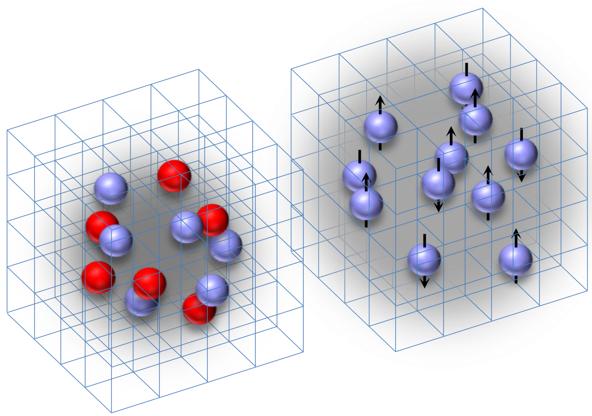
Nuclear physics from lattice effective field theory



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work done in collaboration with

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Hermann Krebs (Bochum)

Ulf-G. Meißner (Bonn/Jülich)

Strong interactions: From methods to structures

Bad Honnef, February 12-16, 2011

Outline

Chiral effective field theory for nucleons

Lattice effective field theory

Lattice interactions and scattering data

Three-nucleon forces

Isospin breaking and Coulomb interaction

Euclidean time projection and auxiliary fields

Ground states for helium-4, beryllium-8, carbon-12

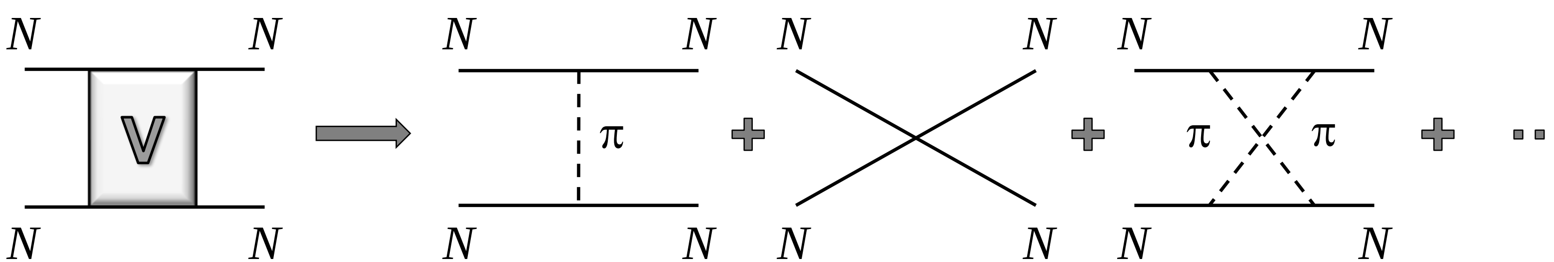
Carbon-12 spectrum and the Hoyle state

Summary and future directions

Chiral EFT for low-energy nucleons

Weinberg, PLB 251 (1990) 288; NPB 363 (1991) 3

Construct the effective potential order by order



V^{OPEP} | Contact interactions $1,\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ | V^{TPEP}

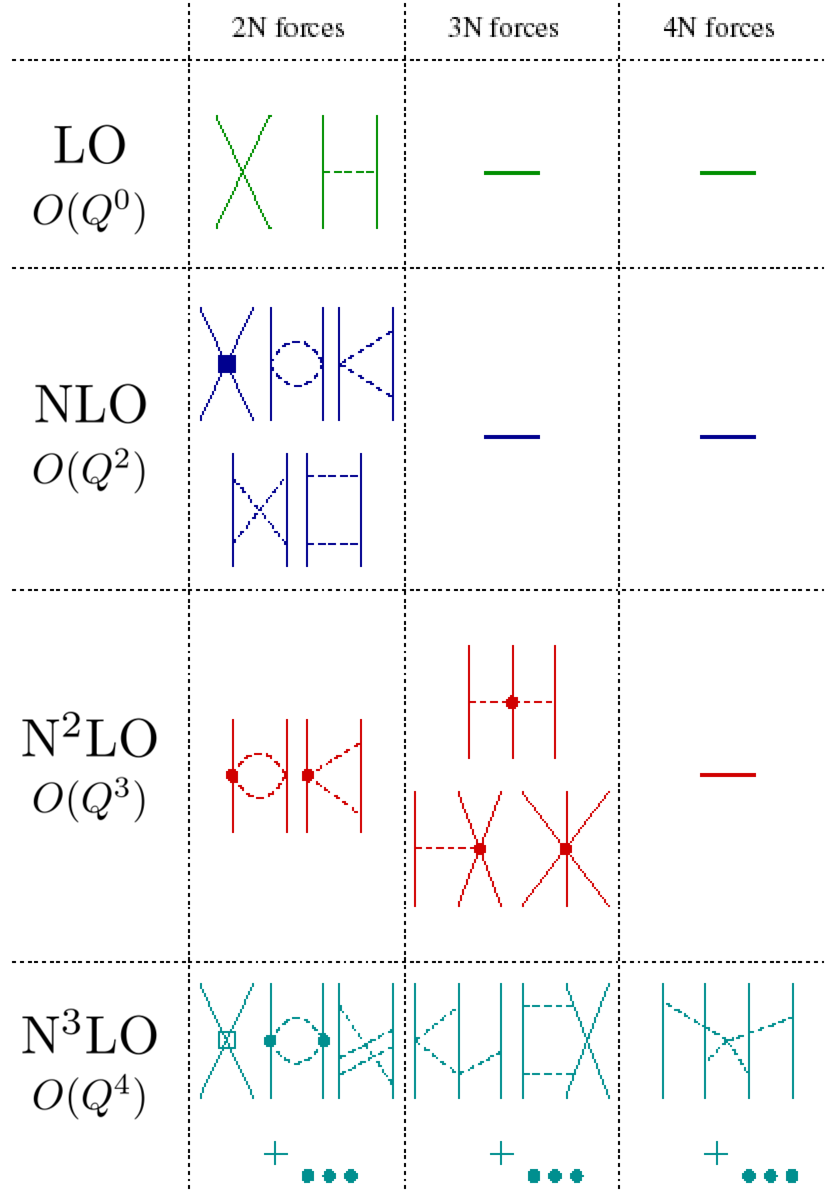
Leading order (LO) | Next-to-leading order (NLO)

Nuclear Scattering Data

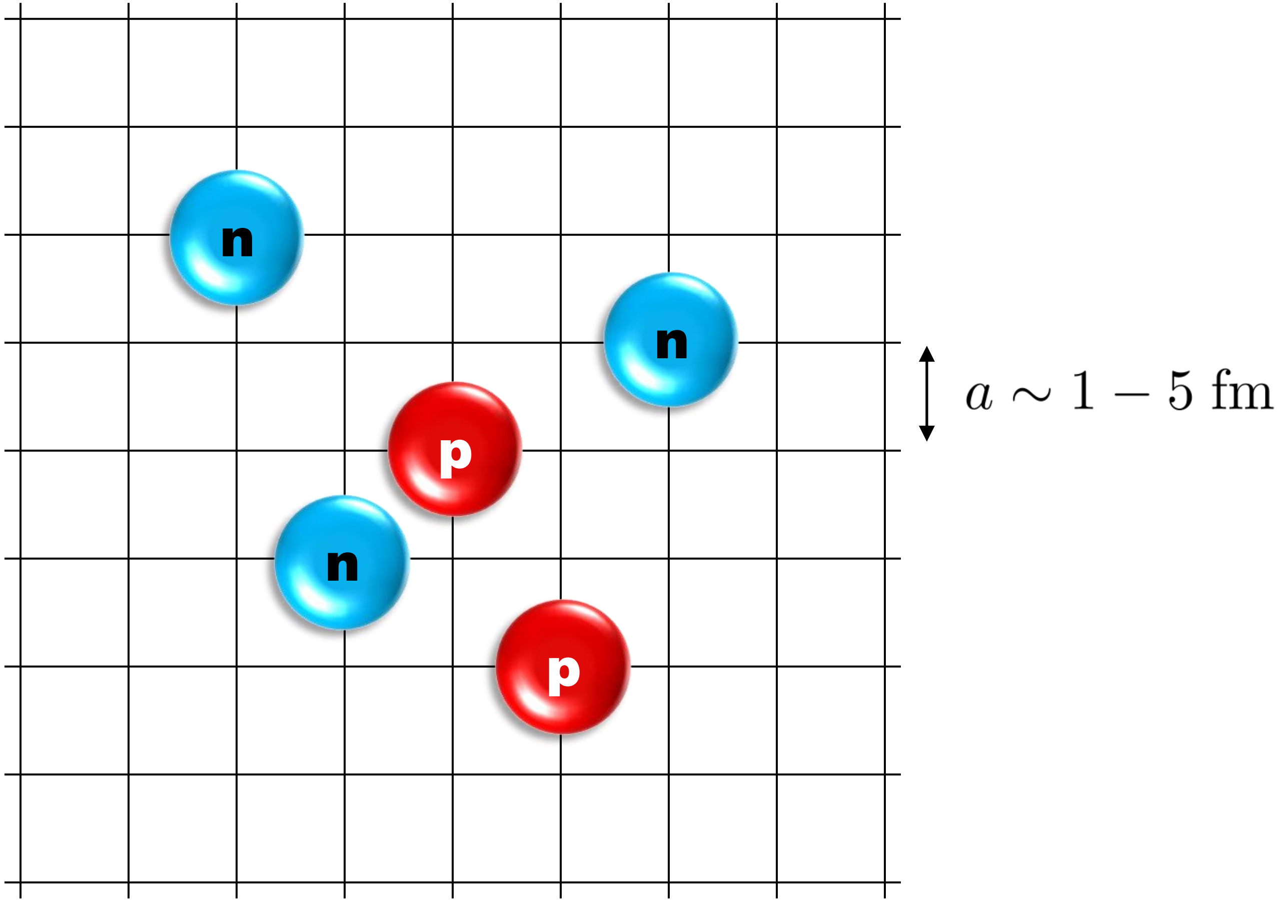
↓

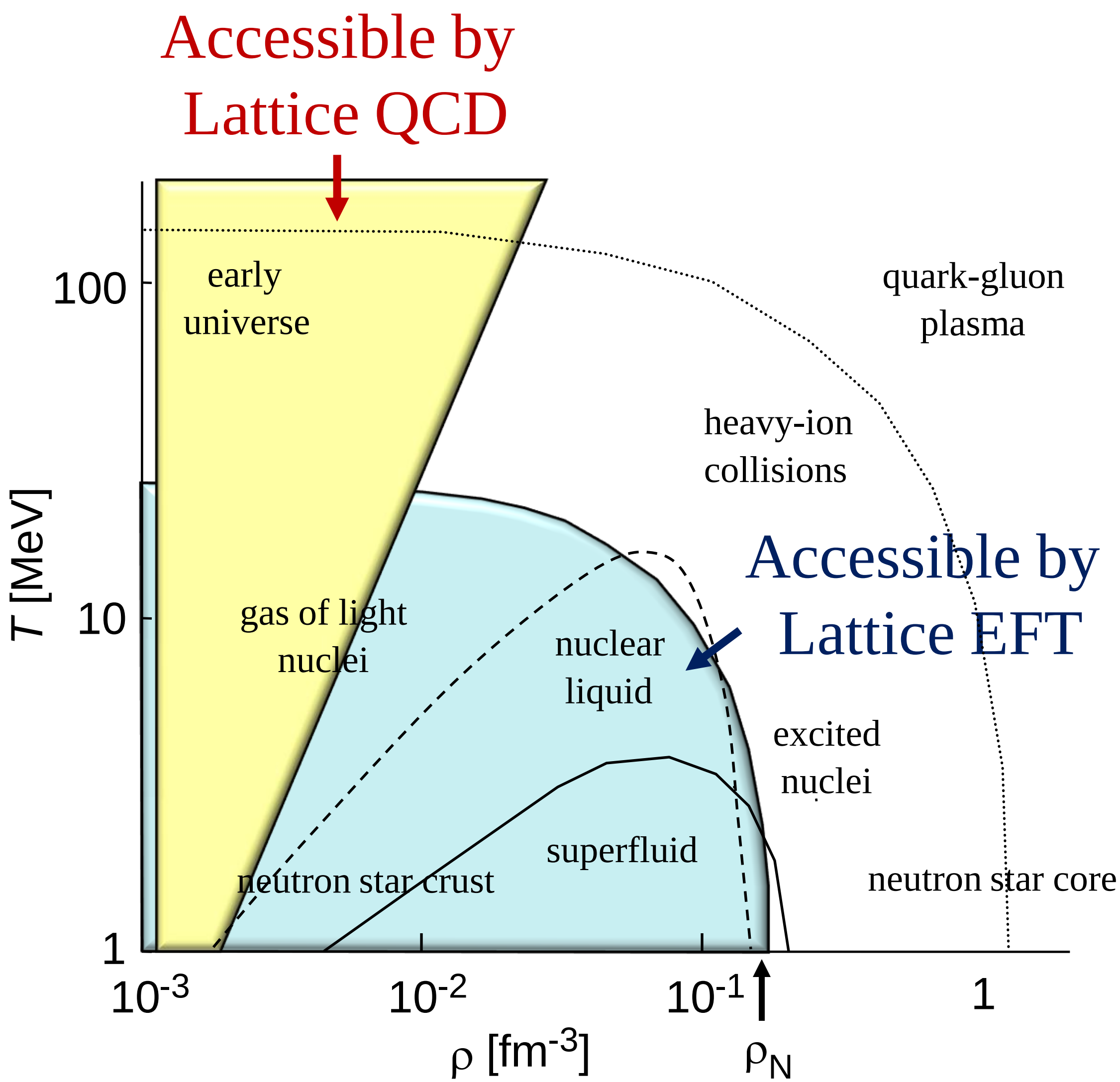
Effective Field Theory

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; Epelbaum et al. ’98, ‘03; Kaiser ’99-’01; Higa et al. ’03; Entem & Machleidt ’01, ‘03…



Lattice EFT for nucleons

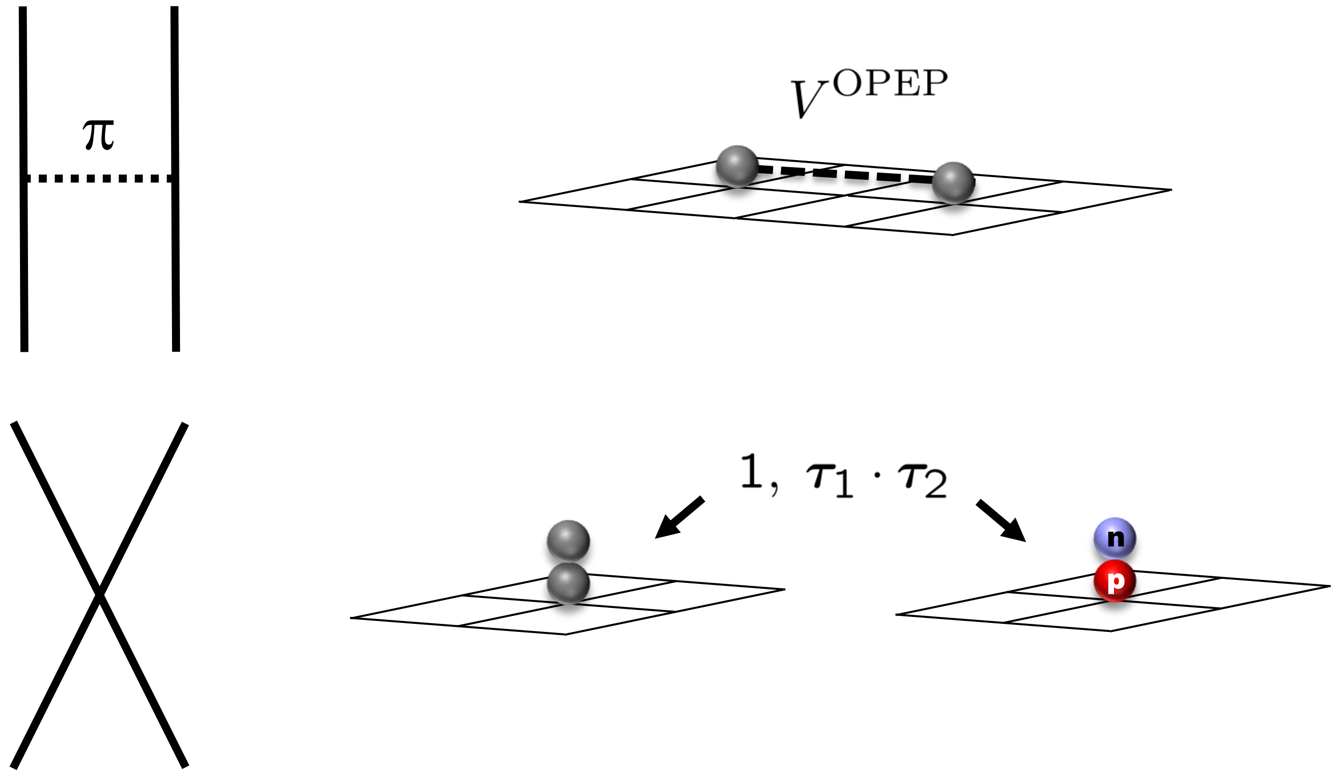




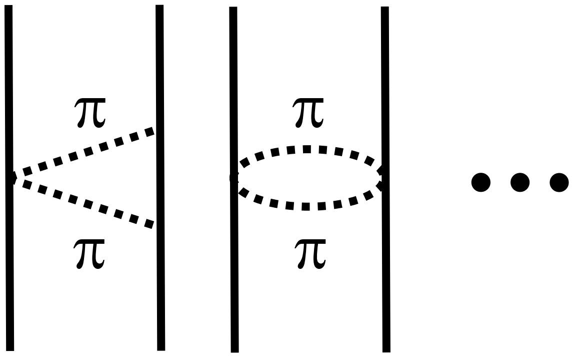
Accessible by
Lattice QCD
early
universe
100
10
1
T [MeV]
quark-gluon
plasma
heavy-ion
collisions
Accessible by
Lattice EFT
gas of light
nuclei
nuclear
liquid
excited
nuclei
superfluid
neutron star crust
neutron star core
10^{-3}
10^{-2}
10^{-1}
1
ρ [fm^{-3}]
ρ_N

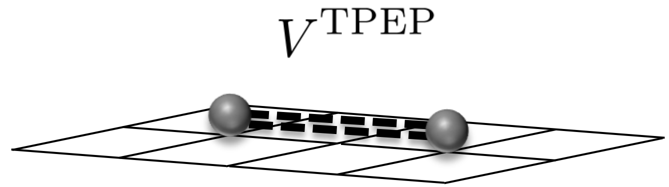
Lattice interactions

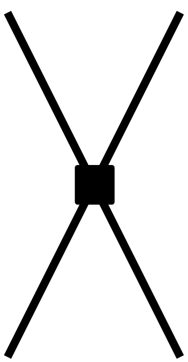
Leading order on the lattice



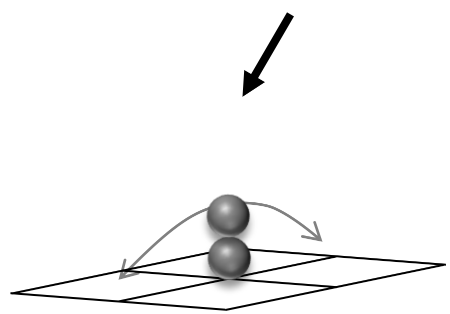
Next-to-leading order on the lattice

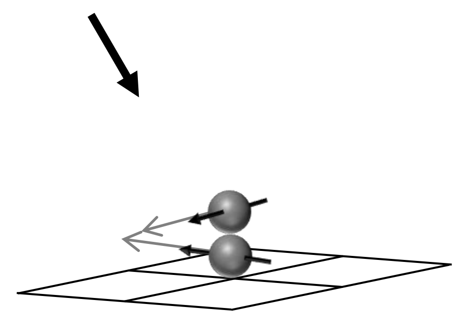




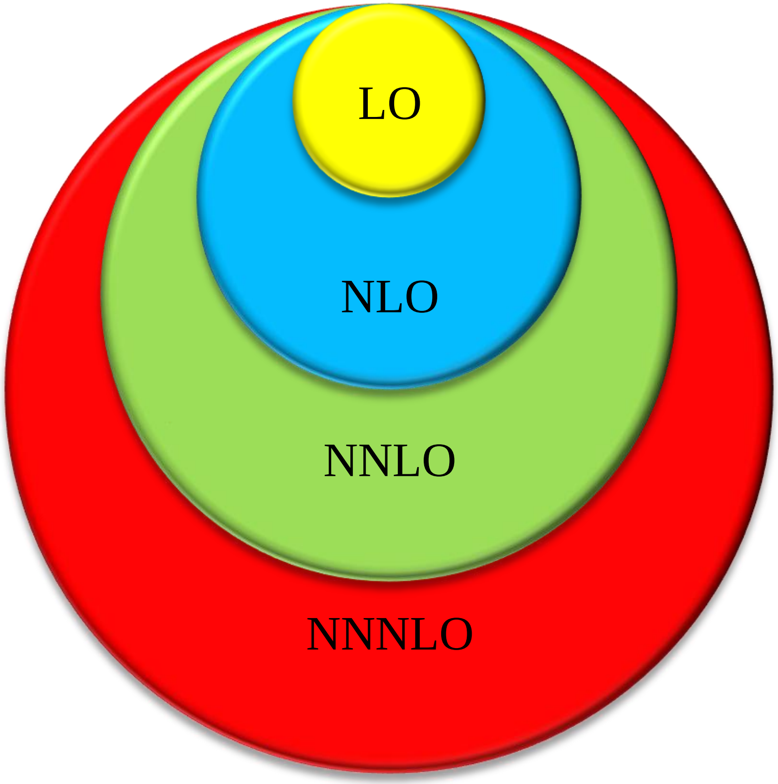


$\vec{\nabla}_1 \cdot \vec{\nabla}_2 \qquad (\vec{\sigma}_1 \cdot \vec{\nabla}_1)\,(\vec{\sigma}_2 \cdot \vec{\nabla}_2) \qquad \cdots$



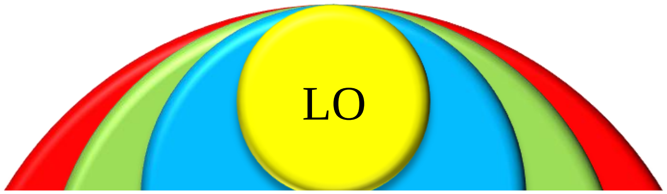


Computational strategy



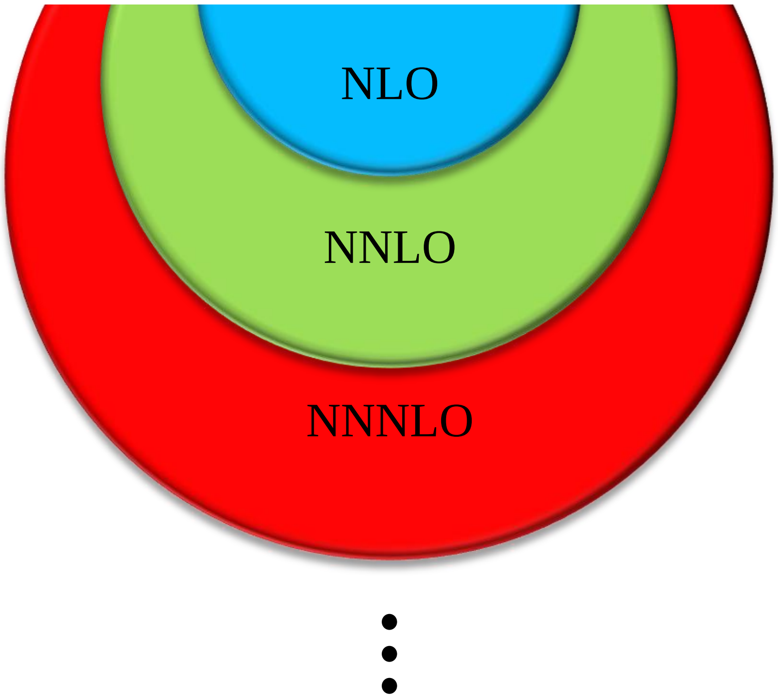
⋮

Non-perturbative – Monte Carlo



"Improved LO"

Perturbative corrections



LO_1: Pure contact interactions

$$\mathcal{A}(V_{\mathrm{LO}_1}) = C + C_I \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \mathcal{A}(V^{\mathrm{OPEP}})$$

LO_2: Gaussian smearing

$$\mathcal{A}(V_{\mathrm{LO}_2}) = C f(\vec{q}^{\,2}) + C_I f(\vec{q}^{\,2}) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \mathcal{A}(V^{\mathrm{OPEP}})$$

LO_3: Gaussian smearing only in even partial waves

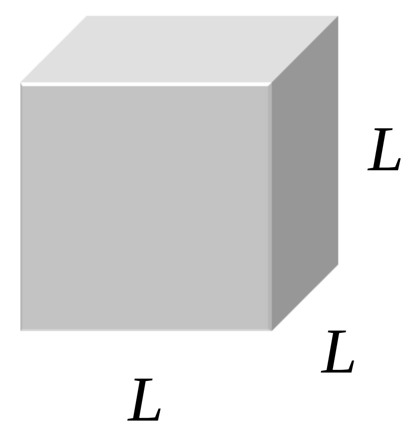
$$\begin{aligned}\mathcal{A}(V_{\mathrm{LO}_3}) = C_{S=0,I=1} f(\vec{q}^{\,2}) \left(\tfrac{1}{4} - \tfrac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \left(\tfrac{3}{4} + \tfrac{1}{4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \\ + C_{S=1,I=0} f(\vec{q}^{\,2}) \left(\tfrac{3}{4} + \tfrac{1}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \left(\tfrac{1}{4} - \tfrac{1}{4}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \\ + \mathcal{A}(V^{\mathrm{OPEP}})\end{aligned}$$

Physical scattering data → Unknown operator coefficients

Lüscher's finite-volume formula

Lüscher, Comm. Math. Phys. 105 (1986) 153; NPB 354 (1991) 531

Two-particle energy levels near threshold in a periodic cube related to phase shifts



$$p \cot \delta_0(p) = \frac{1}{\pi L} S(\eta), \qquad \eta = \left(\frac{Lp}{2\pi}\right)^2$$

$$S(\eta) = \lim_{\Lambda \to \infty} \left[\sum_{\vec{n}} \frac{\theta(\Lambda^2 - \vec{n}^2)}{\vec{n}^2 - \eta} - 4\pi\Lambda \right]$$

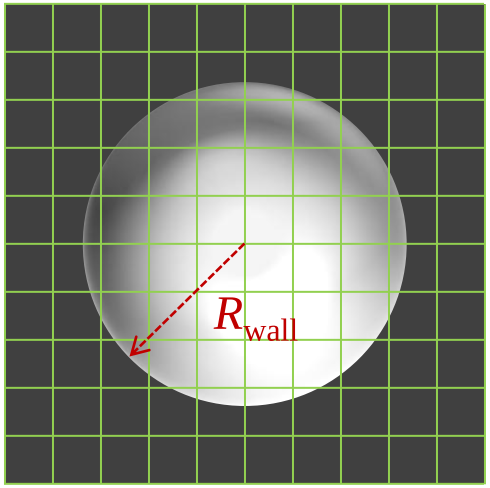
Not so useful for higher total spin and partial wave mixing

Physical scattering data → Unknown operator coefficients

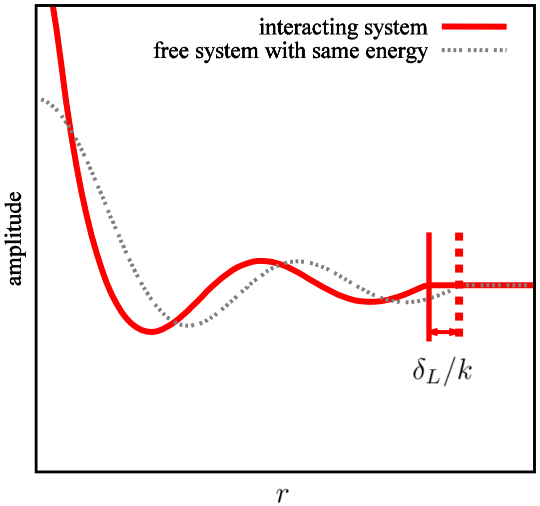
Spherical wall method

Borasoy, Epelbaum, Krebs, D.L., Meißner, EPJA 34 (2007) 185

Spherical wall imposed in the center of mass frame

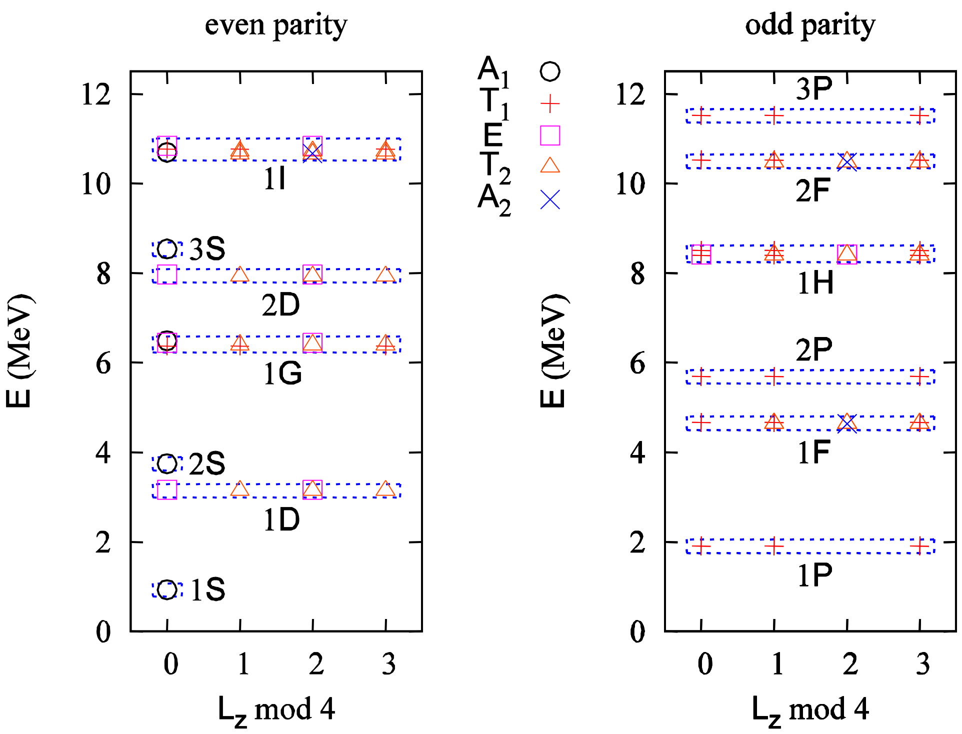


Representation	J_z	Example
A_1	$0 \bmod 4$	$Y_{0,0}$
T_1	$0, 1, 3 \bmod 4$	$\{Y_{1,0}, Y_{1,1}, Y_{1,-1}\}$
E	$0, 2 \bmod 4$	$\left\{Y_{2,0}, \frac{Y_{2,-2}+Y_{2,2}}{\sqrt{2}}\right\}$
T_2	$1, 2, 3 \bmod 4$	$\left\{Y_{2,1}, \frac{Y_{2,-2}-Y_{2,2}}{\sqrt{2}}, Y_{2,-1}\right\}$
A_2	$2 \bmod 4$	$\frac{Y_{3,2}-Y_{3,-2}}{\sqrt{2}}$



Energy levels with hard spherical wall

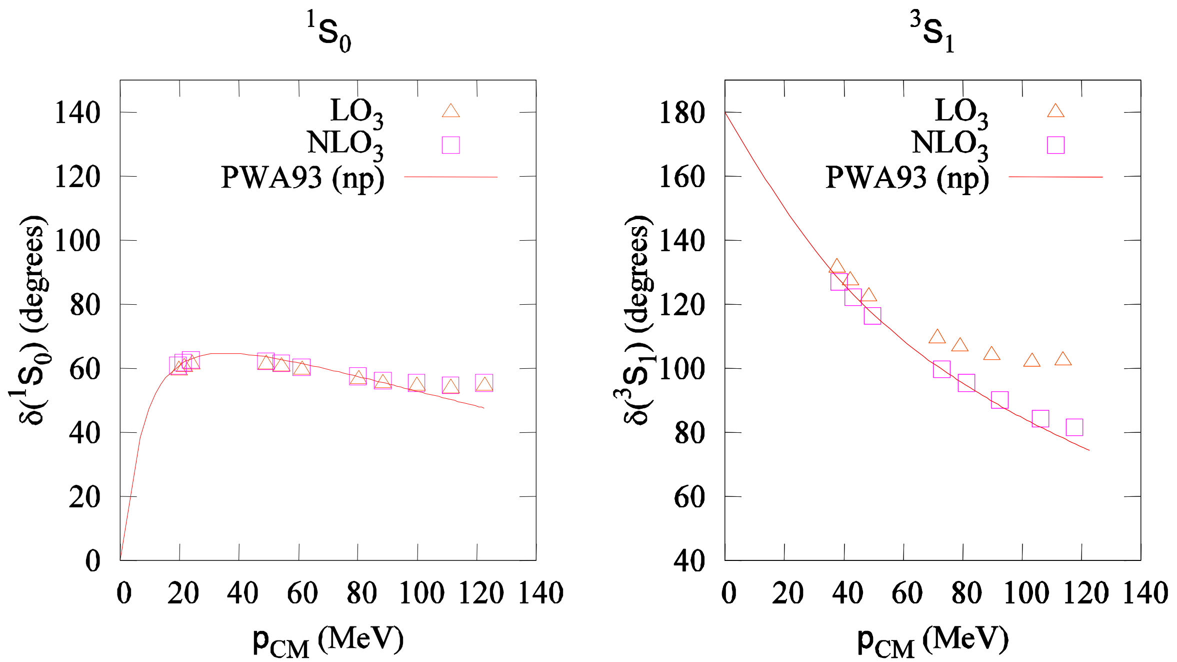
$$R_{\text{wall}} = 10a$$
$$a = 1.97 \text{ fm}$$



Energy shift from free-particle values gives the phase shift

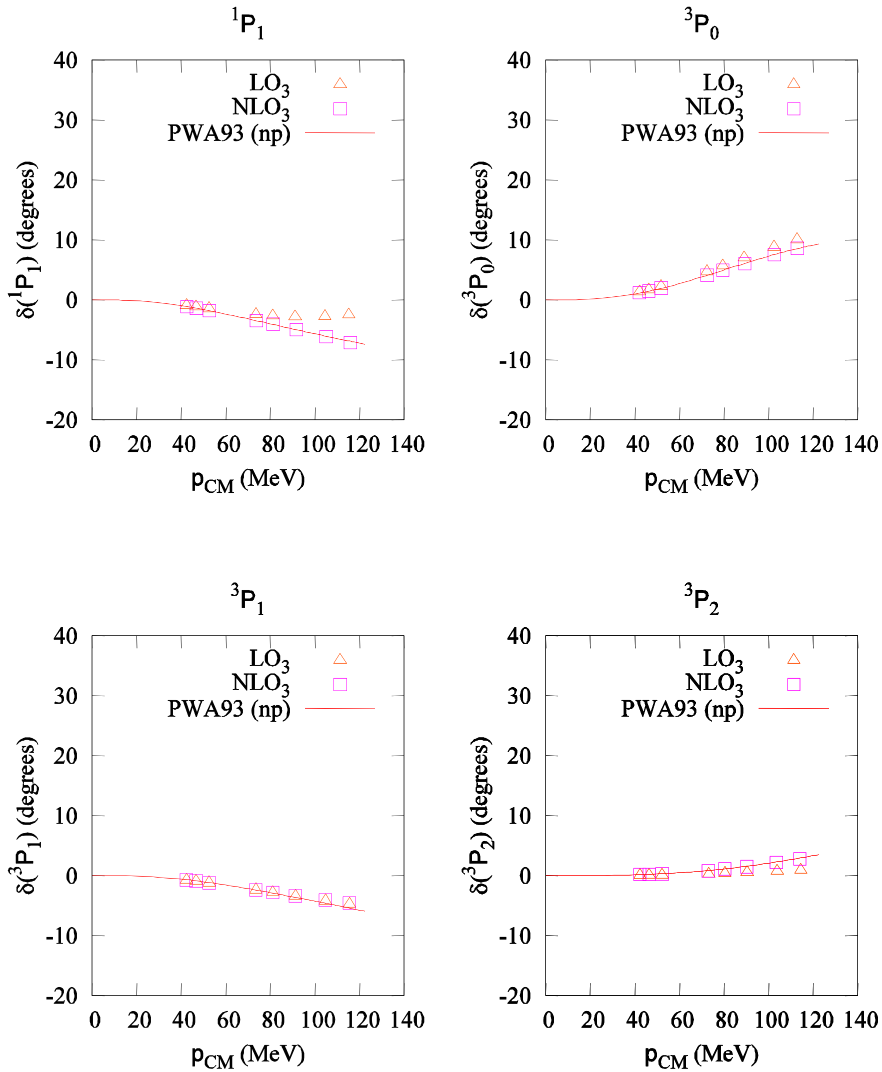
Nucleon-nucleon phase shifts

LO_3: S waves $\quad a = 1.97$ fm



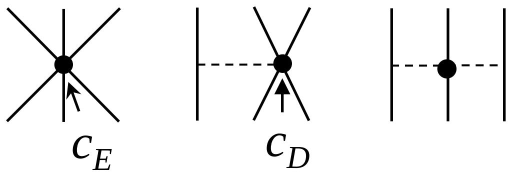
LO_3: P waves

$a = 1.97$ fm

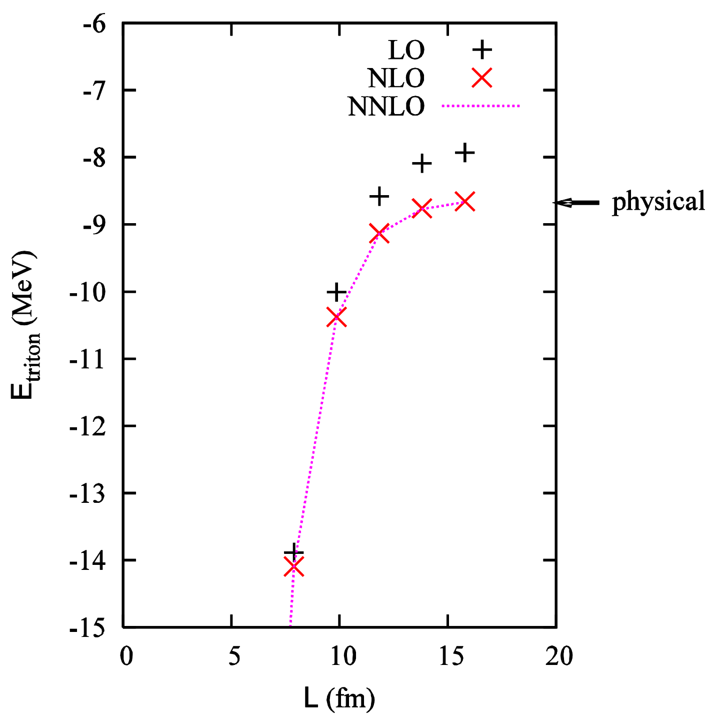


Three-nucleon forces

Two unknown coefficients at NNLO from three-nucleon forces.
Determine c_D and c_E using ^{3}H and ^{4}He binding energies



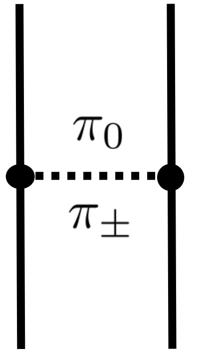
^{3}H binding energy



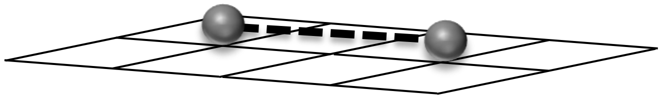
Isospin breaking and Coulomb interaction

Isospin-breaking and power counting *[Friar , van Kolck, PRC 60 (1999) 034006; Walzl, Meißner, Epelbaum NPA 693 (2001) 663; Friar, van Kolck, Payne, Coon, PRC 68 (2003) 024003; Epelbaum, Meißner, PRC72 (2005) 044001…]*

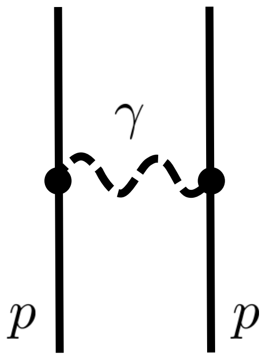
<u>Pion mass difference</u>

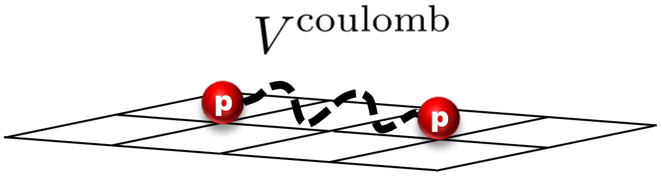


$m_{\pi_0} \neq m_{\pi_\pm}$

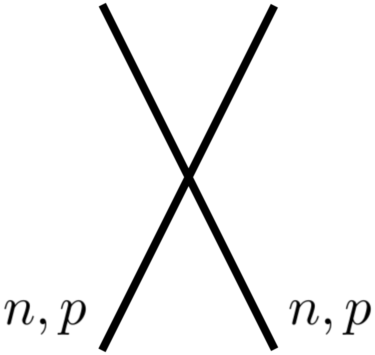


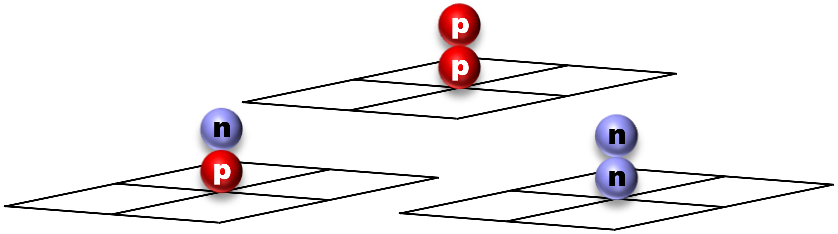
Coulomb potential



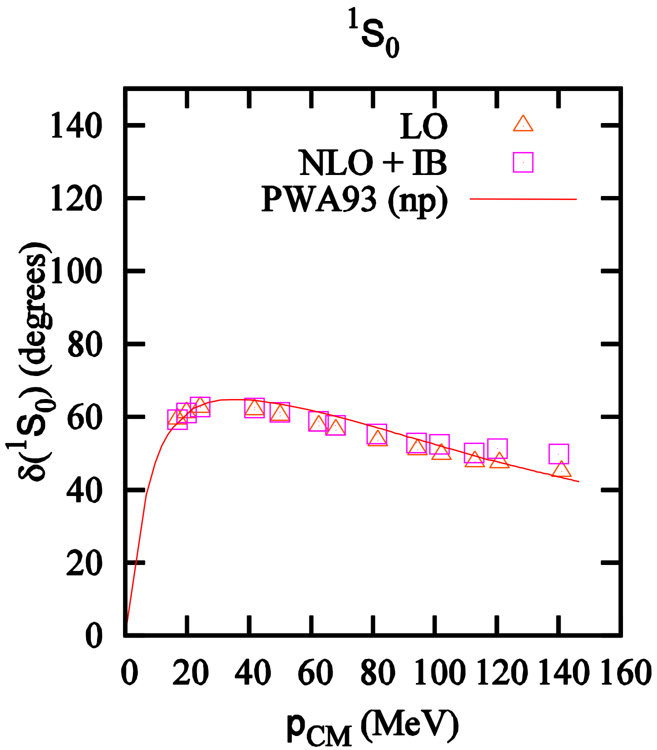


Charge symmetry breaking
Charge independence breaking

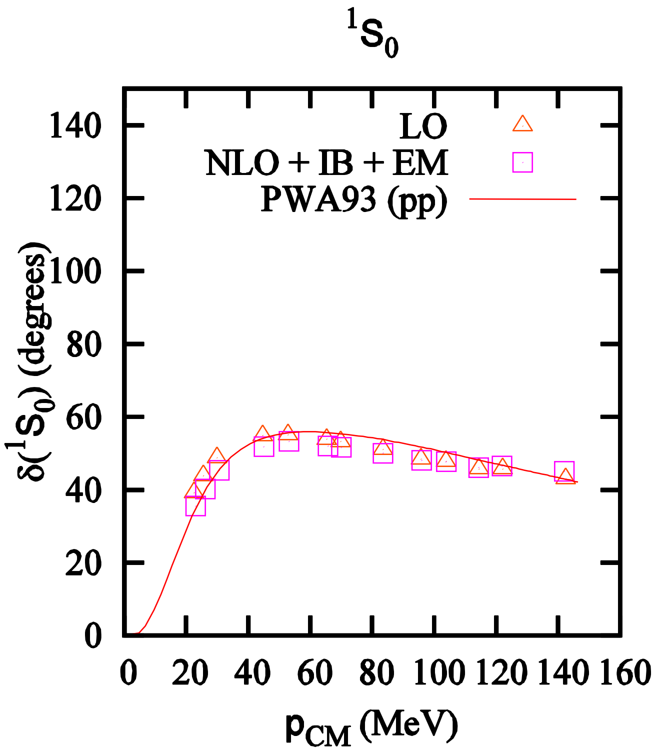




Neutron-proton



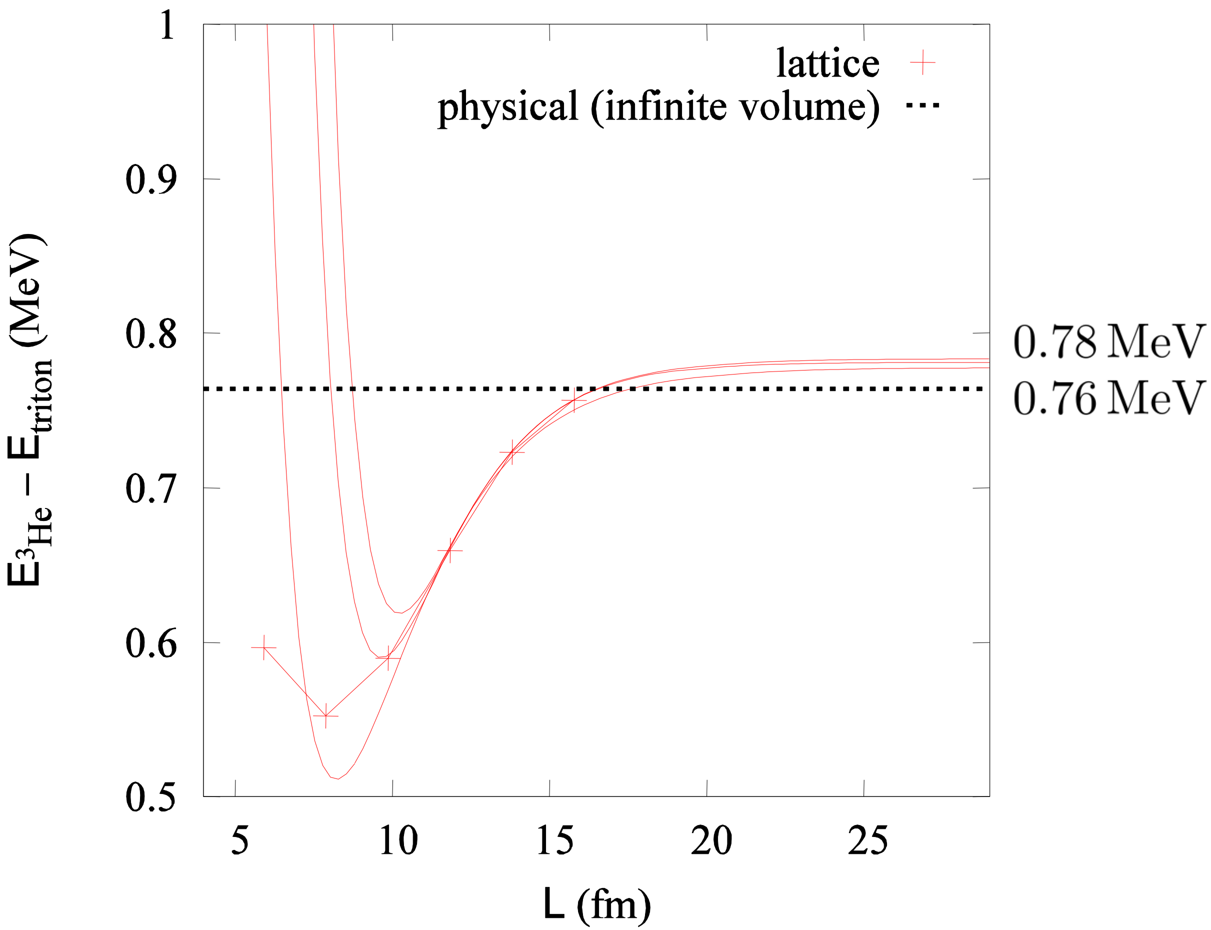
Proton-proton



Epelbaum, Krebs, D.L, Meißner, PRL 104:142501, 2010
Epelbaum, Krebs, D.L, Meißner, EPJA 45 (2010) 335

Triton and Helium-3

$$E_{^3\mathrm{He}} - E_{\mathrm{triton}} = 0.78(5)\ \mathrm{MeV}$$



Euclidean time projection

Let H be the Hamiltonian for a quantum system. We don't know the energies and energy eigenstates, but label them as

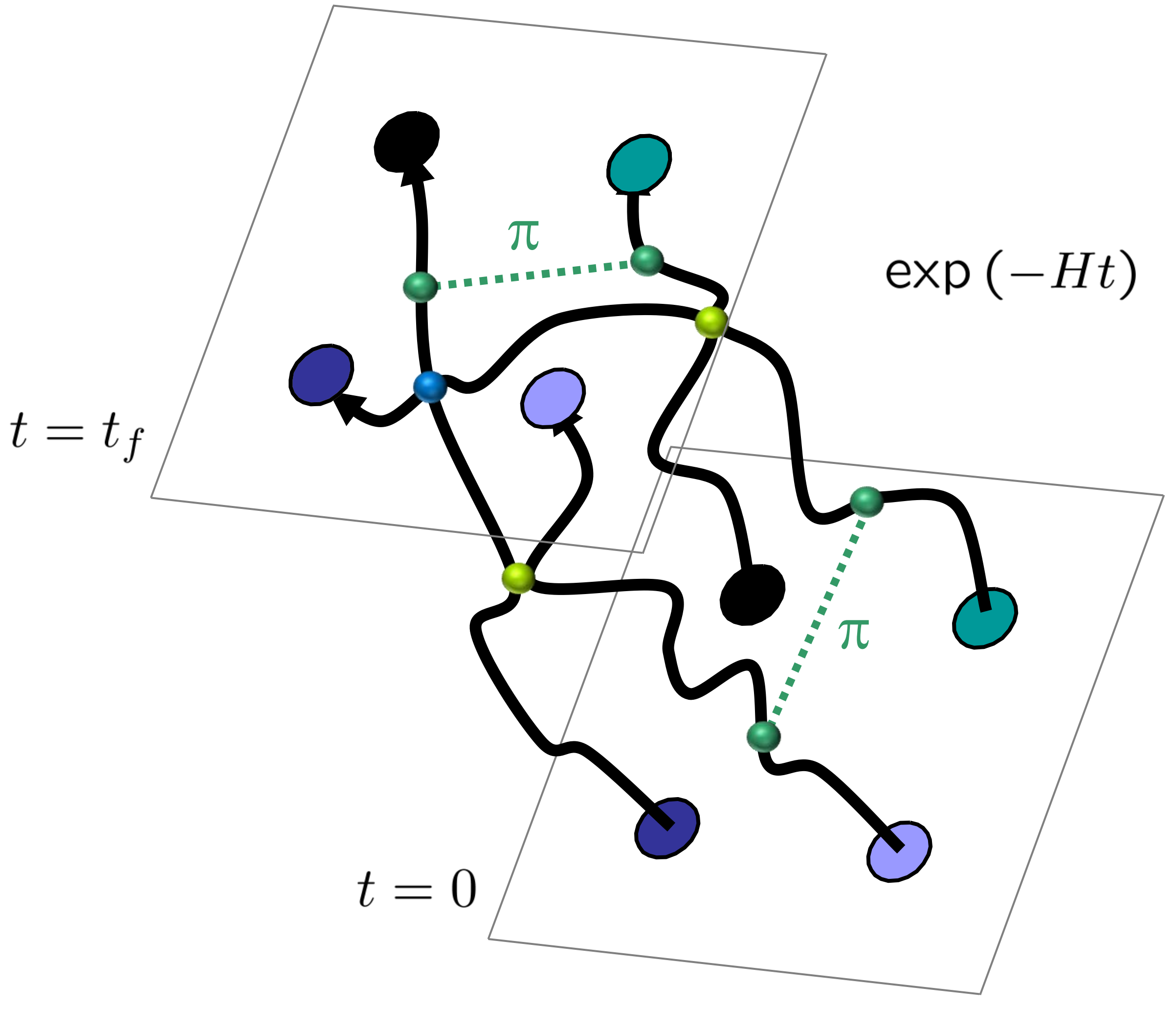
$$H\left|\psi_n\right\rangle = E_n\left|\psi_n\right\rangle$$
$$E_0 < E_1 \leq E_2 \leq \cdots$$

Convenient to work with exponentials of the Hamiltonian

$$\exp(-Ht)\left|\psi_n\right\rangle = \exp(-E_n t)\left|\psi_n\right\rangle$$

If initial state overlap is nonzero, then the ground state dominates as Euclidean time goes to infinity

$$\exp(-Ht)\left|\phi\right\rangle \to \exp(-E_0 t)\left|\psi_0\right\rangle\left\langle\psi_0\right|\phi\rangle$$

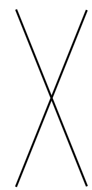


$t = t_f$
π
$\exp(-Ht)$
π
$t = 0$

Auxiliary fields

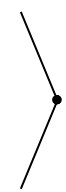
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^{\dagger}N)^{2}\right]$$



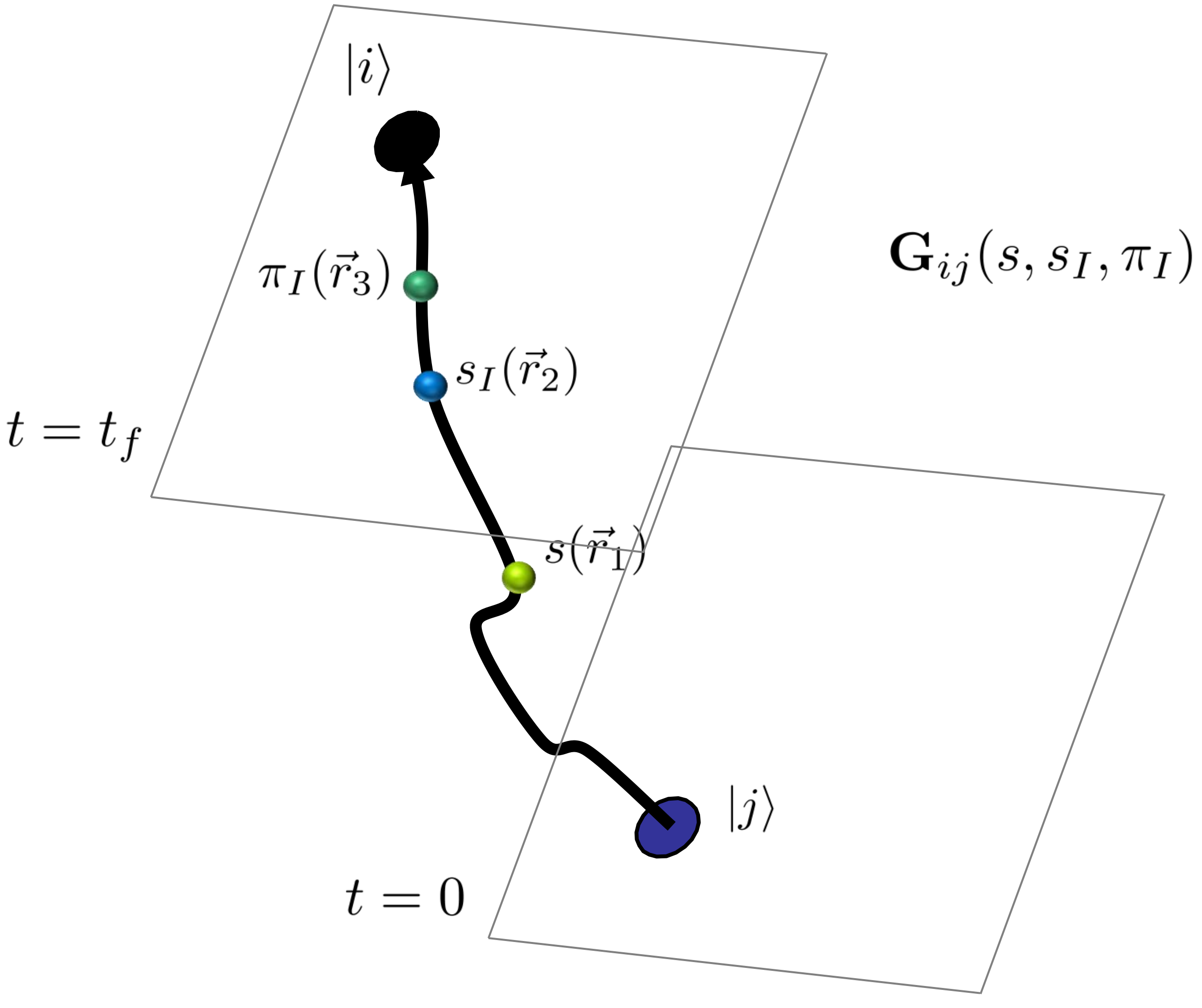
$(N^{\dagger}N)^{2}$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds\,\exp\left[-\frac{1}{2}s^{2} + \sqrt{-C}\,s(N^{\dagger}N)\right]$$



$sN^{\dagger}N$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$t = t_f$
$s(\vec{r}_1)$
$|j\rangle$
$t = 0$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$

Take any initial state with the desired quantum numbers and which is an antisymmetric product of *A* single nucleon states (i.e., a Slater determinant)

$$|\psi_{\text{init}}\rangle = |1\rangle \wedge |2\rangle \wedge \cdots \wedge |A\rangle$$

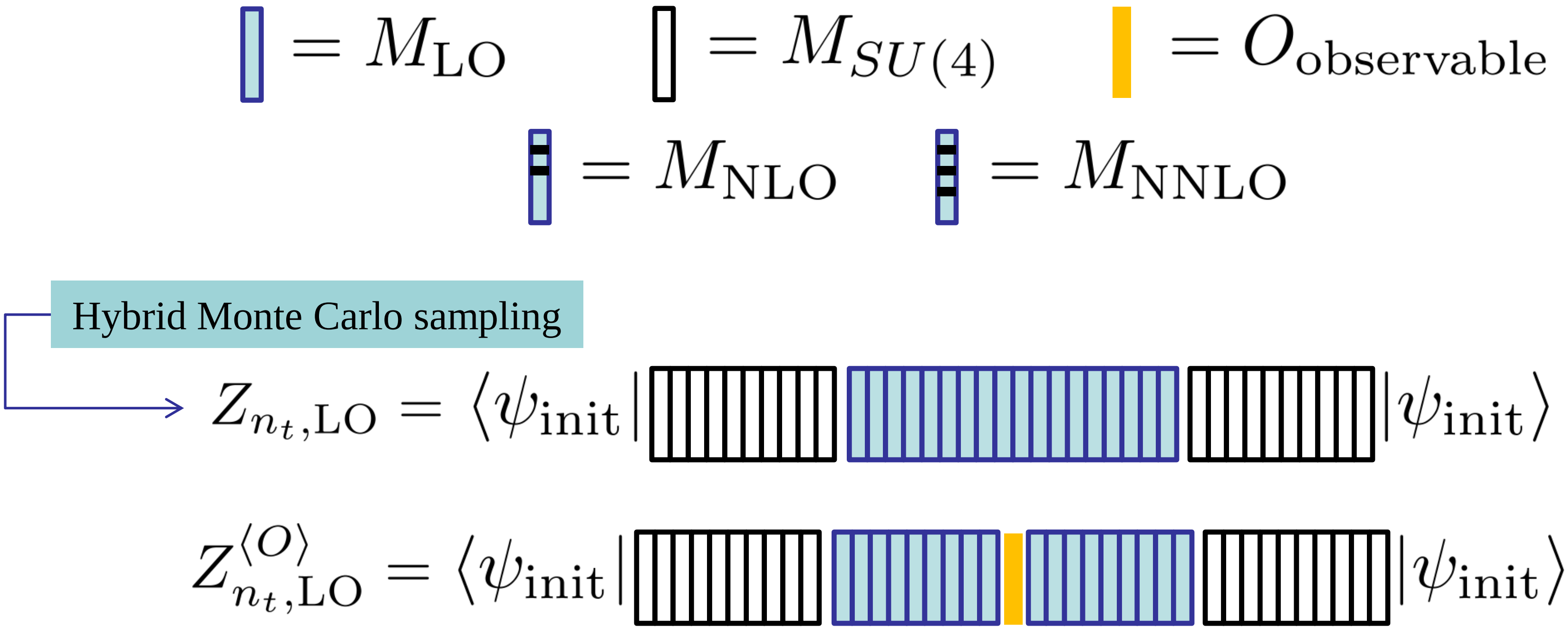
For any configuration of the auxiliary and pion fields,

$$\langle\psi_{\text{init}}| \exp\left[-H(s, s_I, \pi_I)t\right] |\psi_{\text{init}}\rangle = \det \mathbf{G}(s, s_I, \pi_I)$$

$$\mathbf{G}_{ij}(s, s_I, \pi_I) = \langle i| \exp\left[-H(s, s_I, \pi_I)t\right] |j\rangle$$

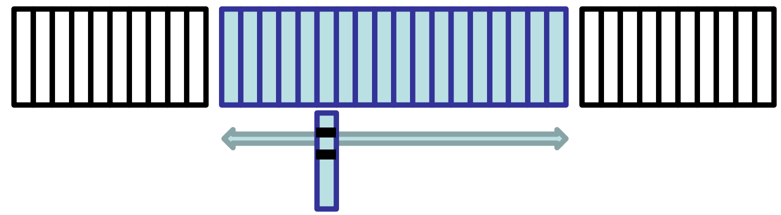
For *A* nucleons, the matrix is *A* by *A*. We use Monte Carlo to integrate over all possible configurations of the auxiliary and pion fields.

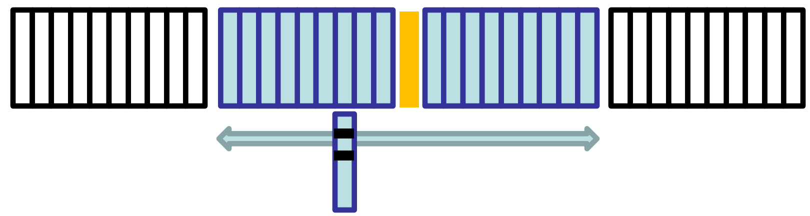
Schematic of calculations



$$e^{-E_{0,\text{LO}} a_t} = \lim_{n_t \to \infty} Z_{n_t+1,\text{LO}} / Z_{n_t,\text{LO}}$$

$$\langle O \rangle_{0,\text{LO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\text{LO}} / Z_{n_t,\text{LO}}$$

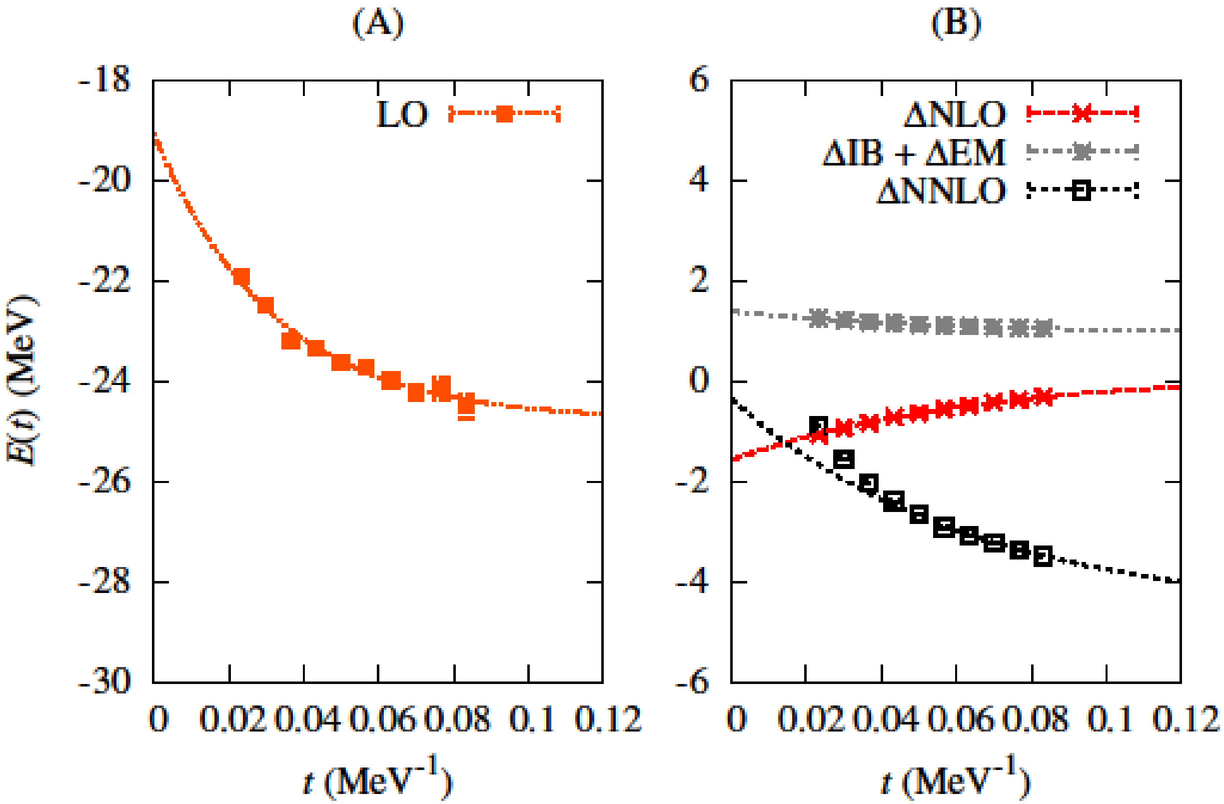
$Z_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} = \langle \psi_{\mathrm{init}} |$  $| \psi_{\mathrm{init}} \rangle$

$$\langle O \rangle_{0,\mathrm{NLO}} = \lim_{n_t \to \infty} Z^{\langle O \rangle}_{n_t,\mathrm{NLO}} / Z_{n_t,\mathrm{NLO}}$$

Ground state of Helium-4

$$L = 9.9\,\text{fm}$$



Epelbaum, Krebs, D.L, Meißner, PRL 104:142501, 2010;
EPJA 45 (2010) 335; arXiv:1101.2547

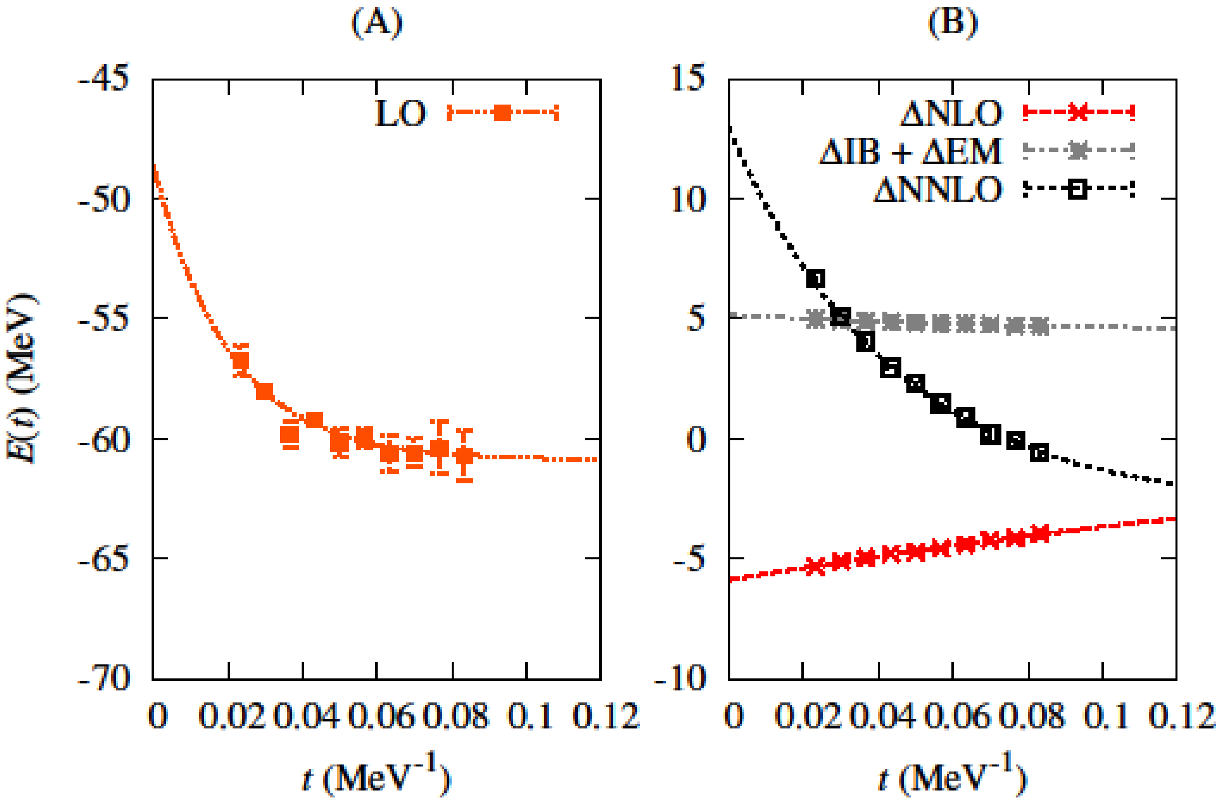
Ground state of Helium-4

$$L = 9.9\,\mathrm{fm}$$

LO ($O(Q^0)$)	–24.8(2) MeV
NLO ($O(Q^2)$)	–24.7(2) MeV
NLO + IB + EM ($O(Q^2)$)	–23.8(2) MeV
NNLO ($O(Q^3)$)	–28.4(3) MeV
Experiment	–28.3 MeV

Ground state of Beryllium-8

$L = 11.8\,\mathrm{fm}$



Epelbaum, Krebs, D.L, Meißner, arXiv:1101.2547

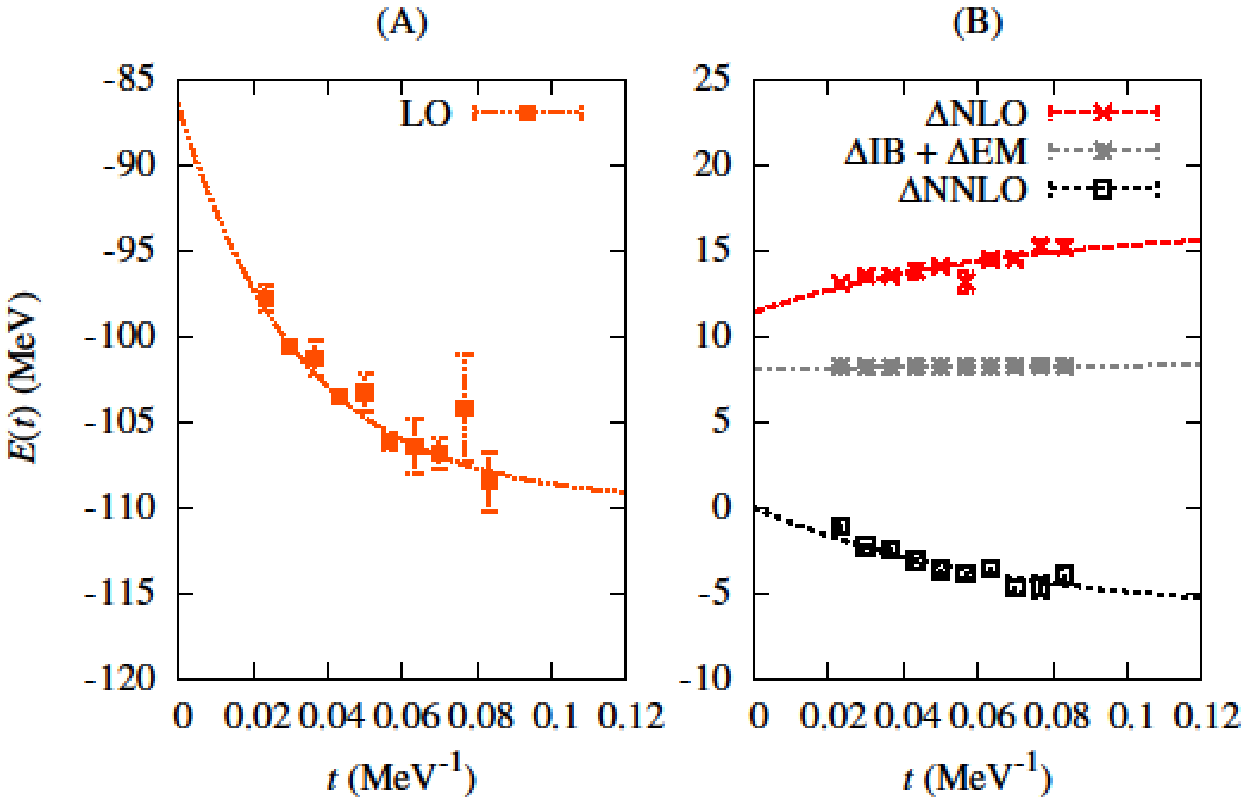
Ground state of Beryllium-8

$$L = 11.8\,\mathrm{fm}$$

LO ($O(Q^0)$)	–60.9(7) MeV
NLO ($O(Q^2)$)	–60(2) MeV
NLO + IB + EM ($O(Q^2)$)	–55(2) MeV
NNLO ($O(Q^3)$)	–58(2) MeV
Experiment	–56.5 MeV

Ground state of Carbon-12

$L = 11.8\,\mathrm{fm}$



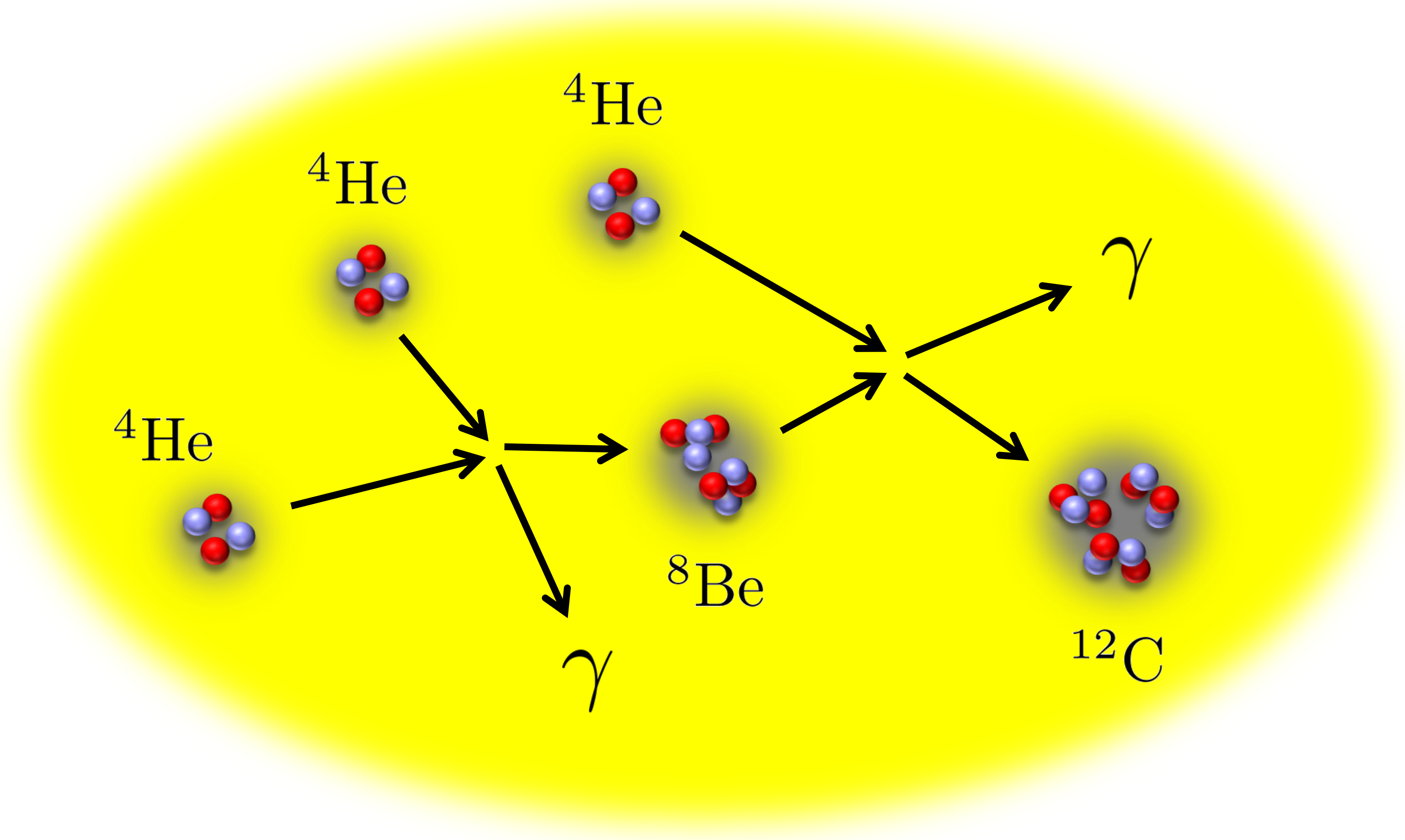
Epelbaum, Krebs, D.L, Meißner, PRL 104:142501, 2010;
EPJA 45 (2010) 335; arXiv:1101.2547

Ground state of Carbon-12

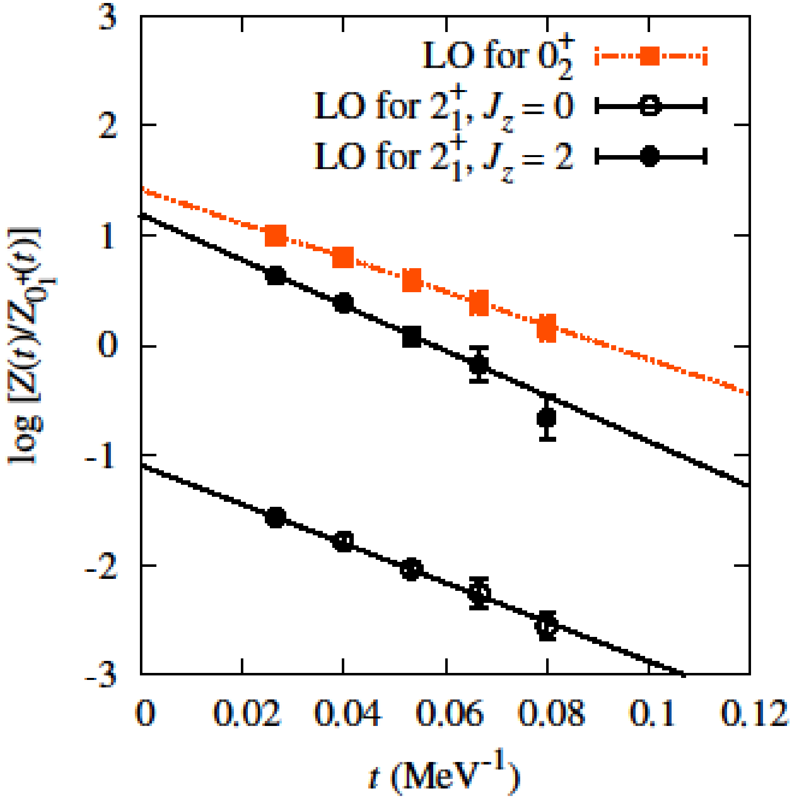
$$L = 11.8\,\mathrm{fm}$$

LO ($O(Q^0)$)	–110(2) MeV
NLO ($O(Q^2)$)	–93(3) MeV
NLO + IB + EM ($O(Q^2)$)	–85(3) MeV
NNLO ($O(Q^3)$)	–91(3) MeV
Experiment	–92.2 MeV

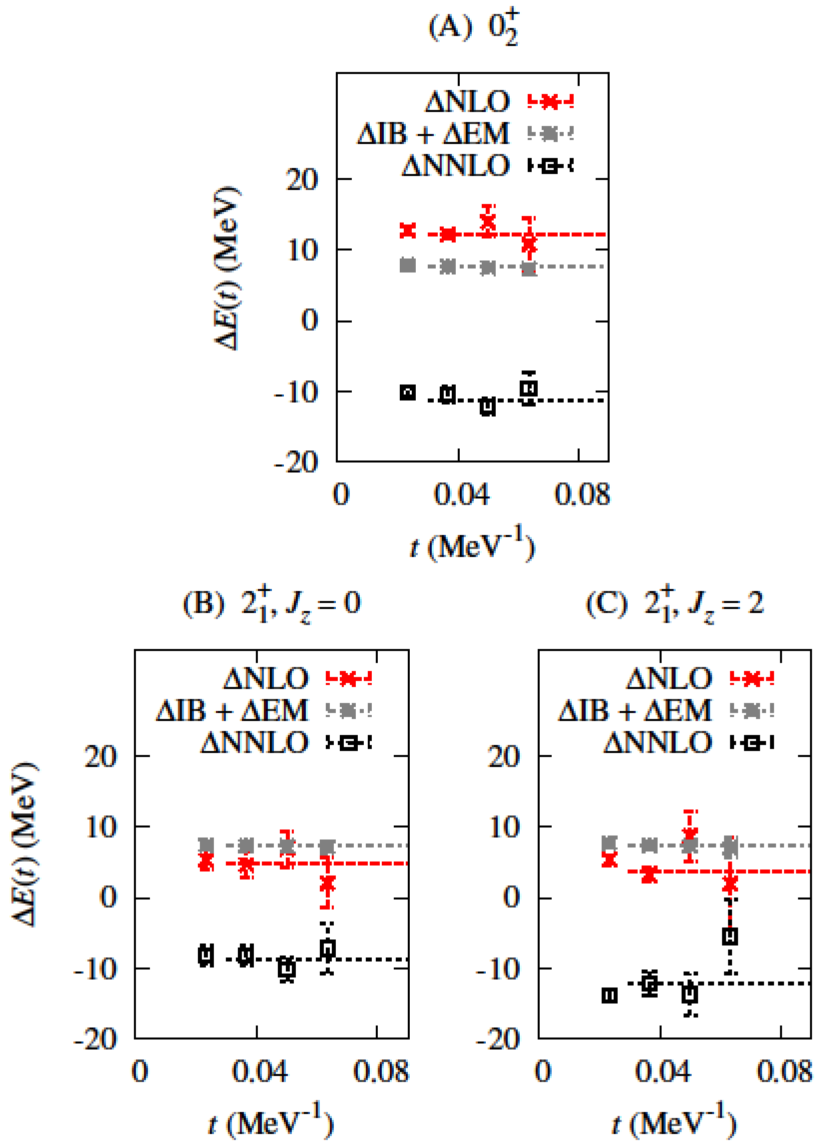
Carbon-12 spectrum and the Hoyle state



Excited state energy gaps



Higher-order corrections

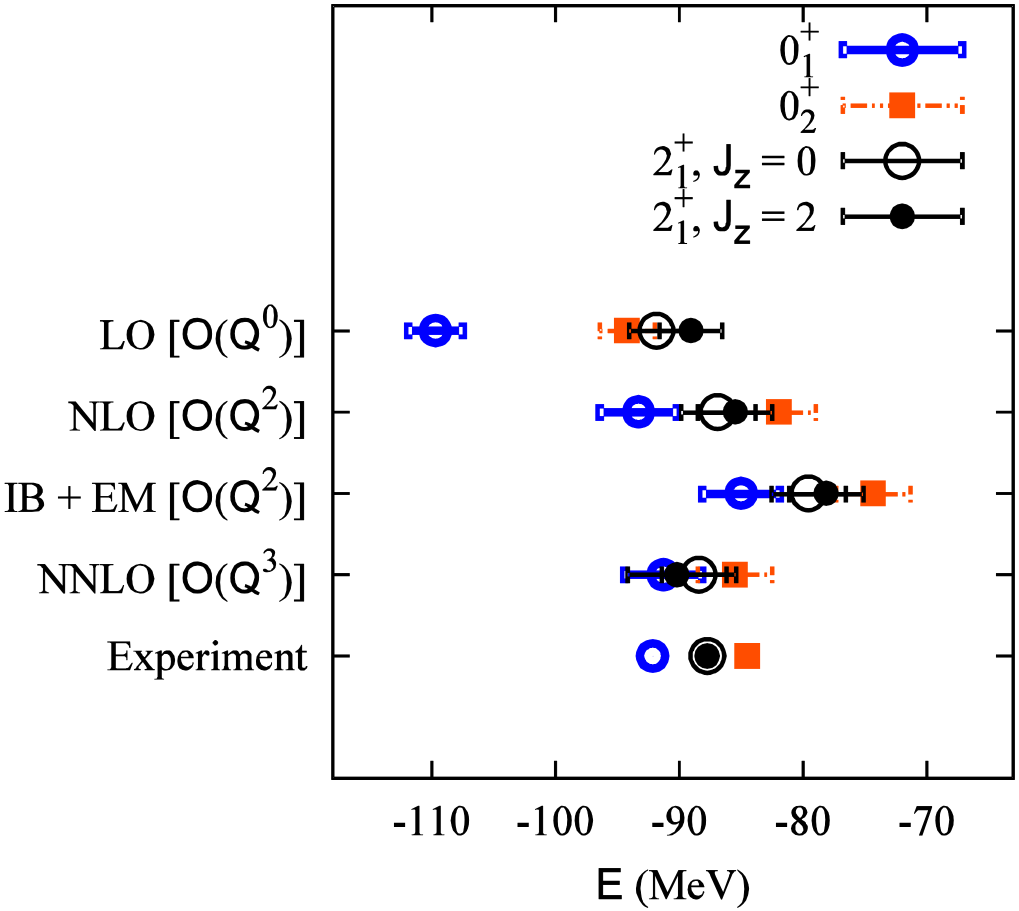


Excited state spectrum of carbon-12

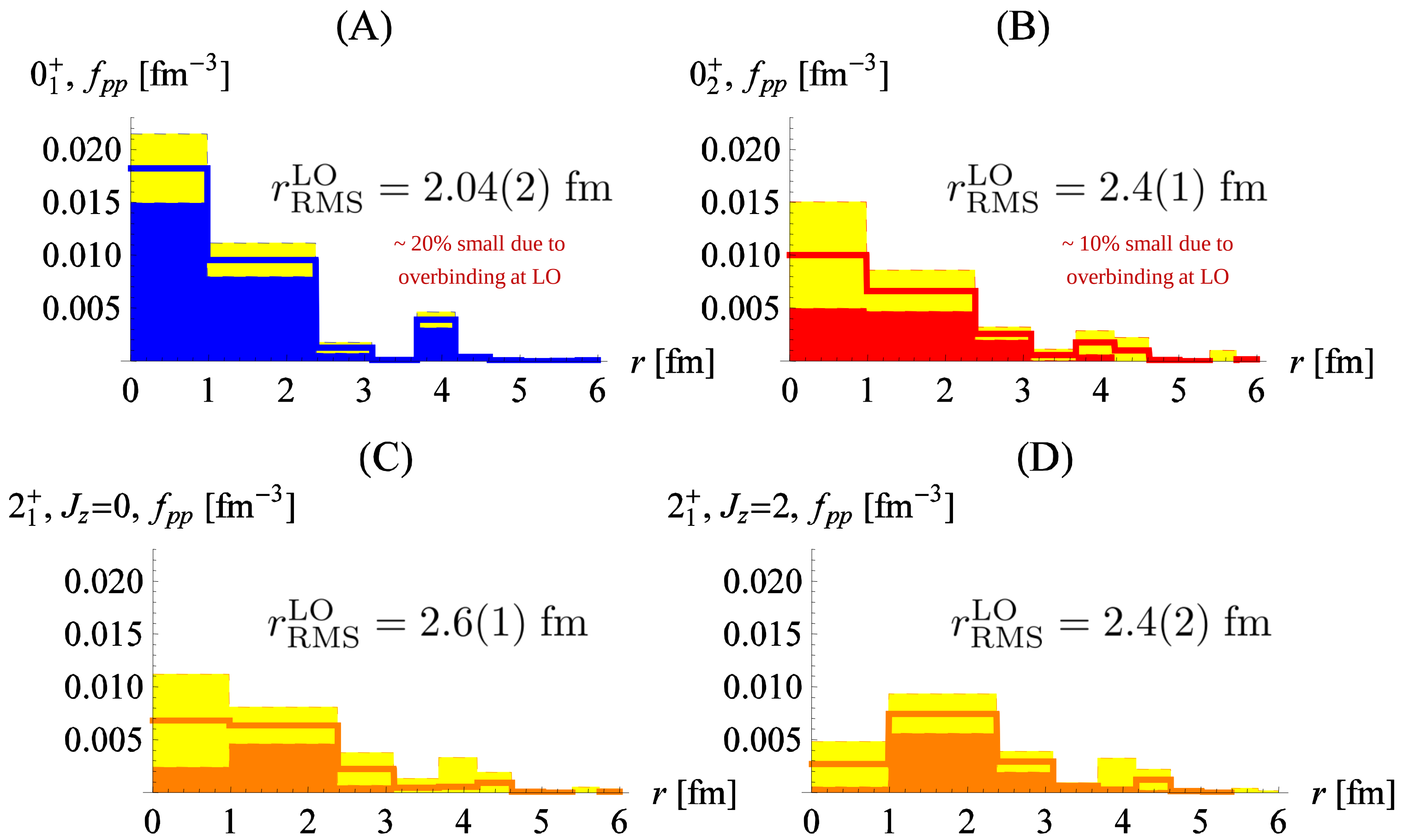
	0_2^+	$2_1^+,\ J_z = 0$	$2_1^+,\ J_z = 2$
LO ($O(Q^0)$)	–94(2) MeV	–92(2) MeV	–89(2) MeV
NLO ($O(Q^2)$)	–82(3) MeV	–87(3) MeV	–85(2) MeV
NLO + IB + EM ($O(Q^2)$)	–74(3) MeV	–80(3) MeV	–78(3) MeV
NNLO ($O(Q^3)$)	–85(3) MeV	–88(3) MeV	–90(4) MeV
Experiment	–84.51 MeV	–87.72 MeV	

First *ab initio* calculation of the Hoyle state

Epelbaum, Krebs, D.L, Meißner, arXiv:1101.2547



Proton-proton radial distribution function at leading order



Summary and future directions

Lattice effective field theory is a relatively new and promising tool that combines the framework of effective field theory and computational lattice methods.

Many topics to explore…

Electromagnetic transitions for carbon-12; spectrum of beryllium-8; alpha clustering in nuclei; three-dimensional profile of the Hoyle state; nitrogen-14; oxygen-16; beryllium-10; transition from S-wave to P-wave pairing in superfluid neutron matter; compressibility of nuclei, weak matrix elements; configurations for general public use; etc.

Extra slides

Initial state for carbon-12

For the ground state calculation of carbon-12 the initial state is a Slater determinant of 12 single-nucleon standing waves in the cubic periodic box.

$$a_{0,0} = a_{\uparrow,p}, a_{0,1} = a_{\uparrow,n}$$

$$a_{1,0} = a_{\downarrow,p}, a_{1,1} = a_{\downarrow,n}$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_1\rangle \propto \delta_{i,0}\delta_{j,0}, \quad \langle 0| a_{i,j}(\vec{n}) |\psi_2\rangle \propto \delta_{i,0}\delta_{j,1},$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_3\rangle \propto \delta_{i,1}\delta_{j,0}, \quad \langle 0| a_{i,j}(\vec{n}) |\psi_4\rangle \propto \delta_{i,1}\delta_{j,1}$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_5\rangle \propto \delta_{i,0}\delta_{j,0} \cos(\tfrac{2n_z\pi}{L}), \quad \langle 0| a_{i,j}(\vec{n}) |\psi_6\rangle \propto \delta_{i,0}\delta_{j,1} \cos(\tfrac{2n_z\pi}{L})$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_7\rangle \propto \delta_{i,1}\delta_{j,0} \cos(\tfrac{2n_z\pi}{L}), \quad \langle 0| a_{i,j}(\vec{n}) |\psi_8\rangle \propto \delta_{i,1}\delta_{j,1} \cos(\tfrac{2n_z\pi}{L})$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_9\rangle \propto \delta_{i,0}\delta_{j,0} \sin(\tfrac{2n_z\pi}{L}), \quad \langle 0| a_{i,j}(\vec{n}) |\psi_{10}\rangle \propto \delta_{i,0}\delta_{j,1} \sin(\tfrac{2n_z\pi}{L})$$

$$\langle 0| a_{i,j}(\vec{n}) |\psi_{11}\rangle \propto \delta_{i,1}\delta_{j,0} \sin(\tfrac{2n_z\pi}{L}), \quad \langle 0| a_{i,j}(\vec{n}) |\psi_{12}\rangle \propto \delta_{i,1}\delta_{j,1} \sin(\tfrac{2n_z\pi}{L})$$

To measure the lowest spin-2 state and Hoyle state (first excited spin-0) we also consider 12 other single-nucleon standing waves

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_1\rangle \propto \delta_{i,0}\delta_{j,0}\cos(\tfrac{2n_x\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_2\rangle \propto \delta_{i,0}\delta_{j,1}\cos(\tfrac{2n_x\pi}{L})$$

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_3\rangle \propto \delta_{i,0}\delta_{j,0}\sin(\tfrac{2n_x\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_4\rangle \propto \delta_{i,0}\delta_{j,1}\sin(\tfrac{2n_x\pi}{L})$$

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_5\rangle \propto \delta_{i,0}\delta_{j,0}\cos(\tfrac{2n_y\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_6\rangle \propto \delta_{i,0}\delta_{j,1}\cos(\tfrac{2n_y\pi}{L})$$

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_7\rangle \propto \delta_{i,0}\delta_{j,0}\sin(\tfrac{2n_y\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_8\rangle \propto \delta_{i,0}\delta_{j,1}\sin(\tfrac{2n_y\pi}{L})$$

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_9\rangle \propto \delta_{i,0}\delta_{j,0}\cos(\tfrac{4n_z\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_{10}\rangle \propto \delta_{i,0}\delta_{j,1}\cos(\tfrac{4n_z\pi}{L})$$

$$\langle 0|\, a_{i,j}(\vec{n})\, |\phi_{11}\rangle \propto \delta_{i,0}\delta_{j,0}\sin(\tfrac{4n_z\pi}{L}), \quad \langle 0|\, a_{i,j}(\vec{n})\, |\phi_{12}\rangle \propto \delta_{i,0}\delta_{j,1}\sin(\tfrac{4n_z\pi}{L})$$

From these 24 single-nucleon standing waves we construct 7 initial states.

$$|\Psi_1\rangle \propto \bigwedge_{k=1,2,\cdots,12} |\psi_k\rangle$$

$$|\Psi_2\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_1\rangle \wedge |\phi_2\rangle \qquad |\Psi_3\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_3\rangle \wedge |\phi_4\rangle$$

$$|\Psi_4\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_5\rangle \wedge |\phi_6\rangle \qquad |\Psi_5\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_7\rangle \wedge |\phi_8\rangle$$

$$|\Psi_6\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_9\rangle \wedge |\phi_{10}\rangle \qquad |\Psi_7\rangle \propto \bigwedge_{k=3,4,\cdots,12} |\psi_k\rangle \wedge |\phi_{11}\rangle \wedge |\phi_{12}\rangle$$

From these 7 initial states we make 4 linear combinations with total momentum zero and even parity. Three of these have $J_z = 0$ and one has $J_z = 2$.

Relative contribution of omitted operators