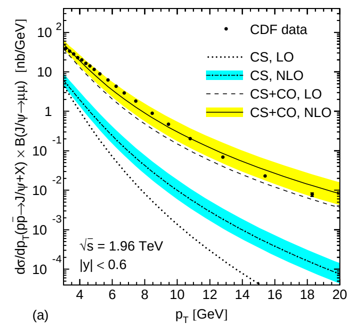
$+$ similar diagrams $\propto \dfrac{4\alpha_s}{3\pi m_c^2}\left(\dfrac{1}{\varepsilon_{UV}} - \dfrac{1}{\varepsilon_{IR}}\right)\cdot$

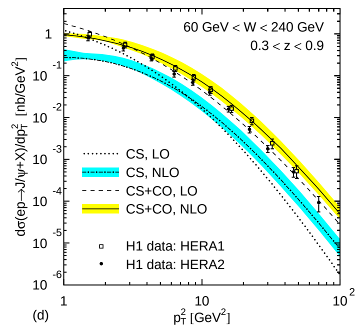


- UV singularity cancelled by renormalization of 4-fermion operat.
- IR singularity cancels soft #3 terms of *P* states.

Combined Fit to Tevatron and HERA (1)

Fit CO LDMEs to p_T distributions from Tevatron II and HERA II:

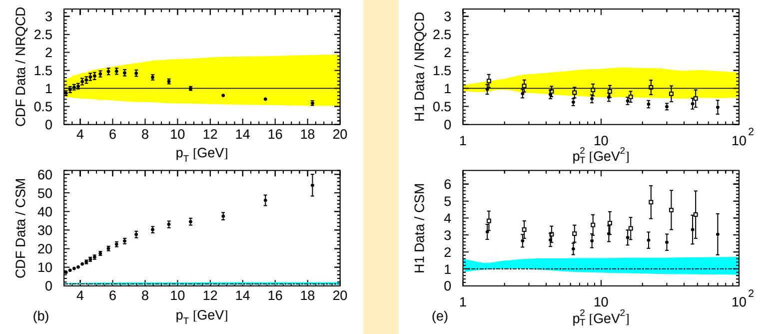




- Best fit values: $\langle \mathcal{O}(^1S_0^{[8]}) \rangle = (0.0450 \pm 0.0072)\ \text{GeV}^3$, $\langle \mathcal{O}(^3S_1^{[8]}) \rangle = (0.00312 \pm 0.00093)\ \text{GeV}^3$, $\langle \mathcal{O}(^3P_0^{[8]}) \rangle = -(0.0121 \pm 0.0035)\ \text{GeV}^5$
- $\propto v^4 \langle O_1(^3S_1) \rangle \rightsquigarrow$ NRQCD velocity scaling rules $\surd$

Combined Fit to Tevatron and HERA (2)

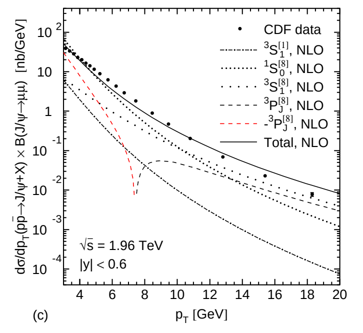
Tevatron II and HERA II data over NRQCD and CSM:

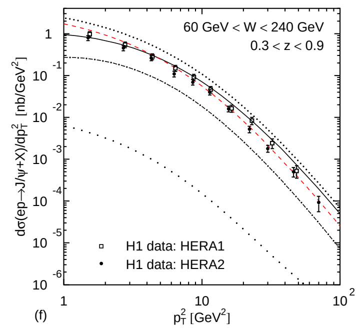


- Tevatron II and HERA II clearly favor NLO NRQCD predictions
- NLO CSM predictions significantly undershoot the data

Combined Fit to Tevatron and HERA (3)

Contribution of individual states:

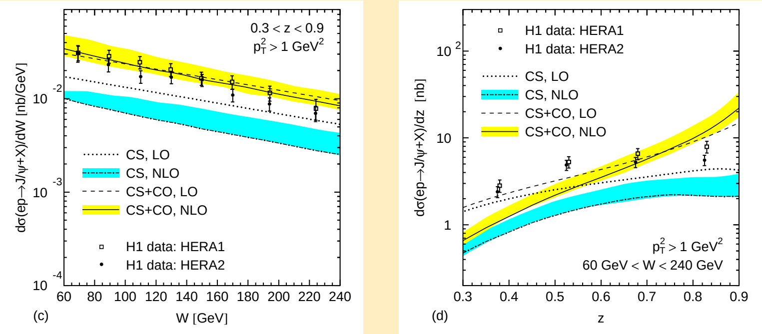




- Hadroprod.: Short-distance $\sigma(c\bar{c}[^3P_J^{[8]}])$ negative for $p_T \gtrsim 7$ GeV
- But: Short-distance cross sections and LDMEs unphysical (NRQCD scale and scheme dependence) $\Longrightarrow$ No problem!

Further Predictions for HERA

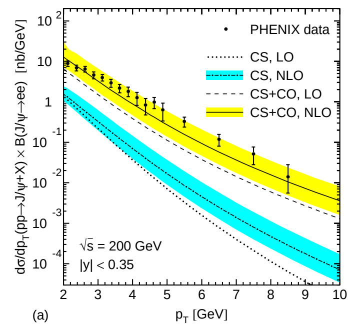
Use LDMEs to make W and z distribution predictions:



- Proton rest frame: $z =$ fraction of photon energy going to J/ψ.
- $z \lessapprox 0.45$: Resolved photoproduction important (not yet included).
- W distribution also well described. Data not part of the fit.

Predictions for RHIC

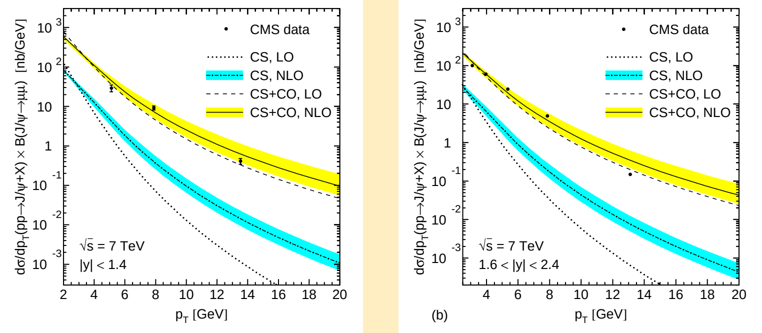
Use LDMEs to make prediction for RHIC:



- Also RHIC data well described by CS+CO.
- Like at Tevatron: CS orders of magnitudes below the data.
- These data not part of the fit, outcome not trivial.

Predictions for LHC

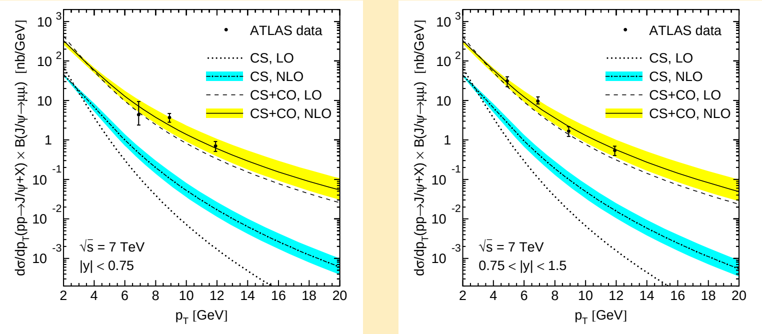
Use LDMEs to make predictions for CMS:



- Also CMS data well described by CS+CO.
- Like at Tevatron: CS orders of magnitudes below the data.
- These data not part of the fit, outcome not trivial.

Predictions for LHC (2)

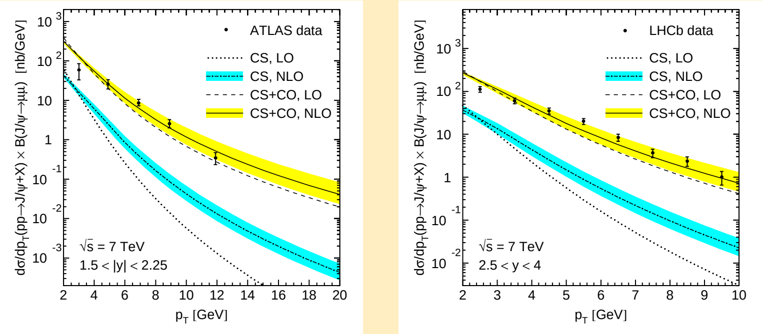
Use LDMEs to make predictions for ATLAS:



- Also ATLAS data well described by CS+CO.
- Like at Tevatron: CS orders of magnitudes below the data.
- These data not part of the fit, outcome not trivial.

Predictions for LHC (3)

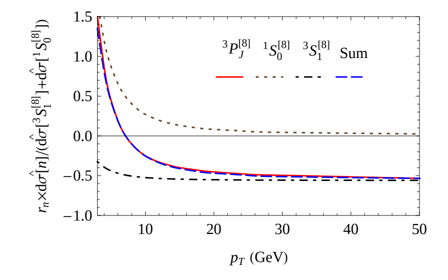
Use LDMEs to make predictions for ATLAS and LHCb:

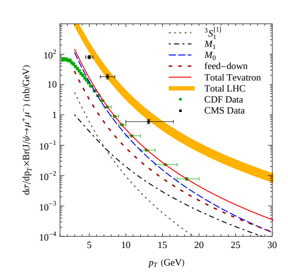


- Also ATLAS and LHCb data well described by CS+CO.
- Like at Tevatron: CS orders of magnitudes below the data.
- These data not part of the fit, outcome not trivial.

Comparison with Ma, Wang, Chao arXiv:1009.3655

Fit only to Tevatron data with $p_T > 7$ GeV, including feed-down

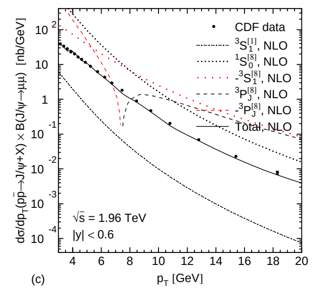




- Observe that $d\hat{\sigma}(^3P_J^{[8]}) \approx r_0 d\hat{\sigma}(^1S_0^{[8]}) + r_1 d\hat{\sigma}(^3S_1^{[8]})$ with $r_0 = 3.9$ and $r_1 = -0.56$
- Define $M_0 = \langle \mathcal{O}(^1S_0^{[8]})\rangle + \frac{r_0}{m_c^2}\langle \mathcal{O}(^3P_0^{[8]})\rangle$ and $M_1 = \langle \mathcal{O}(^3S_1^{[8]})\rangle + \frac{r_1}{m_c^2}\langle \mathcal{O}(^3P_0^{[8]})\rangle$
- Substitute $\langle \mathcal{O}(^1S_0^{[8]})\rangle \to M_0$ and $\langle \mathcal{O}(^3S_1^{[8]})\rangle \to M_1$ and discard $d\hat{\sigma}(^3P_J^{[8]})$.
- Fit yields $M_0 = (7.4 \pm 1.9) \times 10^{-2}$ GeV^3 and $M_1 = (0.05 \pm 0.02) \times 10^{-2}$ GeV^3 with $\chi^2/\mathrm{d.o.f.} = 0.33$
- Cf. $M_0 = (2.47 \pm 0.93) \times 10^{-2}$ GeV^3 and $M_1 = (0.59 \pm 0.13) \times 10^{-2}$ GeV^3 from BK

Significance of HERA data

Fit only to Tevatron data with $p_T > 3$ GeV, excluding feed-down

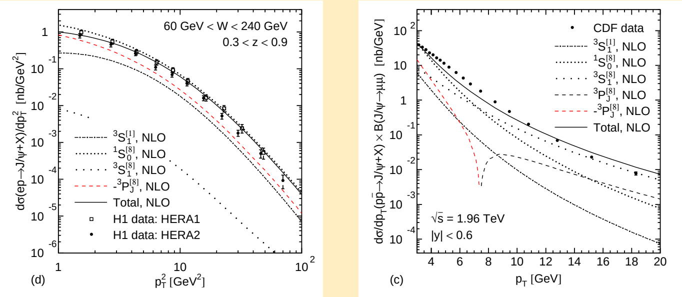


10^{-2} GeV^{3+2L}	BK default	Tevatron only
$\langle\mathcal{O}(^1S_0^{[8]})\rangle$	4.50 ± 0.72	59
$\langle\mathcal{O}(^3S_1^{[8]})\rangle$	0.312 ± 0.093	-6.6
$\langle\mathcal{O}(^3P_0^{[8]})\rangle$	-1.21 ± 0.35	-29

- CO MEs too large in magnitude $\rightsquigarrow$ NRQCD velocity scaling rules violated
- Substantial fine tuning $\rightsquigarrow$ unnatural

Significance of p_T cut on Tevatron data

Fit to Tevatron $p_T > 7$ GeV and HERA data, excluding feed-down

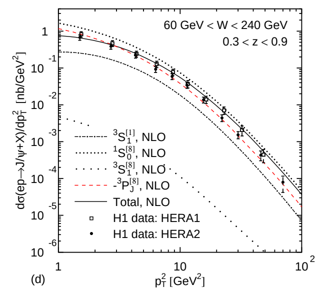


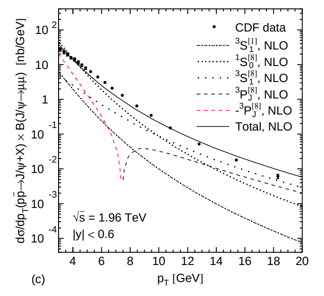
10^{-2} GeV^{3+2L}	BK default	Tevatron $p_T > 7$ GeV
$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$	4.50 ± 0.71	2.9
$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$	0.312 ± 0.093	0.4
$\langle \mathcal{O}(^3P_0^{[8]}) \rangle$	-1.21 ± 0.35	-0.58

- Shift of p_T cut on Tevatron data from 3 GeV to 7 GeV less dramatic
- Thanks to stabilizing influence of HERA data

Significance of feed-down

Fit to HERA and Tevatron data w/ feed-down subtracted





- feed-down/prompt $\approx (32 - 0.62\frac{p_T}{\mathrm{GeV}})\%$ from Tevatron I, $\approx 15\%$ at HERA from MC

10^{-2} GeV^{3+2L}	BK default	feed-down subtracted
$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$	4.50 ± 0.72	3.1
$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$	0.312 ± 0.093	0.23
$\langle \mathcal{O}(^3P_0^{[8]}) \rangle$	-1.21 ± 0.35	-0.82

- Feed-down corrections $\ll$ theoretical uncertainties

Comparison with MWC arXiv:1009.3655 (2)

Summary of M_0 and M_1 in (10^{-2} GeV^3) from BK and MWC

authors	data	feed-down	M_0	M_1
BK	default	—	2.5	0.59
BK	HERA $p_T > \sqrt{5}$ GeV	—	2.5	0.60
BK	Tevatron $p_T > 7$ GeV	—	1.9	0.54
BK	default	$\surd$	1.7	0.43
BK	only Tevatron	—	8.7	0.62
BK	only Tevatron $p_T > 7$ GeV	—	9.3	0.30
MWC	only Tevatron $p_T > 5$ GeV	$\surd$	5.2	0.16
MWC	only Tevatron $p_T > 7$ GeV	$\surd$	7.4	0.05

- Lower p_T cuts on HERA or Tevatron data marginal in joint fit
- Feed-fown corrections moderate $\rightsquigarrow$ no qualitative change
- Exclusion of HERA data $\rightsquigarrow$ fit greatly underconstrained

Comparison with MWC arXiv:1009.3655 (3)

Errors on M_0 and M_1 in 3-parameter fit to only Tevatron $p_T > 7$ GeV

Covariance matrix V_{ij} defined through

$$(V^{-1})_{ij} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial x_i \partial x_j}, \qquad (x_1,x_2,x_3) = (\langle\mathcal{O}(^1S_0^{[8]})\rangle, \langle\mathcal{O}(^3S_1^{[8]})\rangle, \langle\mathcal{O}(^3P_0^{[8]})\rangle)$$

has eigenvalues and eigenvectors:

$$\begin{aligned} \lambda_1 &= 0.127: & \vec{v}_1 &= (0.864, -0.114, -0.491) \\ \lambda_2 &= 1.78\times 10^{-6}: & \vec{v}_2 &= (0.502, 0.0968, 0.860) \\ \lambda_3 &= 4.69\times 10^{-8}: & \vec{v}_3 &= (0.0507, 0.989, -0.141) \end{aligned}$$

Cf. $\vec{v}_{M_0} = (0.500, 0, 0.866)$ and $\vec{v}_{M_1} = (0, 0.970, -0.242)$ from 2-parameter fit of MWC

$\Delta M_0 = (\vec{v}_{M_0}^T V \vec{v}_{M_0})^{1/2}/(\vec{v}_{M_0})_1 = 0.95\times 10^{-2}$

$\Delta M_1 = (\vec{v}_{M_1}^T V \vec{v}_{M_1})^{1/2}/(\vec{v}_{M_1})_2 = 0.29\times 10^{-2}$

I.e. 3-parameter fit yields: $M_0 = (9.3 \pm 0.95)$ and $M_1 = (0.30 \pm 0.29)$ in 10^{-2} GeV^3

$\rightsquigarrow$ almost 100% error on M_1 in 3-parameter fit

Cf. $M_0 = (7.4 \pm 1.9)$ and $M_1 = (0.05 \pm 0.02)$ from 2-parameter fit by MWC

- M_1 corresponds mostly to $\vec{v}_3$, but contains small admixtures of $\vec{v}_1$ and $\vec{v}_2$
- $\vec{v}_1$ very badly constrained $\rightsquigarrow$ large error on M_1
- Not exhibited in 2-parameter fit, where variations orthogonal to M_0 and M_1 are forbidden

Comparison with MWC arXiv:1009.3655 (4)

Errors on $\langle\mathcal{O}(^1S_0^{[8]})\rangle$, $\langle\mathcal{O}(^3S_1^{[8]})\rangle$ and $\langle\mathcal{O}(^3P_0^{[8]})\rangle$ in 3-parameter fits

10^{-2} GeV^{3+2L}	BK default	Tevatron $p_T > 7$ GeV only
$\langle\mathcal{O}(^1S_0^{[8]})\rangle$	4.50 ± 0.72	25.1 ± 30.7
$\langle\mathcal{O}(^3S_1^{[8]})\rangle$	0.312 ± 0.093	-1.96 ± 4.07
$\langle\mathcal{O}(^3P_0^{[8]})\rangle$	-1.21 ± 0.35	-9.09 ± 17.48

Correlation matrix V_{ij} of default fit has eigenvalues and eigenvectors:

$\lambda_1 = 6.39 \times 10^{-5}$: $\vec{v}_1 = (0.893, -0.111, -0.435)$
$\lambda_2 = 1.92 \times 10^{-7}$: $\vec{v}_2 = (0.440, 0.0193, 0.898)$
$\lambda_3 = 4.00 \times 10^{-8}$: $\vec{v}_3 = 0.0910, 0.994, -0.0659$

- No such strong hierarchy among eigenvalues
- Fit results meaningful

Summary (1)

Our project: Test NRQCD

- NRQCD provides rigorous factorization theorem for production and decay of heavy quarkonia: Include color-octet (CO) states.
- But: Need to proof universality of CO LDMEs.
- This work: Technological breakthrough: After 14 years finally NLO NRQCD photo- and hadroproduction calculation.

Our Results:

- CSM predictions: Could verify all previous results:
  CS contributions far below data in all considered experiments.
- Fitted CO LDMEs to p_T distributions at Tevatron and HERA.
- Used LDMEs for RHIC, LHC and HERA W and z distributions
  $\Longrightarrow$ CS+CO: Good agreement with data.

Summary (2)

Discussion of Fit:

- NLO hadroproduction P states: Shape changes, even negative.
  $\Longrightarrow$ Fit all three CO LDMEs (not linear combination like at LO).
- Negative unphysical quantities no problem.
- Photo- and hadroproduction consistently described by NRQCD.

Still to be done:

- Include resolved photoproduction.
- Extend analysis to e^+e^- collisions (LEP, B factories).
- Include feed-down processes.
- Do polarization analysis.