Electric properties of halo nuclei using EFT

Daniel Phillips
Ohio University

Work done in collaboration with H.-W. Hammer

arXiv:1001.1511 and “in preparation”
see also Rupak & Higa arXiv:1101.0207

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Outline

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- Example 1: Halo EFT for Carbon-19
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    - E1 transition from s-state to p-state
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- By this definition the deuteron is the lightest halo nucleus, and the pionless EFT for few-nucleon systems is a specific case of halo EFT.

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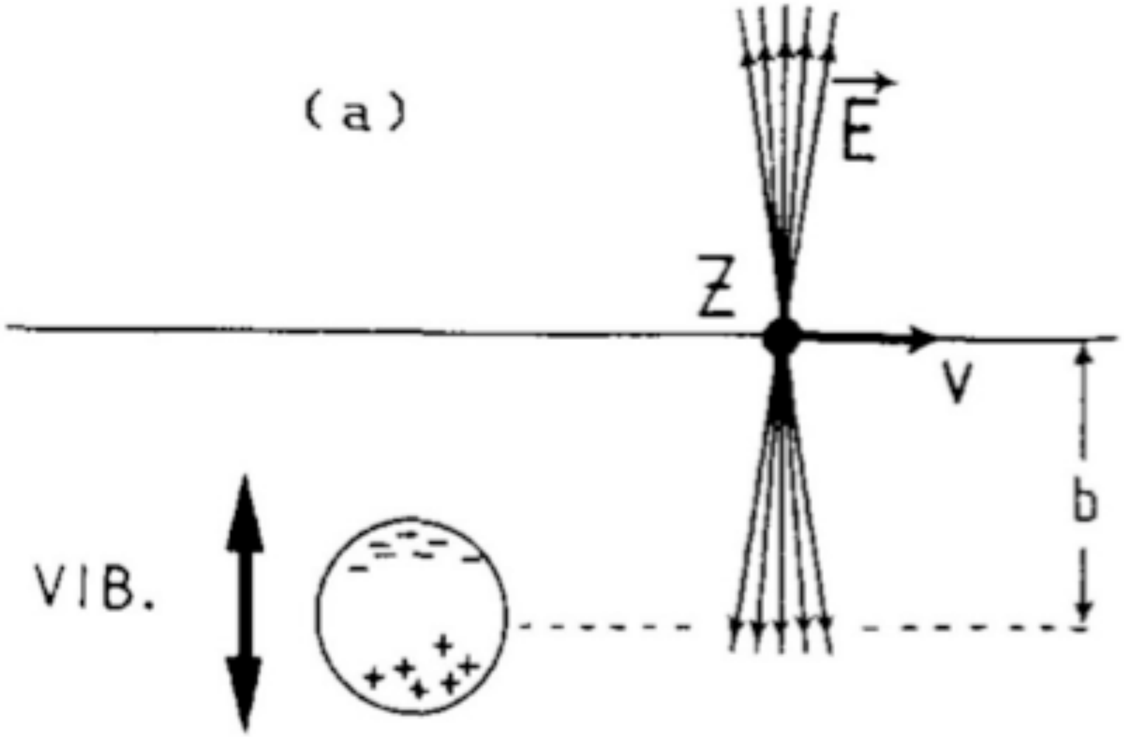
Bertulani, arXiv:0908.4307



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- Do with different Z, different nuclear sizes, different energies to test systematics

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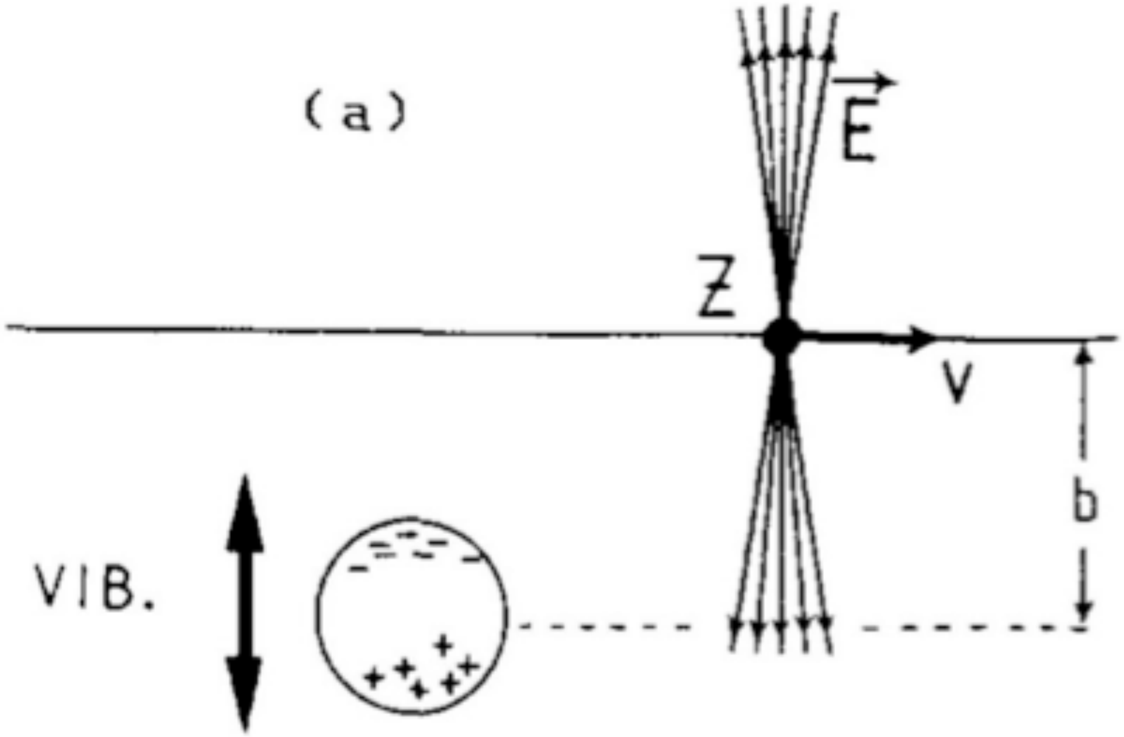

From disintegration to E1 strength

- Coulomb excitation dissociation cross section (p.v. b>>R_{target})

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- Virtual photon numbers computable in terms of relative velocity, equivalent photon frequency, impact parameter

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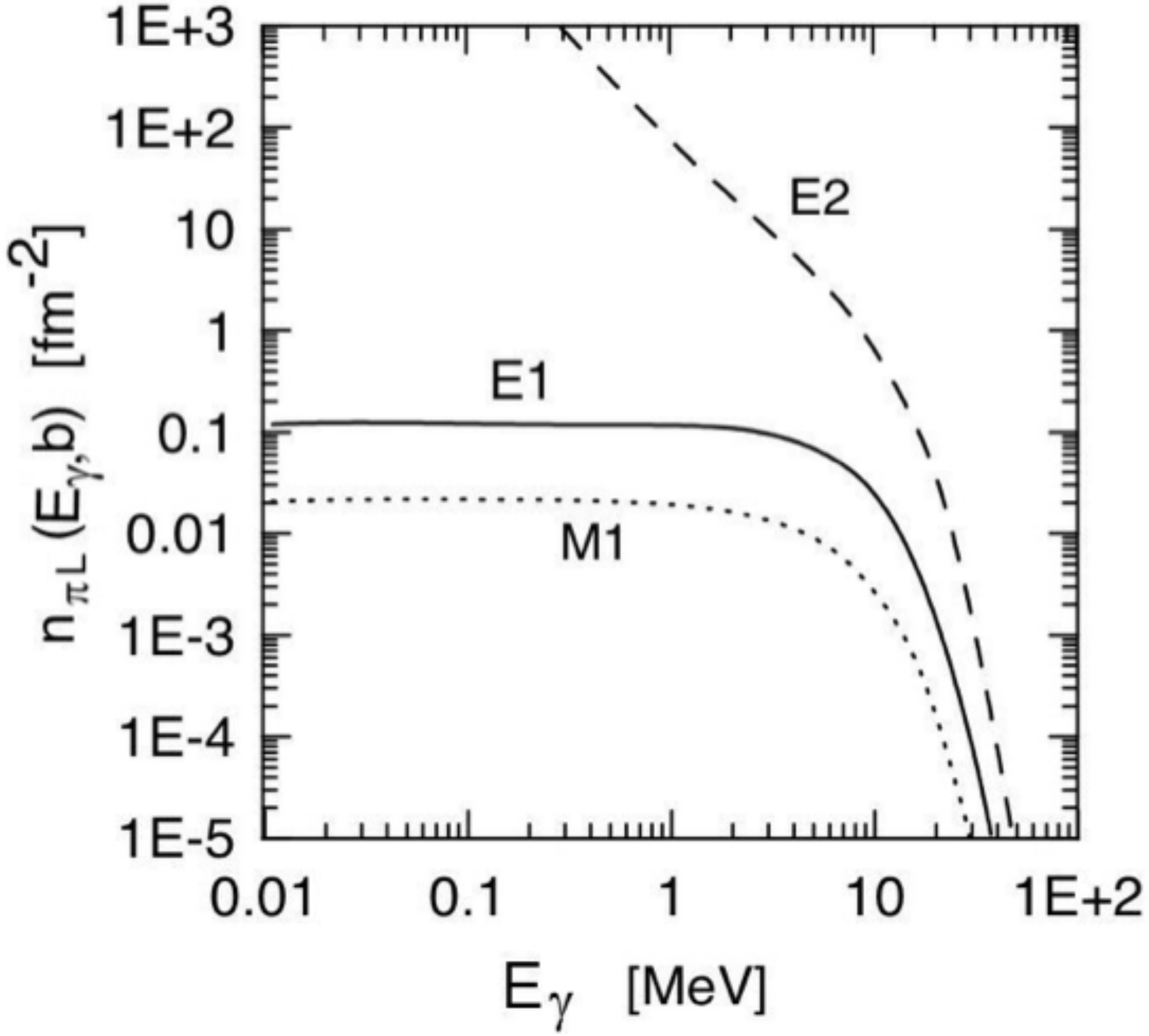

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- $\sigma_\gamma^{\pi L}(E_\gamma)$ can then be extracted: it's the (total) cross section for dissociation of the nucleus due to the impact of photons of multipolarity πL.

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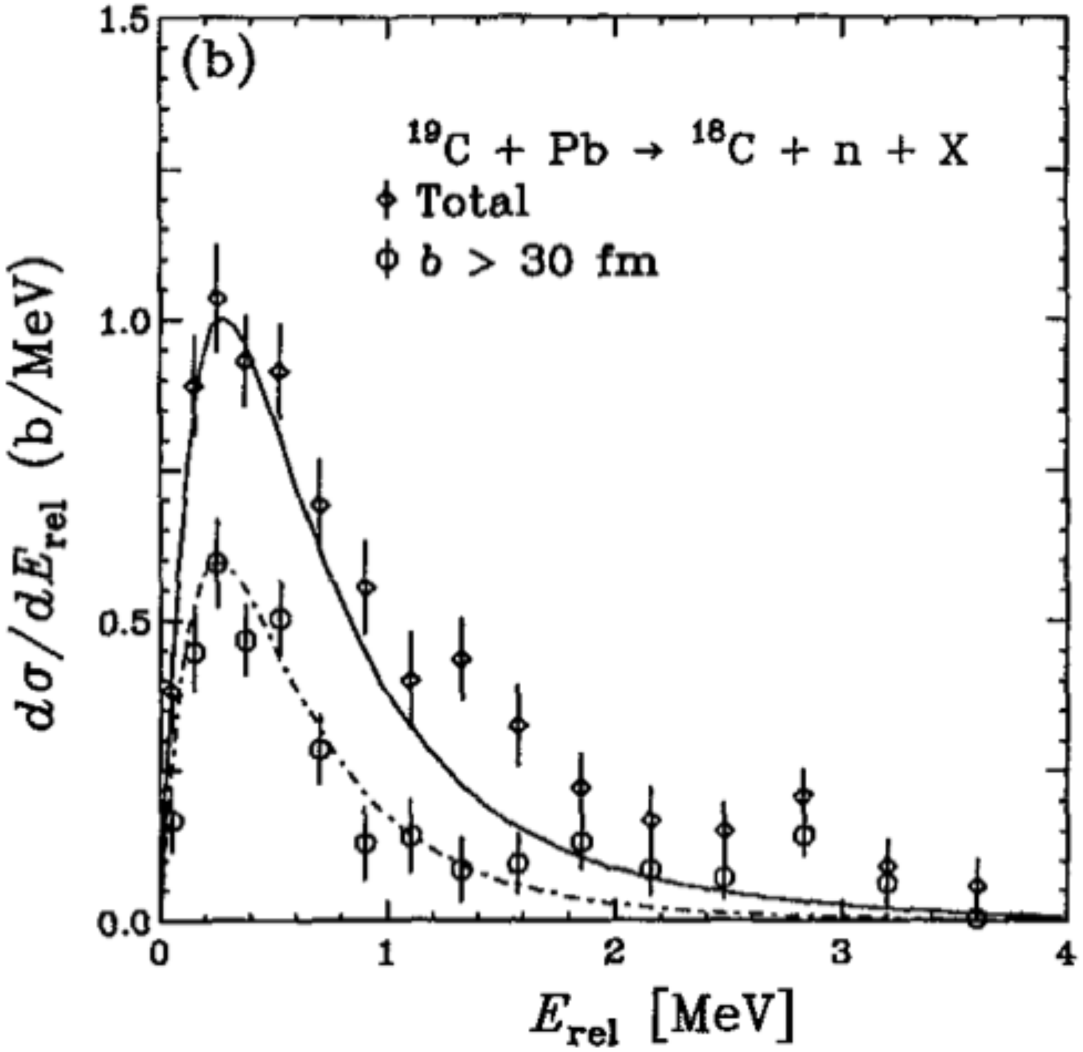

Nakamura et al. (2003)

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- Situation is different for P-wave state in^{11}Be, but that comes later....

Lagrangian I: shallow s-wave state

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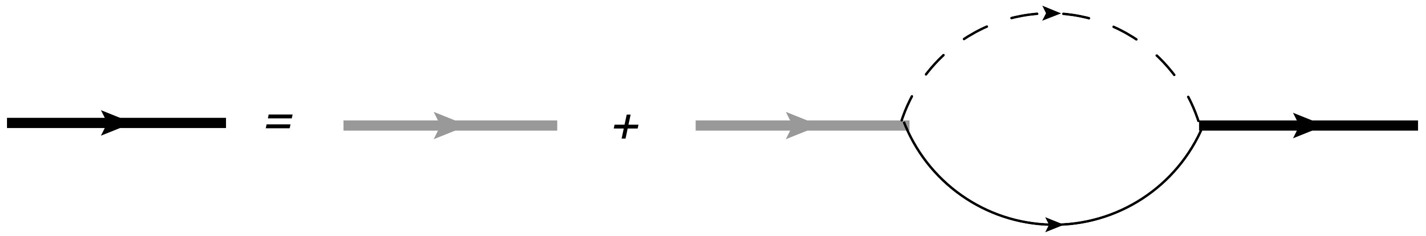
- ...if coefficients natural. But that’s a testable assumption.

Dressing the s-wave state

Kaplan, Savage, Wise; van Kolck; Gegelia;
Birse, Richardson, McGovern



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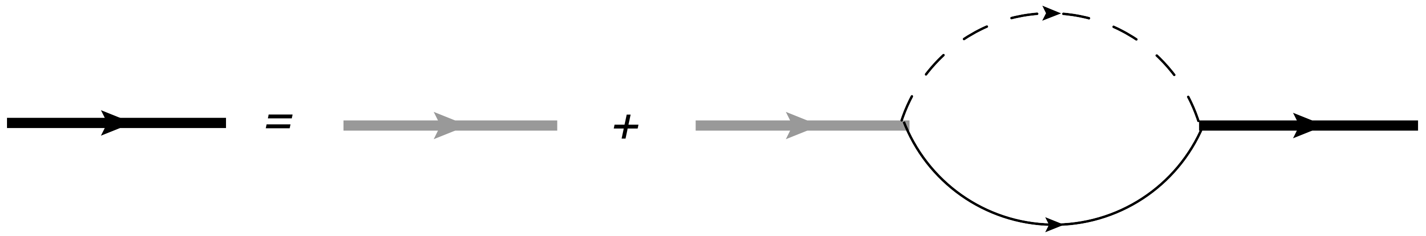

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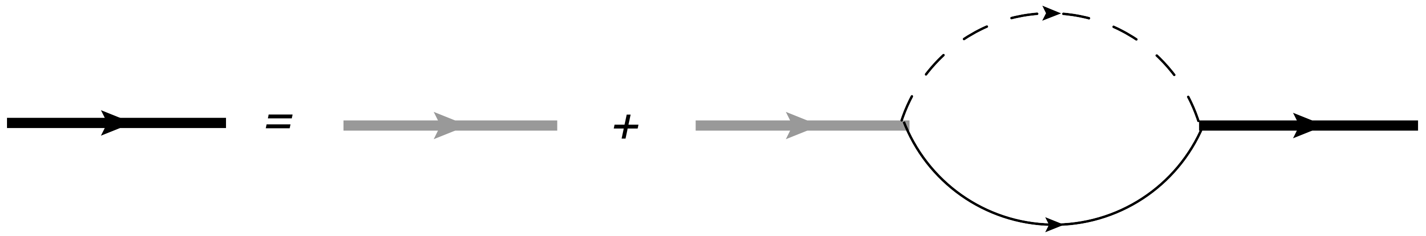

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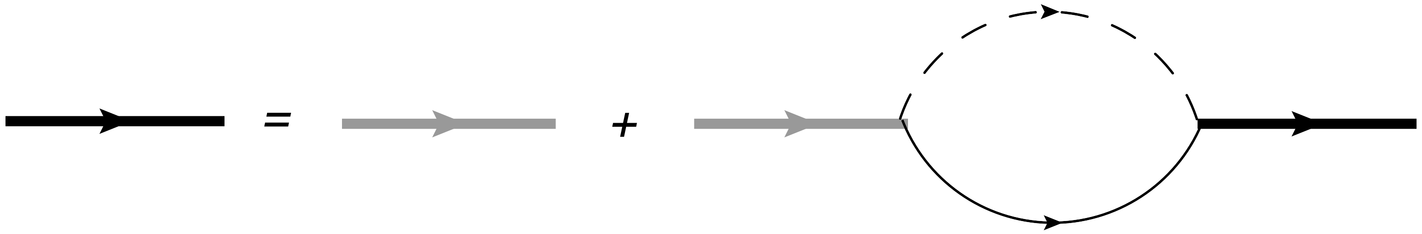

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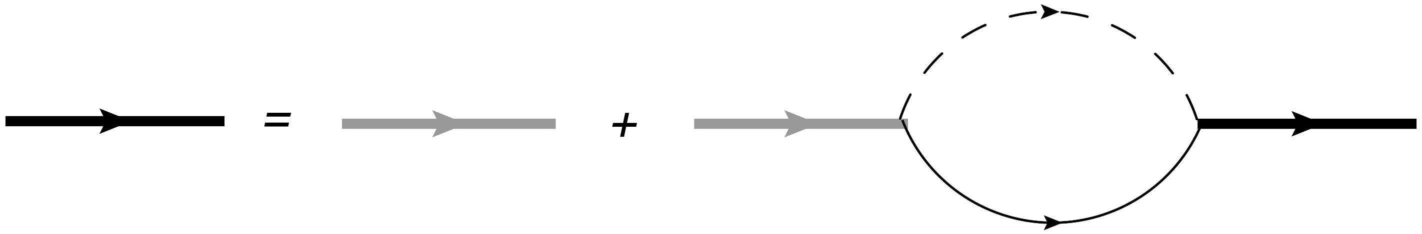

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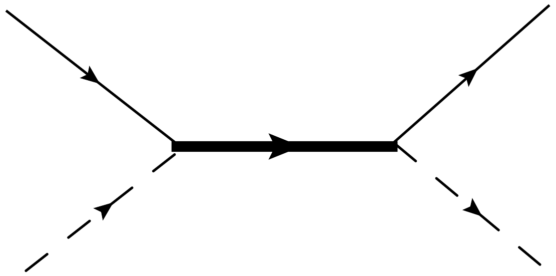

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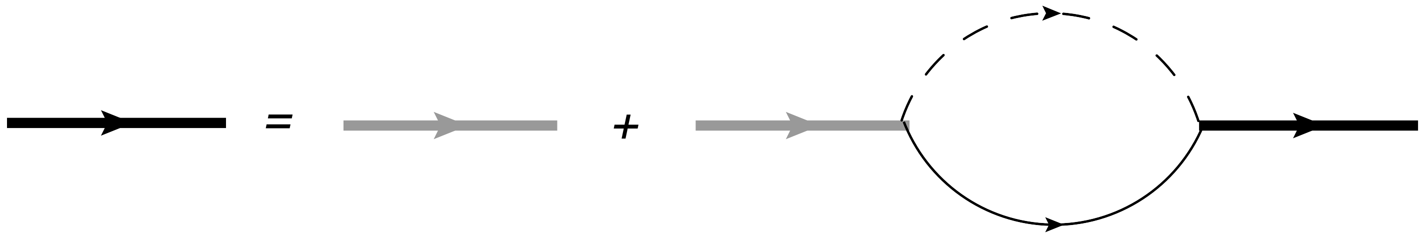

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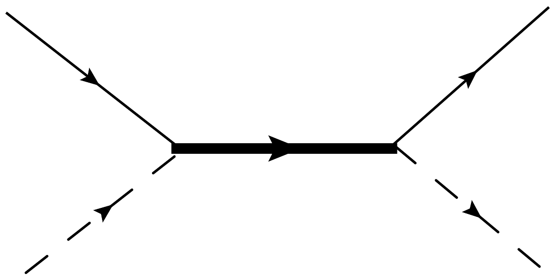

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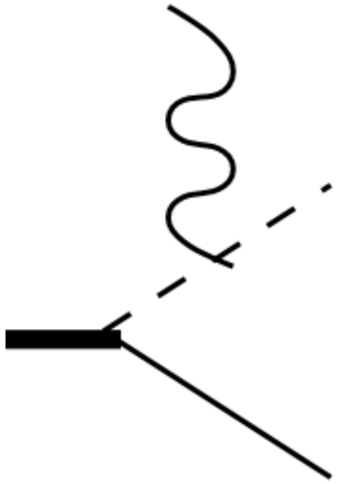

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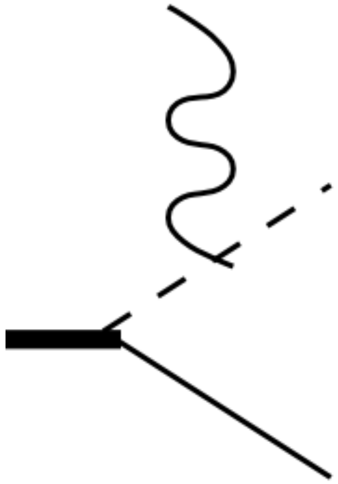

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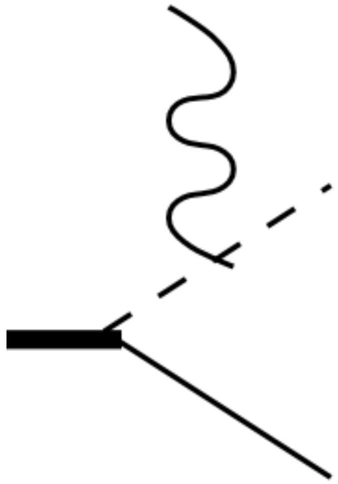

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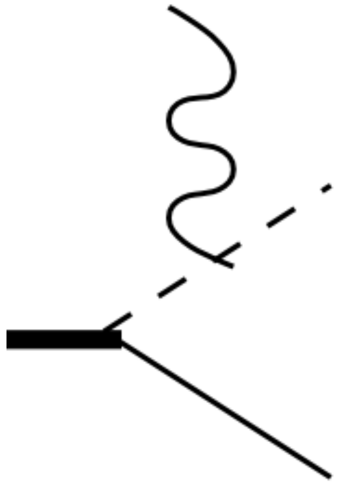

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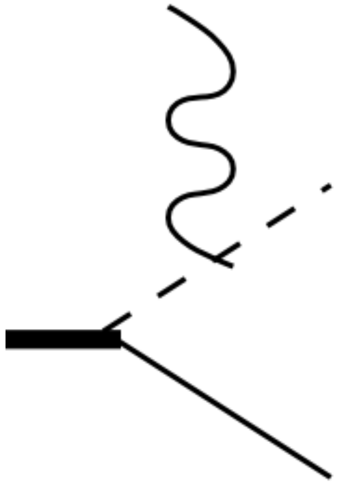

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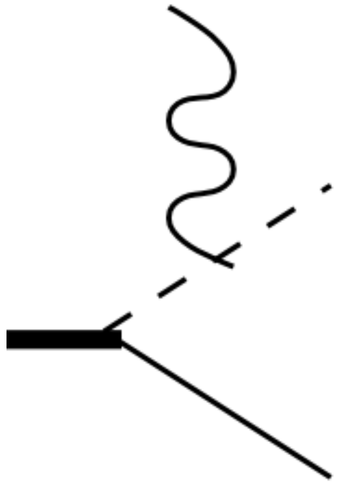

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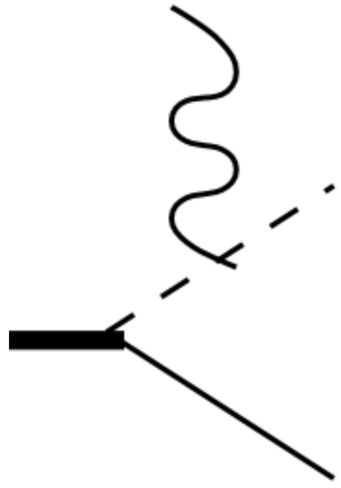

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- Consistent with short-distance piece of FSI loop due to P-wave interactions

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  - r_0 fixed from fitting height of peak at NLO

$$\sim A_0^2 = \frac{2\gamma_0}{1 - r_0\gamma_0}$$

  - γ_0 determines peak position

Results

Data: Nakamura et al., PRL, 1999

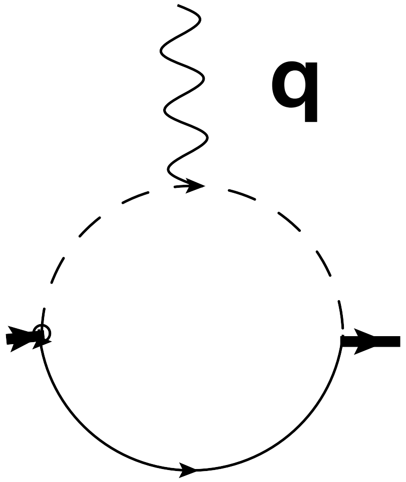
- Observable is dB(E1)/dE: E1 strength for transition to a core + neutron state, per unit energy, as function of energy (momentum) of the outgoing nc pair
  - Multiply by $N_{E1}(E_\gamma)$:
  - r_0 fixed from fitting height of peak at NLO

$$\sim A_0^2 = \frac{2\gamma_0}{1 - r_0\gamma_0}$$

  - γ_0 determines peak position
- Determine S-wave ^{18}C-n scattering parameters from dissociation data.

S-wave form factor

$-i\Gamma_c(\mathbf{q})$



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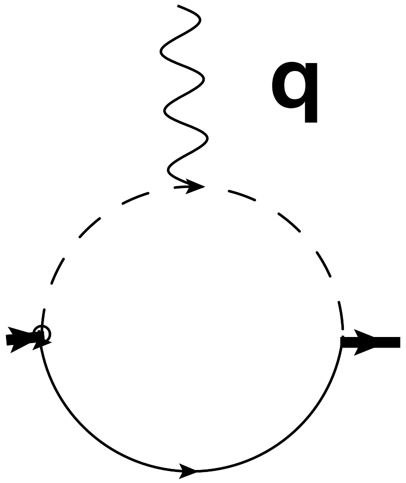

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$f=m/M_{nc}=m_R/M$

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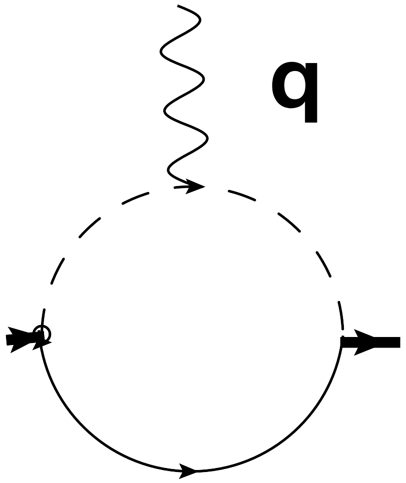

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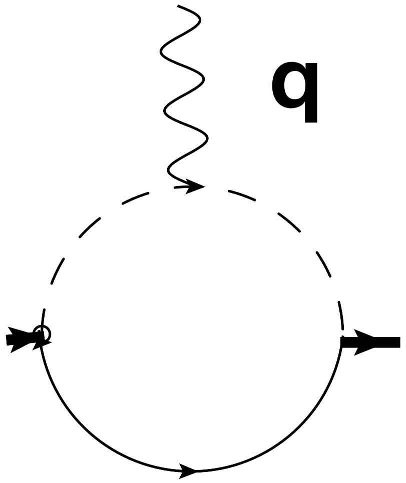

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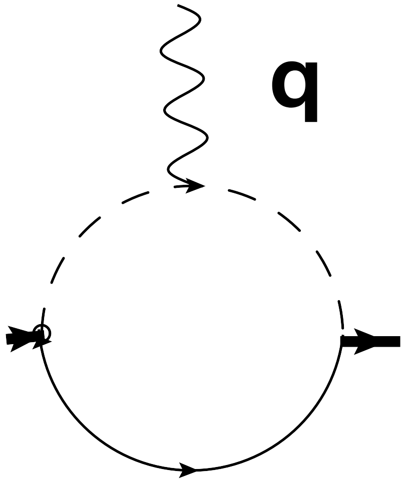

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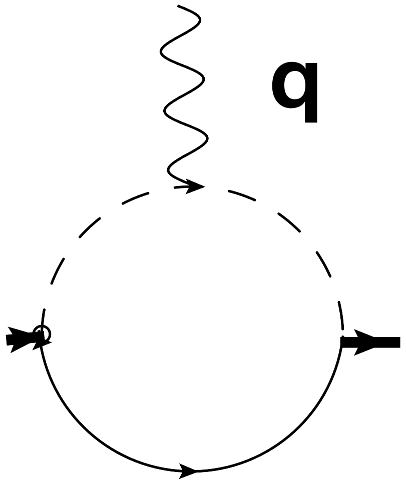

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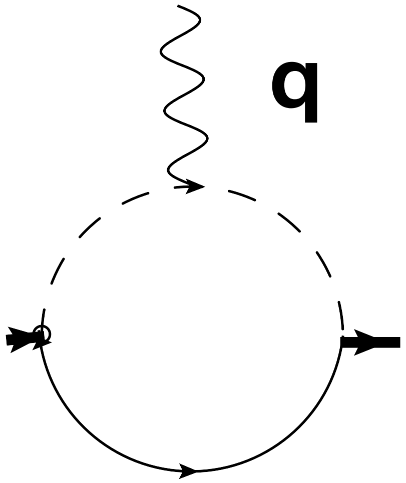

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$(<r_E^2>_{C19} - <r_E^2>_{C18})^{1/2} = 0.23 + 0.08$ fm

LO NLO

Beryllium-11 as a (one-neutron)* halo nucleus

---

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- First excitation in ^{10}Be: 3.4 MeV, ^{10}Be ground state is 0^+
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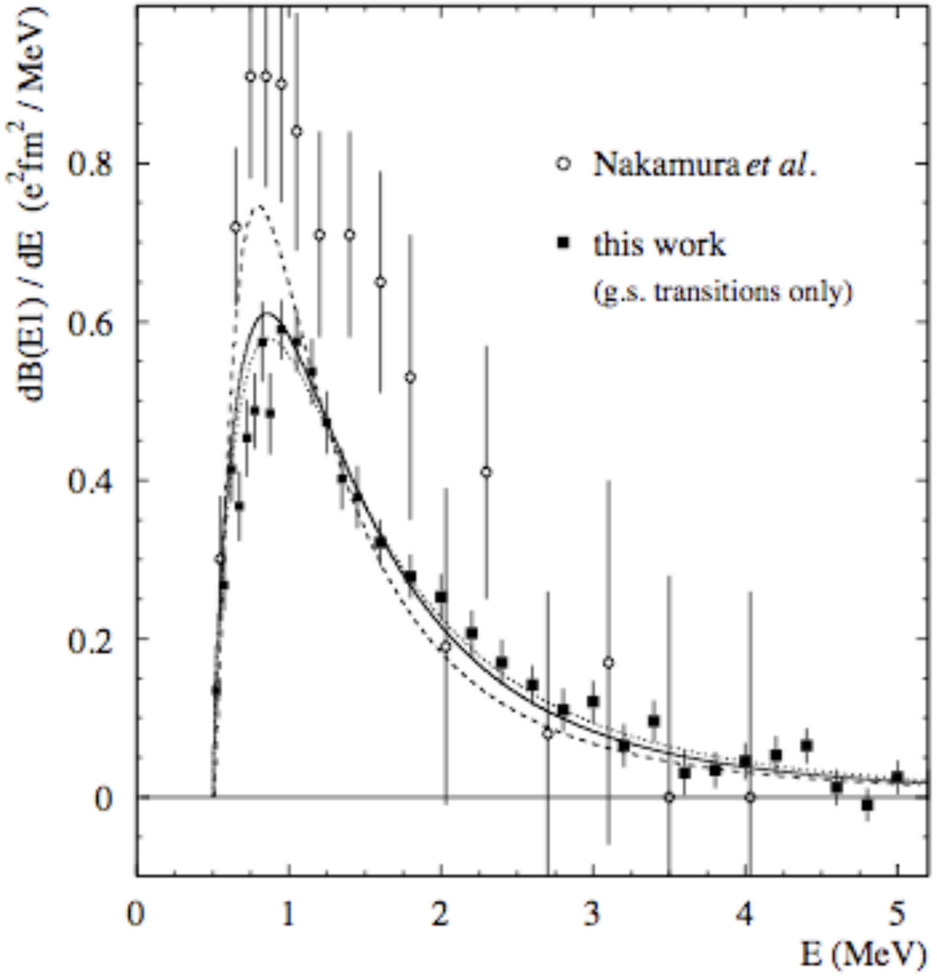

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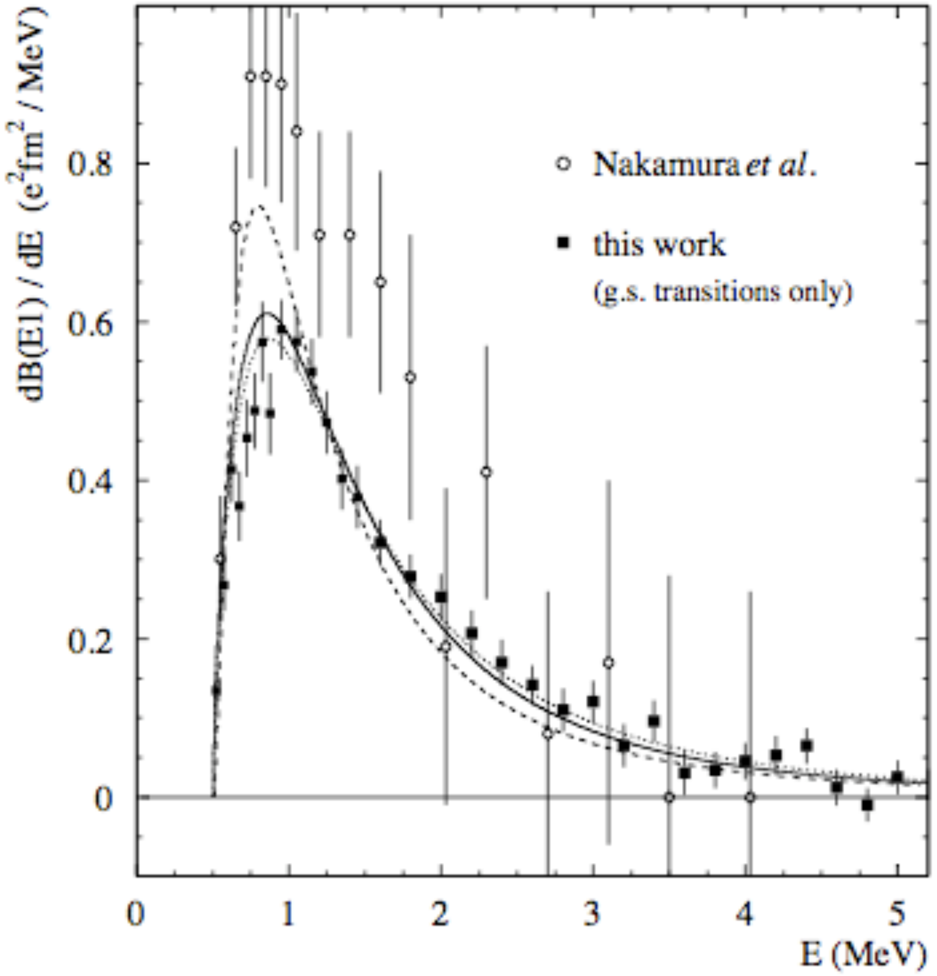

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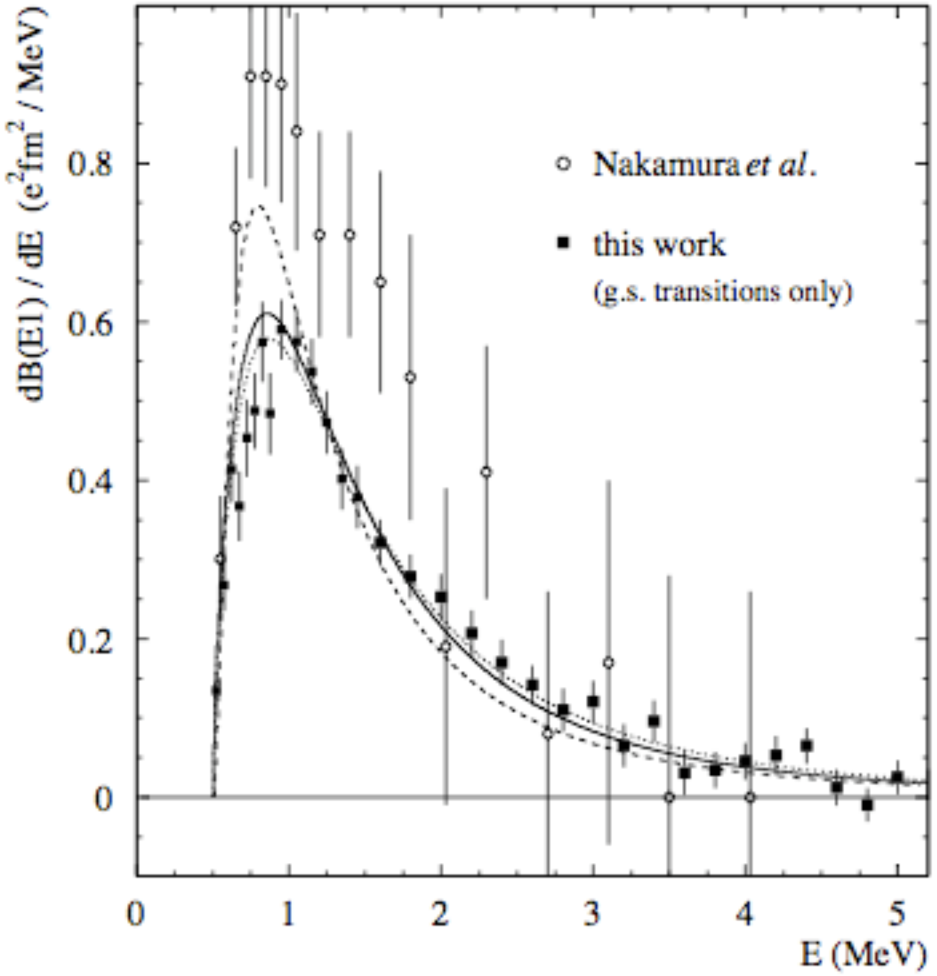

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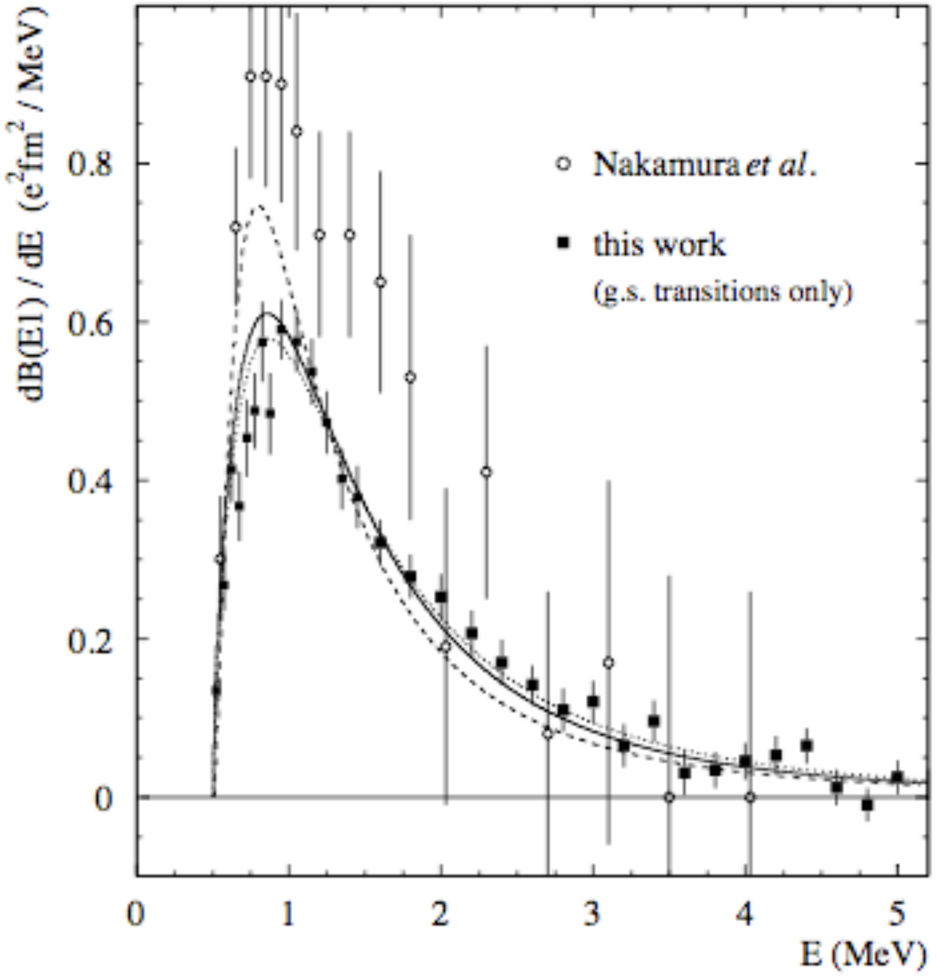

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c.f. atomic-physics measurement of radii

Noerterhaueser et al., PRL (2009)

Lagrangian II: shallow S- and P-states

$$
\begin{aligned}
\mathcal{L} \quad = \quad & c^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) c + n^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) n \\
& + \sigma^\dagger \left[\eta_0 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_0 \right] \sigma + \pi_j^\dagger \left[\eta_1 \left(i\partial_t + \frac{\nabla^2}{2M_{nc}} \right) + \Delta_1 \right] \pi_j \\
& - g_0 \left[\sigma n^\dagger c^\dagger + \sigma^\dagger n c \right] - \frac{g_1}{2} \left[\pi_j^\dagger (n \; i\overleftrightarrow{\nabla}_j \; c) + (c^\dagger \; i\overleftrightarrow{\nabla}_j \; n^\dagger) \pi_j \right] \\
& - \frac{g_1}{2} \frac{M - m}{M_{nc}} \left[\pi_j^\dagger \; i\overrightarrow{\nabla}_j \; (nc) - i\overleftrightarrow{\nabla}_j \; (n^\dagger c^\dagger) \pi_j \right] + \ldots \,,
\end{aligned}
$$

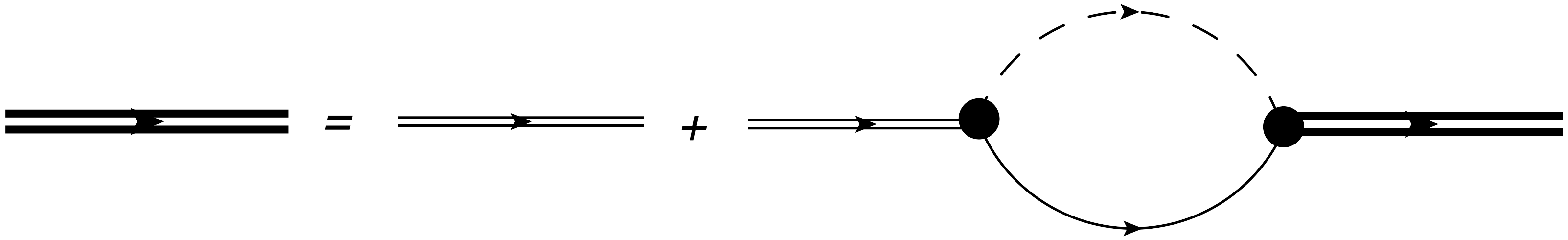
- c, n: “core”, “neutron” fields. c: boson, n: fermion.

- σ, π_j: S-wave and P-wave fields

- Compute power of non-minimal EM couplings by NDA with rescaled fields.

Dressing the P-wave state

Bertulani, Hammer, van Kolck (2002); Bedaque, Hammer, van Kolck (2003)



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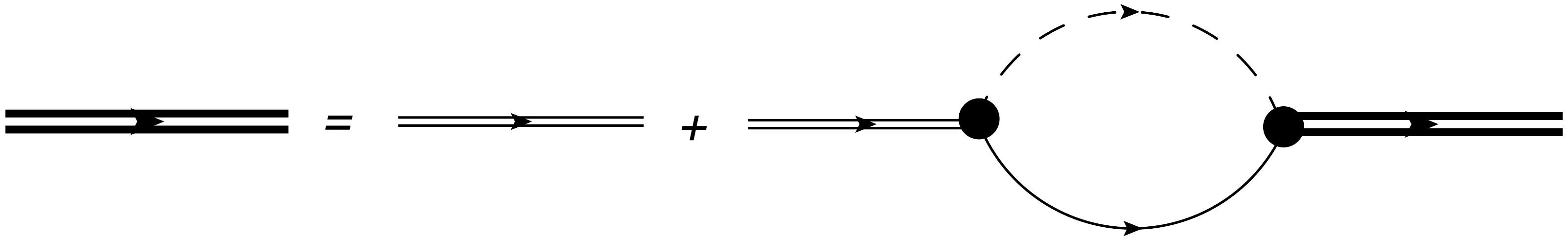

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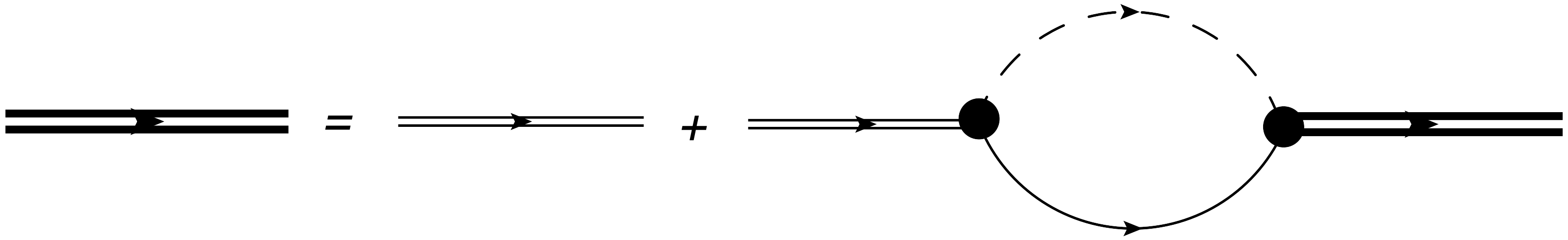

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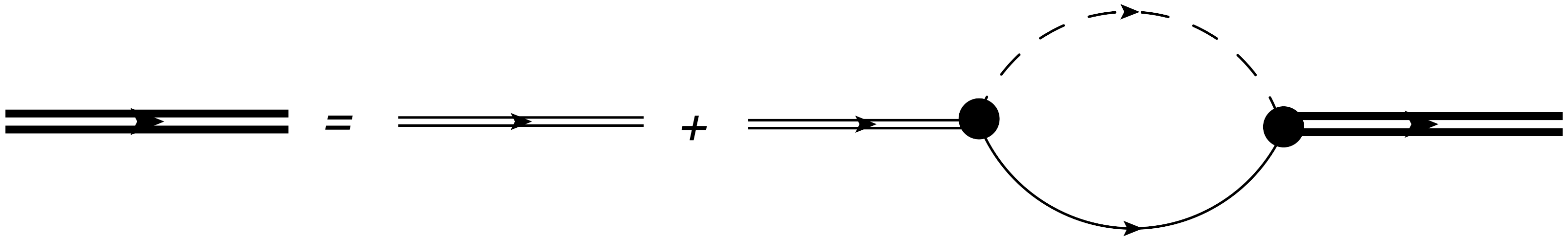

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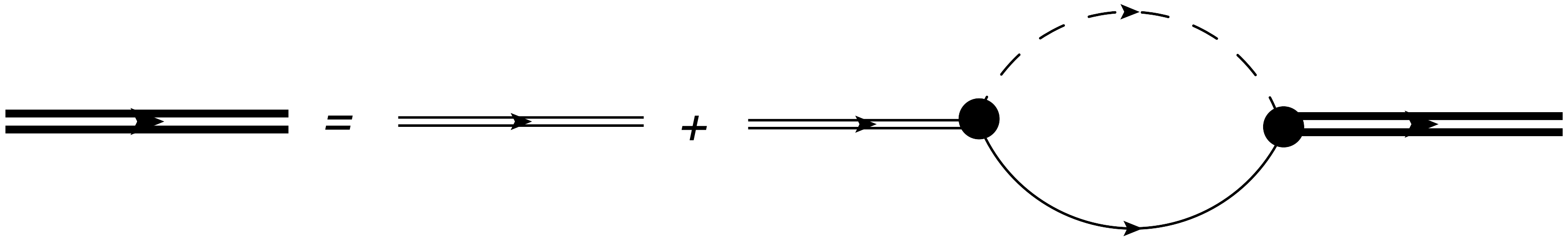

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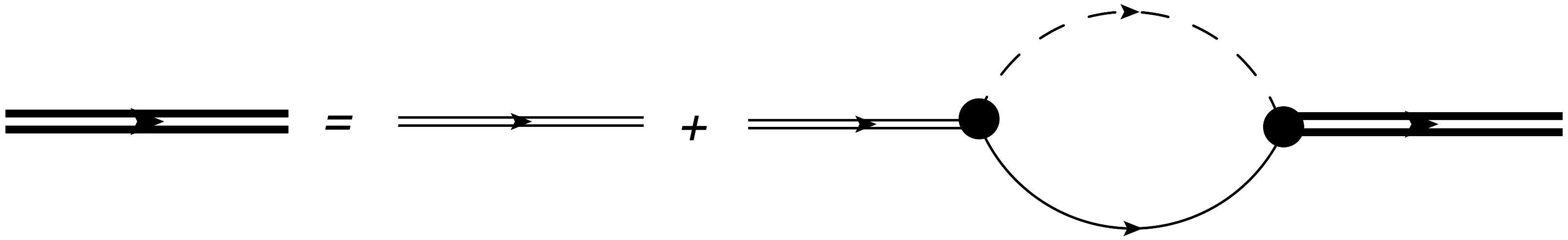

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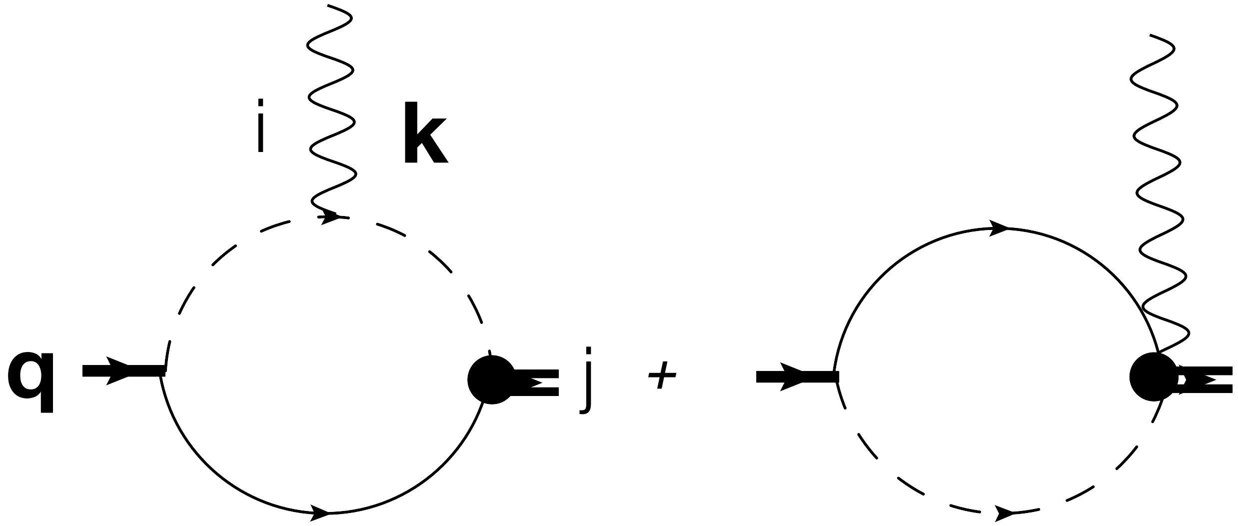
- No propagation of experimental errors here, but it’s easy to do

Irreducible S-to-P vertex: bound-to-bound transition

$-i\Gamma_{j\mu}(k)$



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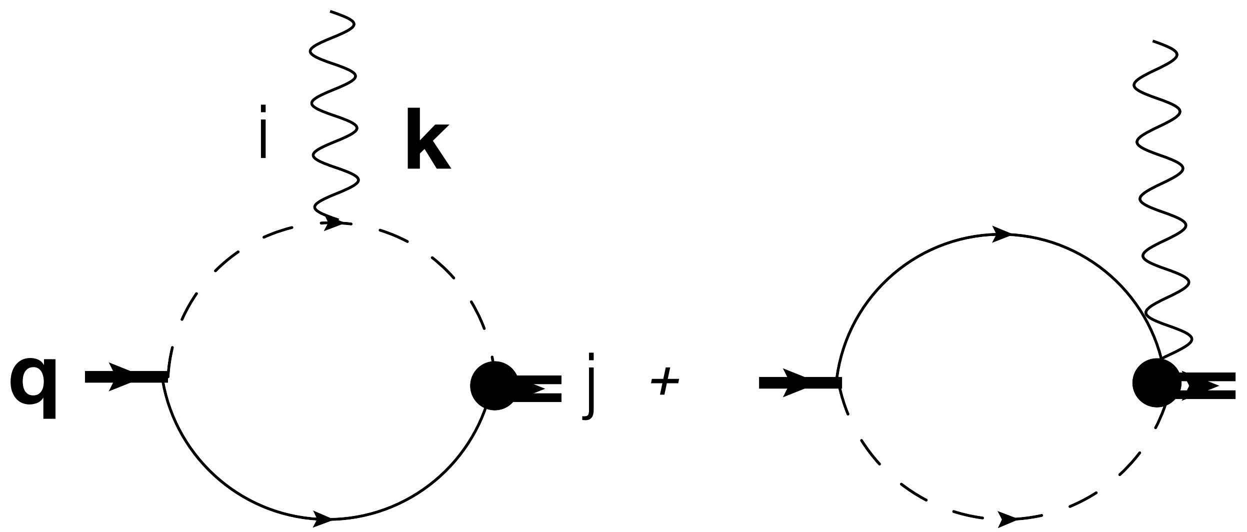

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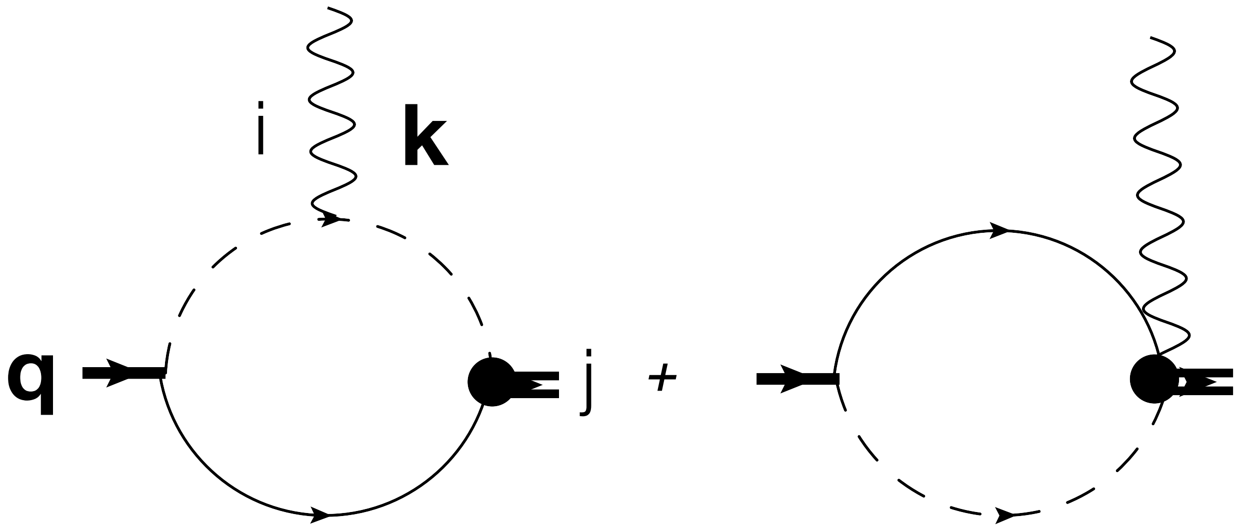

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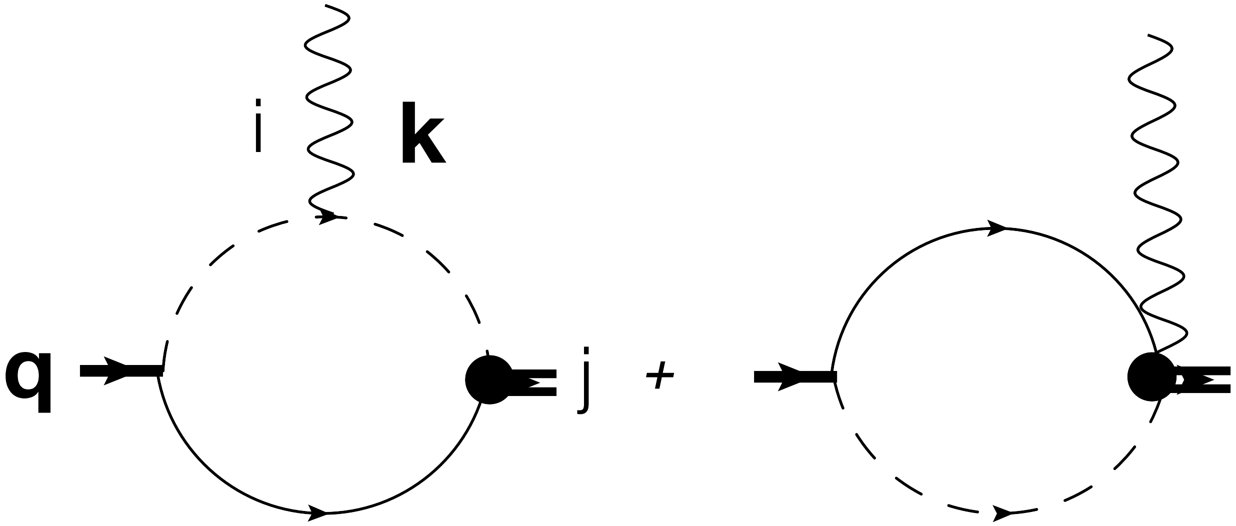

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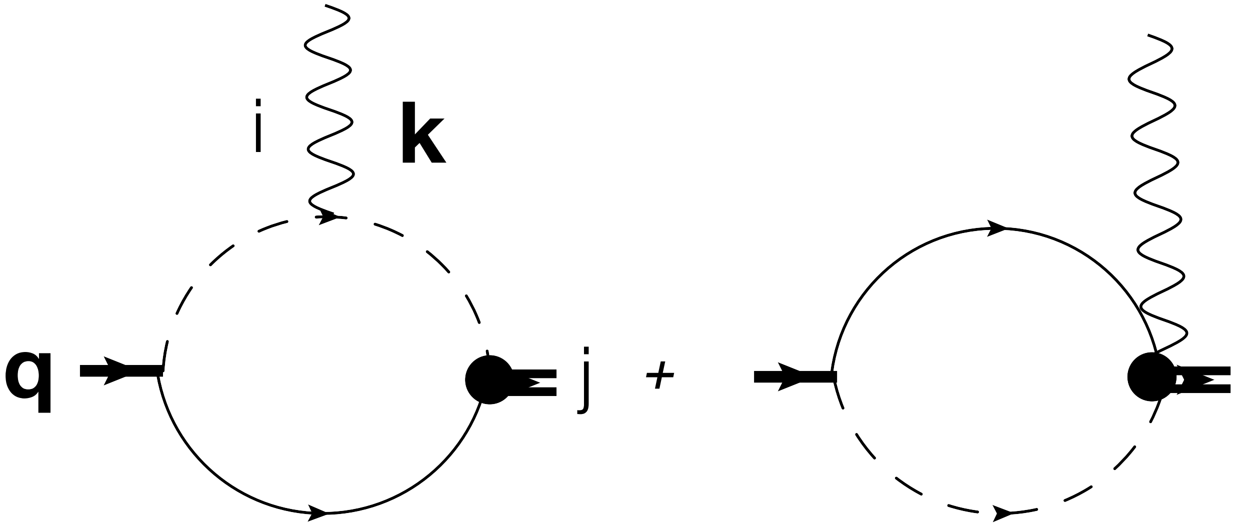

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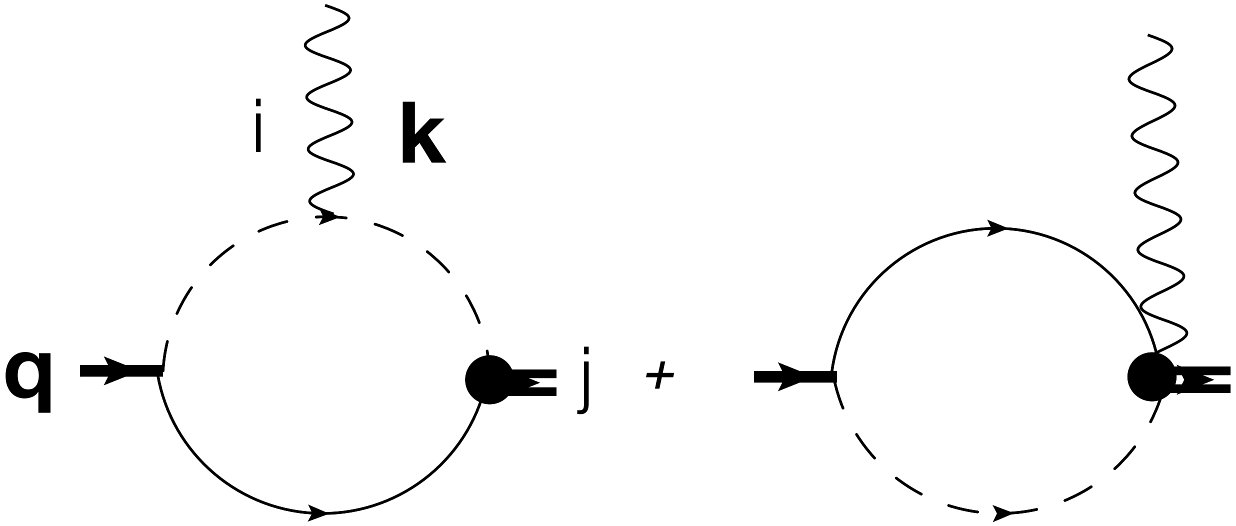

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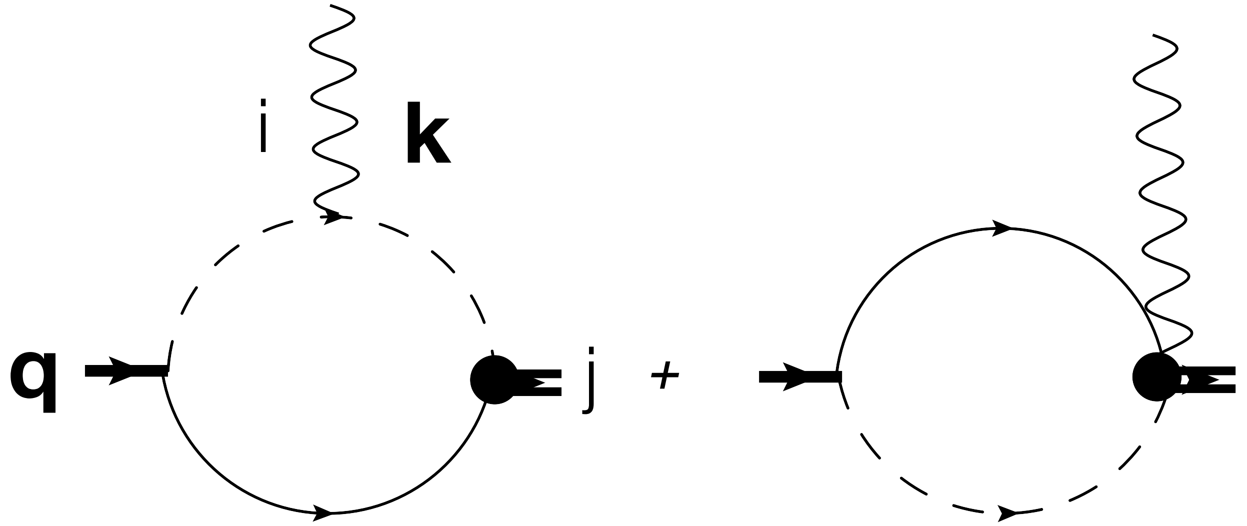
Irreducible S-to-P vertex: bound-to-bound transition

$-i\Gamma_{j\mu}(k)$



$k^0=\omega$

Divergences cancel, as they should

$$\Gamma_{ji} = \delta_{ji}\Gamma_E + k_j q_i \Gamma_M$$

for $\mathbf{k}\cdot\mathbf{q}=0$; $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$

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$$\to \mathbf{k}_j \int dr u_1(r) r u_0(r) \text{ as } |\mathbf{k}| \to \mathbf{0}$$

E1 matrix element

Converting to result for S-to-P transition

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- Also first contribution of physics at scale R_{core} occurs at NLO

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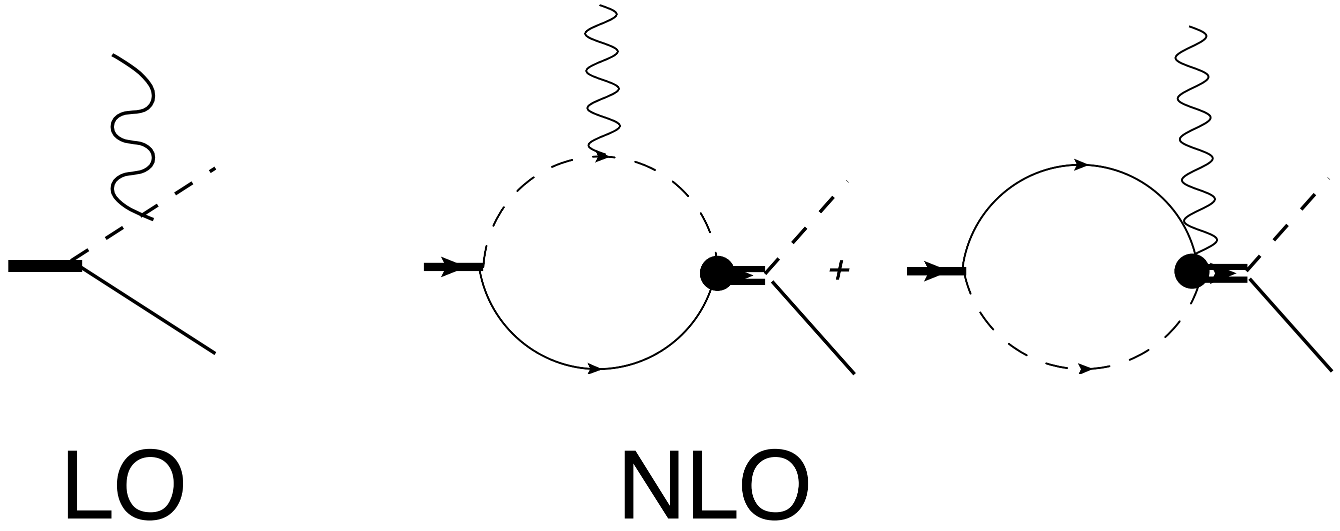

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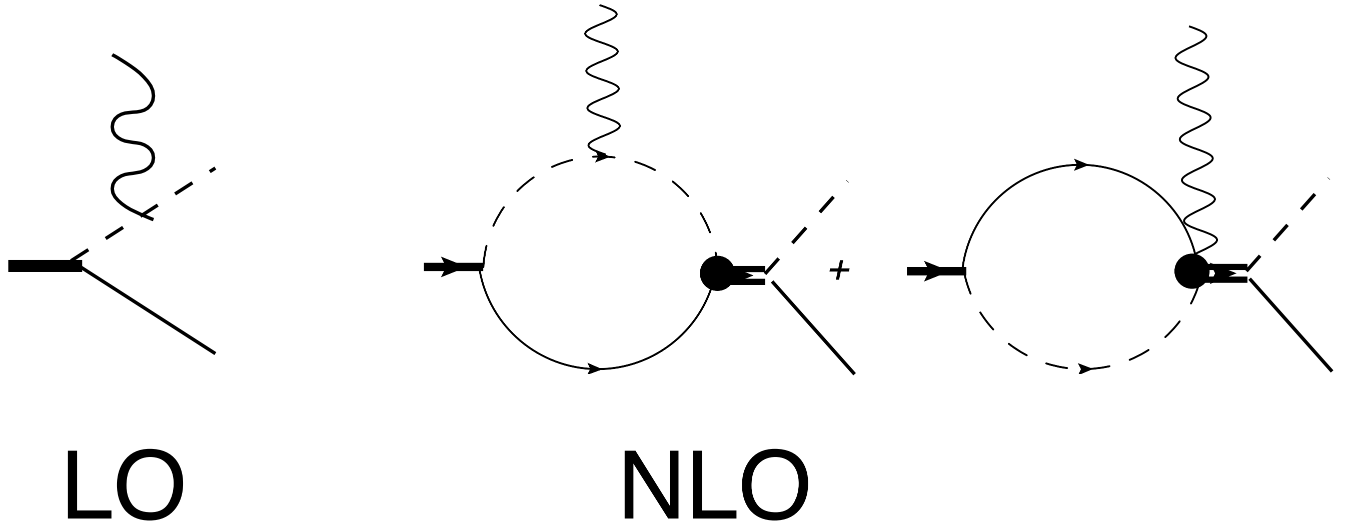

- Also get corrections to A_0 (a.k.a. wf renormalization) at NLO

Coulomb dissociation: formulae

c.f. Rupak & Higa arXiv:1101.0207

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- Higher-order corrections to phase shift at NNLO. Appearance of S-to-${}^2P_{1/2}$ E1 counterterm also at that order.

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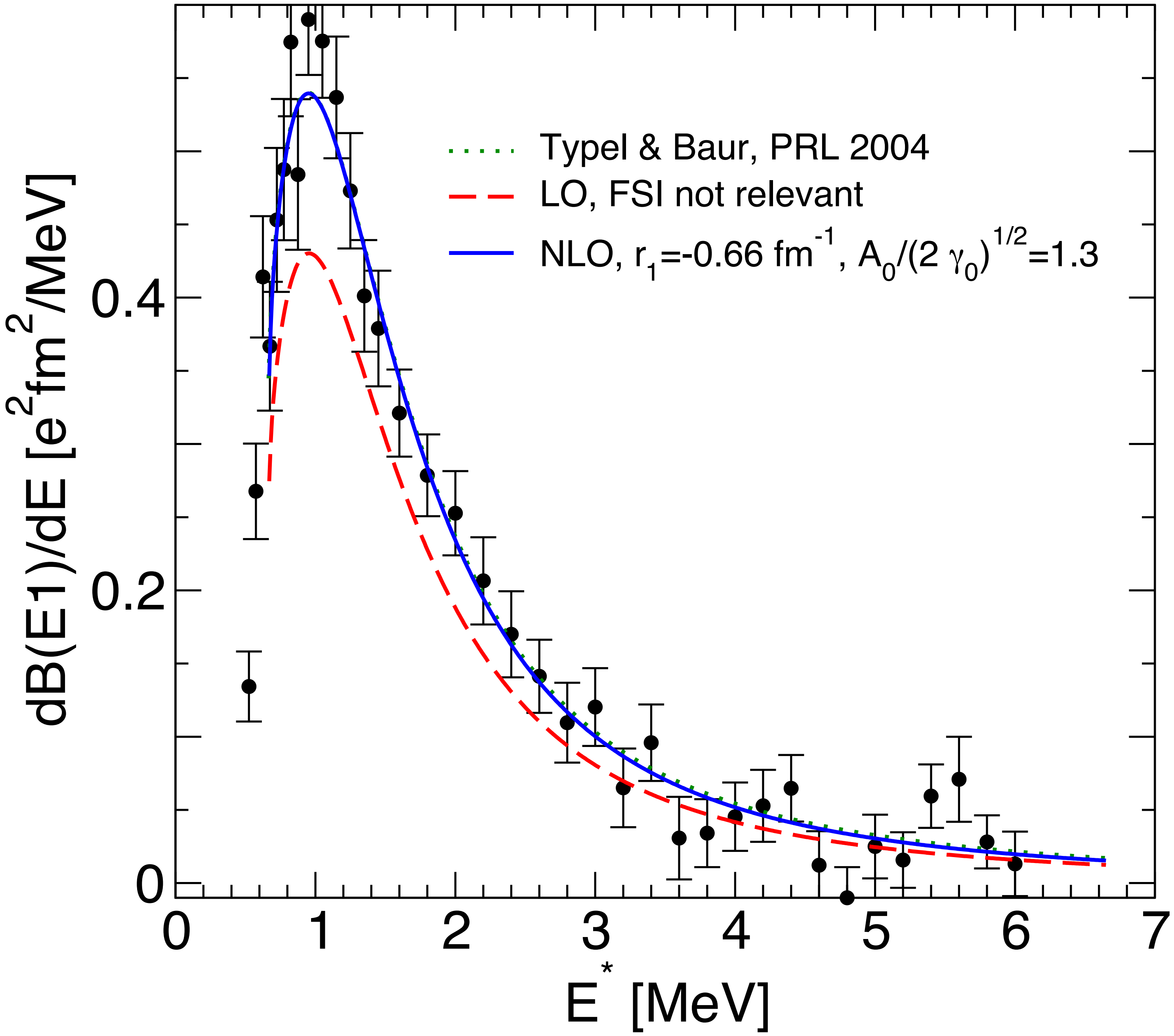

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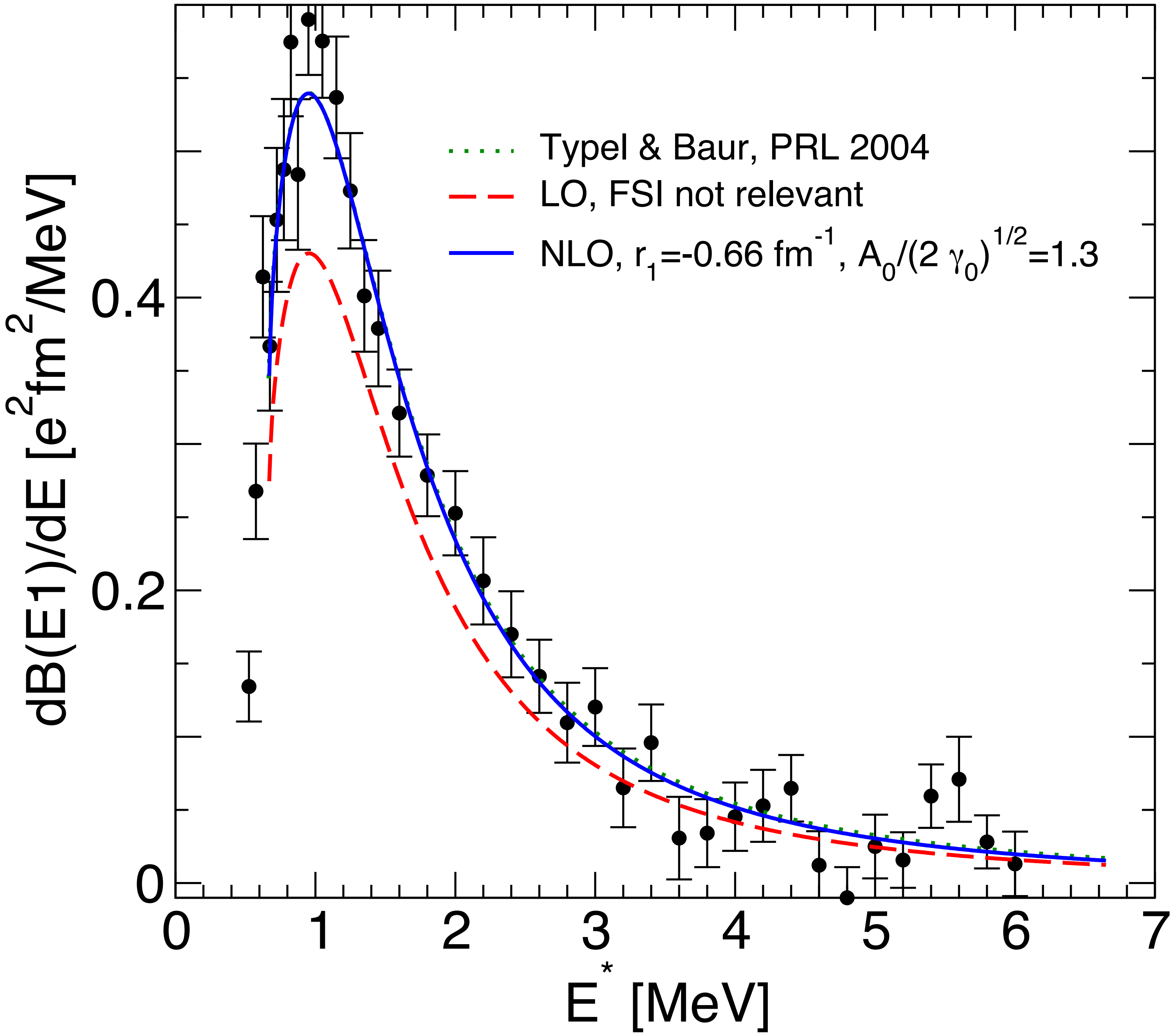

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$$x = \gamma_1/\gamma_0; \quad y = p'/\gamma_0$$

$$\frac{d\mathrm{B(E1)}}{dE} = \frac{48}{\pi^2 B_0}\frac{y^3}{(y^2+1)^4}\left[e^2 Q_c^2 \Delta\langle r_E^2\rangle^{(\sigma)} - \frac{3\pi}{4}B(E1)\frac{(1+x)^4(1+3y^2)}{(y^2+x^2)(1+2x)^2}\right]$$

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- Other one- (and two-?) neutron (?and proton) halos await: “universality”.