Coupled channel approach to pion-nucleon scattering

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Basics of πN scattering

- Pion-nucleon scattering has a long history in strong interaction physics
- Early attempts to constrain πN scattering amplitude from first principles:

causality $\rightarrow$ analyticity $\rightarrow$ dispersion relations

conservation of probability $\rightarrow$ unitarity

locality, analyticity, Lorentz invariance $\rightarrow$ crossing symmetry $\rightarrow$ relation between scattering data and off-shell matrix elements

Basics of πN scattering

- “Partially conserved axial current” $\longrightarrow$ current algebra $\longrightarrow$ Weinberg’s prediction for pion-nucleon scattering lengths

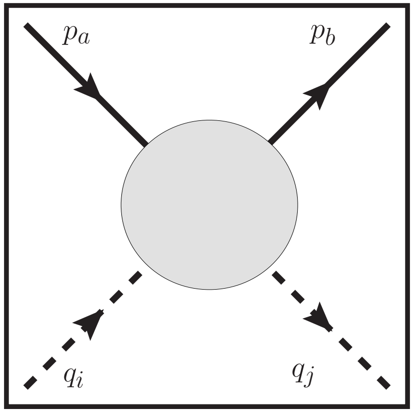
$$a_{I=1/2} = -2a_{I=3/2} = \frac{M_\pi}{4\pi F_\pi^2}$$

Weinberg (1966) ; Tomozawa (1966)

- Extension of current algebra to Chiral Perturbation Theory $\longrightarrow$ Description of pion-nucleon scattering near the reaction threshold

Gasser, Sainio, Svarc (1988); Fettes, Meißner, Steininger (1998); Becher, Leutwyler (2001); Mai, Bruns, Kubis, Meißner (2009) ...

Basics of πN scattering



- Incoming (outgoing) nucleon: $p_a \quad (p_b)$
  Incoming (outgoing) pion: $q_i \quad (q_j)$
- Mandelstam variables:

$$
\begin{aligned}
s &= (p_a + q_i)^2 = (p_b + q_j)^2 \\
t &= (p_b - p_a)^2 = (q_j - q_i)^2
\end{aligned}
$$

- On-shell scattering operator can be parametrized by two scalar functions A and B :

$$
T_{\text{on}} = A(s,t) + \frac{1}{2}(\not{q}_j + \not{q}_i) B(s,t)\,.
$$

- In general, A and B are matrices in the space of particle channels.

Basics of πN scattering

- We will concentrate mainly on the s-waves, so we have to project out states of definite angular momentum and parity:

$$A_\ell := \int_{-1}^{1} dz\, P_\ell(z)\, A(s,t(z))$$

$$B_\ell := \int_{-1}^{1} dz\, P_\ell(z)\, B(s,t(z))$$

- The partial wave amplitudes are then given by

$$-f_{\ell\pm} = \frac{\sqrt{E+m}}{16\pi\sqrt{s}}\left(A_\ell + \frac{1}{2}((\sqrt{s}-m)B_\ell + B_\ell(\sqrt{s}-m))\right)\sqrt{E+m}$$
$$+ \frac{\sqrt{E-m}}{16\pi\sqrt{s}}\left(-A_{\ell\pm1} + \frac{1}{2}((\sqrt{s}+m)B_{\ell\pm1} + B_{\ell\pm1}(\sqrt{s}+m))\right)\sqrt{E-m}\,.$$

- Recall that all objects are matrices in channel space.
- Total angular momentum $j = \ell \pm \frac{1}{2}$.

Chiral Perturbation Theory and its unitarization

- For massless quarks, $\mathcal{L}_{\rm QCD}$ shows chiral symmetry: left- and righthanded quark fields $q_{L,R}$ can be rotated independently in flavor space.
- Masses of the light quarks (u, d, s) can be treated as a small perturbation on a typical hadronic scale $\Lambda_{had} \sim 1\,\mathrm{GeV}$. Heavy quarks are integrated out.
- We know that the full chiral symmetry must be broken down spontaneously to its vectorial subgroup,

$$SU(3)_L \times SU(3)_R \to SU(3)_V \, .$$

- Pions, Kaons and Eta are interpreted as (pseudo-)Goldstone bosons of this spontaneously broken approximate symmetry.

Chiral Perturbation Theory and its unitarization

- The symmetries and symmetry breaking patterns of $\mathcal{L}_{\rm QCD}$ can be encoded in an **effective** Lagrangian for the lowest lying mesons (M) and baryons (B) :

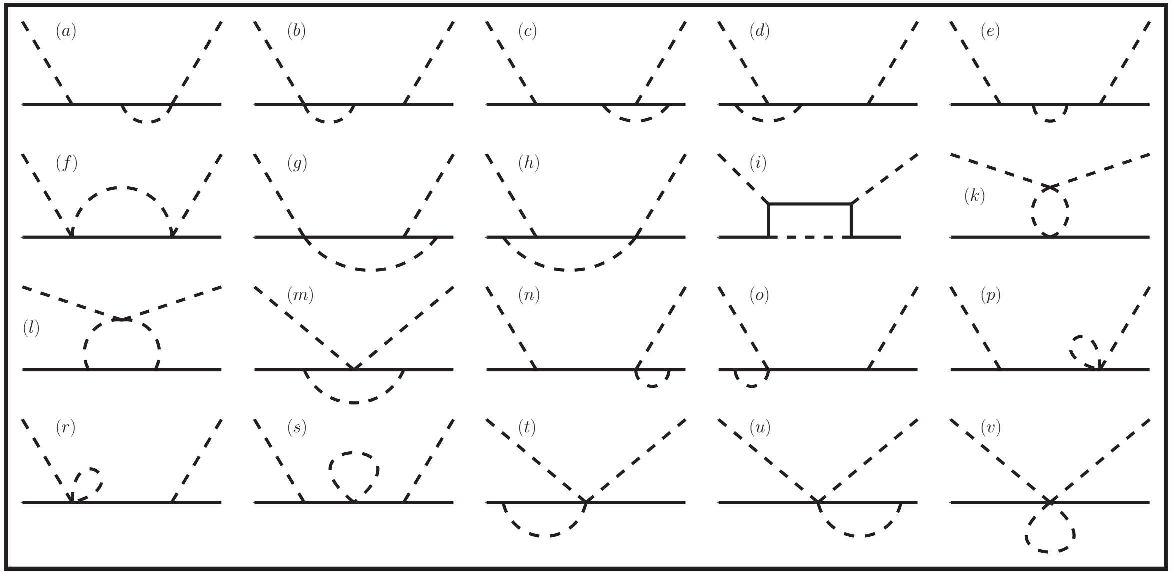
$$\mathcal{L}_M^{eff} = \mathcal{L}_M^{(2)} + \mathcal{L}_M^{(4)} + \cdots = \frac{F_0^2}{4}\langle \nabla_\mu U^\dagger \nabla^\mu U \rangle + \frac{F_0^2}{2} B_0 \langle \mathcal{M}(U + U^\dagger)\rangle + \cdots$$

$$\mathcal{L}_{MB}^{eff} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \cdots = \langle \bar{B}(i\not{D} - m_0)B \rangle - \frac{D/F}{2}\langle \bar{B}\gamma^\mu \gamma_5 [u_\mu, B]_\pm \rangle + \cdots$$

- U , u_μ collect the meson fields (π, K, η) in a convenient way, B collects the baryon fields (N, Σ, Λ, Ξ).
- $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$, m_0 is the baryon mass in the chiral limit.
- Greens functions are evaluated in a low energy expansion in small meson momenta q and the quark masses $m_q = \mathcal{O}(q^2)$. Subleading terms are suppressed by powers of $\mathcal{O}(q/\Lambda_{had})$.

Chiral Perturbation Theory and its unitarization

- We can now achieve a low-energy expansion of the pion-nucleon scattering amplitude.
- Tree level: Weinberg-Tomozawa contact term from $\mathcal{L}_{MB}^{(1)}$ ($\longrightarrow$ leading terms for scattering lengths) and $s-$ and $u-$channel Born graphs.
- Leading one-loop level: Contact terms from $\mathcal{L}_{MB}^{(2,3)}$ and loop graphs:



Chiral Perturbation Theory and its unitarization

- The perturbative approach is rigorous and well-defined, but the possible energy range where it can be applied is limited to the region near the πN threshold.
- **Resonance** phenomena can **not** be described in this framework ($S11(1535)$, $S11(1650)\cdots$).

- **Unitarity** of the S-matrix...

$$S^\dagger S = \mathbf{1} \quad \Rightarrow \quad \mathrm{Im}(f_{\ell\pm})^{-1} = -q_{cms} \,,$$

- ...is satisfied only perturbatively, up to the order one is working.

Chiral Perturbation Theory and its unitarization

- Instead of setting the higher-order terms to zero, we can adjust them so that two-body unitarity is satisfied **exactly**.
- Unitarity statement suggests the form

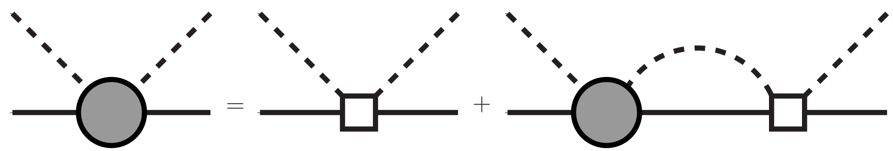
$$f_{\ell\pm} = \frac{1}{\mathbf{K}^{-1} - i\, q_{cms}} \qquad \text{for the PWA.}$$

- Matrix $\mathbf{K}$ is not unique ($\rightarrow$ model-dependence!) , should be related to terms from ChPT Lagrangian.
- There are many different versions of “Unitarized ChPT”.

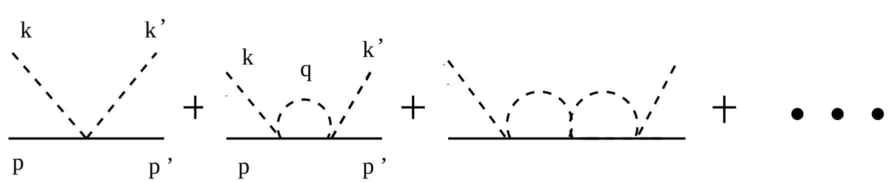
- Can we somehow maintain a one-to-one relation to Feynman graphs ?

Bethe-Salpeter equation (BSE)

- UChPT for the meson-baryon system
- coupled channel dynamics including the ground state octets: channels $\pi^0 p\,,\ \pi^+ n\,,\ \eta p\,,\ K^+\Lambda\,,\ K^+\Sigma^0\,,\ K^0\Sigma^+$
- Two-body unitarity is guaranteed by the Bethe-Salpeter equation (BSE) for the scattering operator T :



$$T(k', k; P) = V(k', k; P) + \int \frac{d^d q}{(2\pi)^d} V(k', q; P)\, iS(P-q)\Delta(q) T(q, k; P)$$



$P = p + k = p + k'$

Bethe-Salpeter equation (BSE)

- What do we take for the potential V in the BSE ?
- kernel (driving term) V is approximated by contact-term interaction from $\mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)}$

  - full inclusion of **Born graphs** leads to severe difficulties:

- Iteration of s-channel Born graphs give contributions to baryon self energies
  - difficult to keep track of mass renormalizations in this nonperturbative setting.
- Iteration of u-channel Born graphs yield complicated multiloop graph topologies
  - no (analytic) solution to this problem is known.

Bethe-Salpeter equation (BSE)

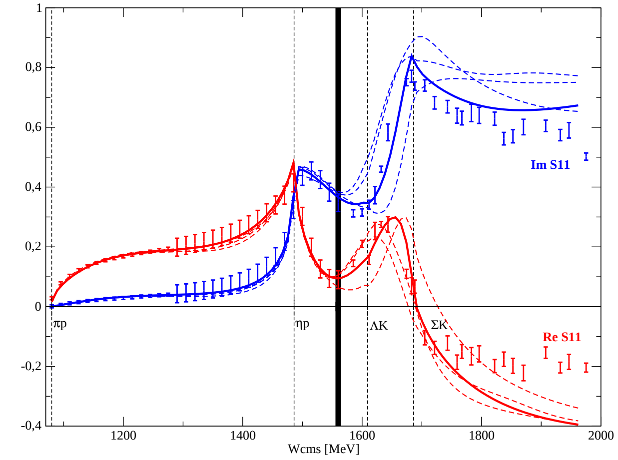
- free parameters in our ansatz:
  LECs $b_0\,,\,b_D\,,\,b_F\,,\,b_1\,,\ldots\,,\,b_{11}$
  and subtraction constants for loop integrals
- Solution T with this V has 20 independent operator structures:

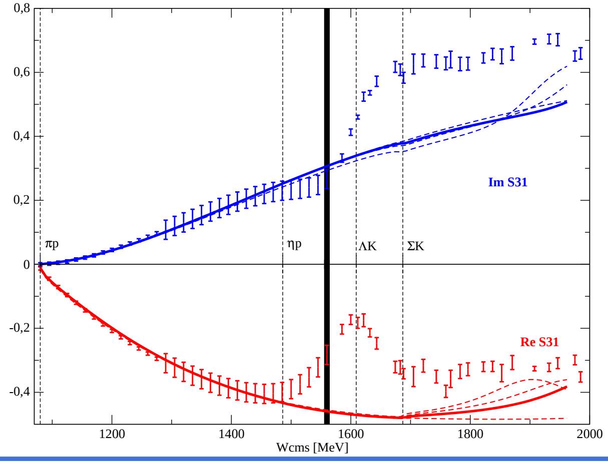
$$\begin{aligned}
T &= T_1(k'\cdot k) + T_2(k'\cdot k)\not{P} + T_3\not{k}'\not{k} + T_4\not{k}'\not{P}\not{k} + T_5\not{k}'(P\cdot k) \\
&+ T_6(k'\cdot P)\not{k} + T_7\not{k}'\not{P}(P\cdot k) + T_8(k'\cdot P)\not{P}\not{k} \\
&+ T_9(k'\cdot P)(P\cdot k) + T_{10}\not{p}(k'\cdot P)(P\cdot k) \\
&+ T_{11}\not{k}' + T_{12}\not{k}'\not{P} + T_{13}(k'\cdot P) + T_{14}\not{P}(k'\cdot P) \\
&+ T_{15}\not{k} + T_{16}\not{P}\not{k} + T_{17}(P\cdot k) + T_{18}\not{P}(P\cdot k) + T_{19}\not{P} + T_{20}\,.
\end{aligned}$$

T_i are scalar matrices in channel space and depend only on $P^2 \equiv s$.

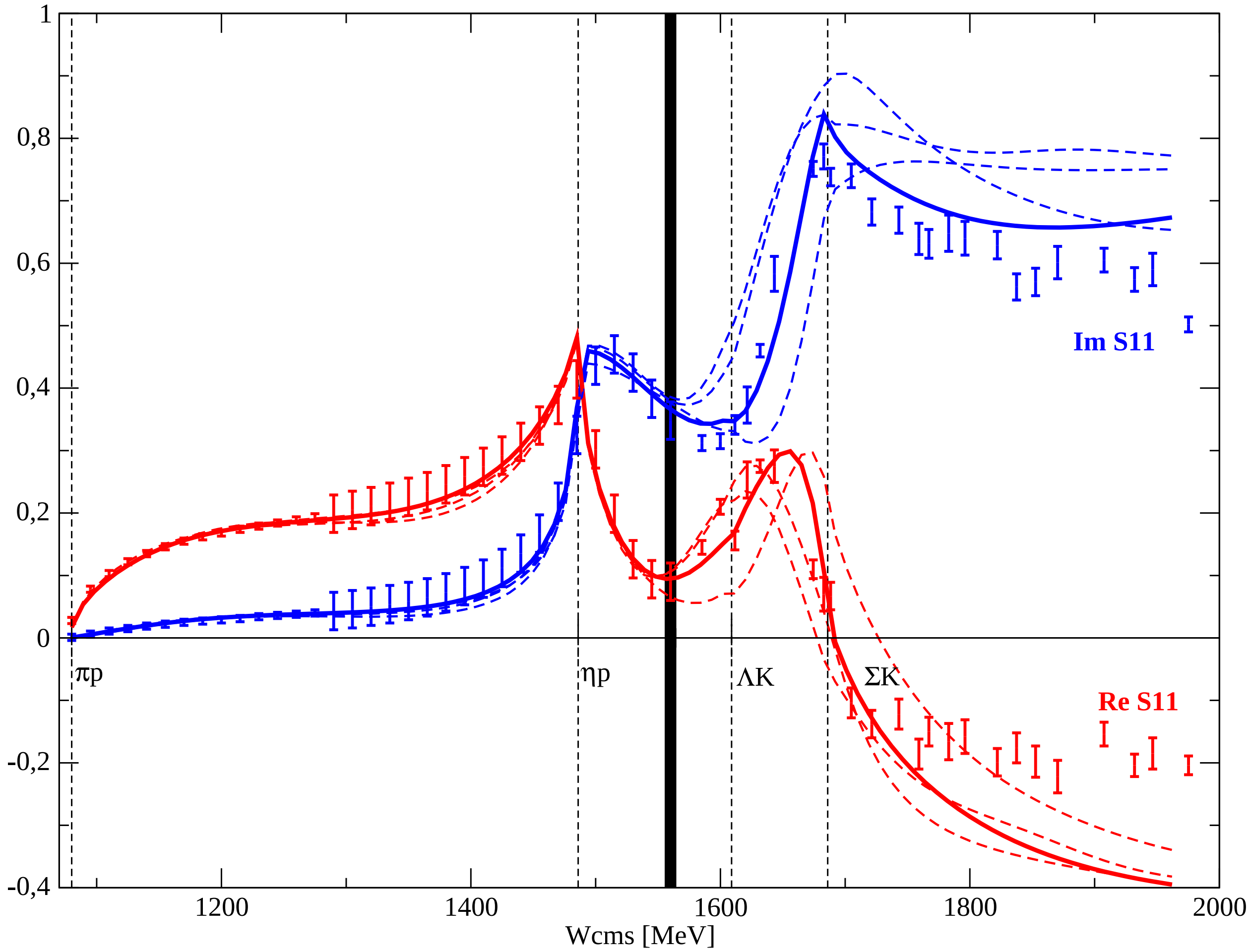
Results for the s-wave amplitudes

- BSE is solved as a linear system for the matrices $T_i(s)$
- $T_i(s)$ depend on LECs b_i and (regularized) scalar loop integrals
- Free parameters are fixed by a fit to s-wave πN scattering partial wave amplitudes
- Examples for fits to πN scattering exhibiting $S_{11}(1535)$ and $S_{11}(1650)$ resonances, dynamically generated in our approach
- $\Delta(1620)\,S_{31}$ can not be dynamically generated.
- Plotted is the dimensionless quantity $q_{cms} f_{0+}$ over $W = \sqrt{s}$ for $I = (1/2)\,,\,(3/2)$; real and imaginary part.





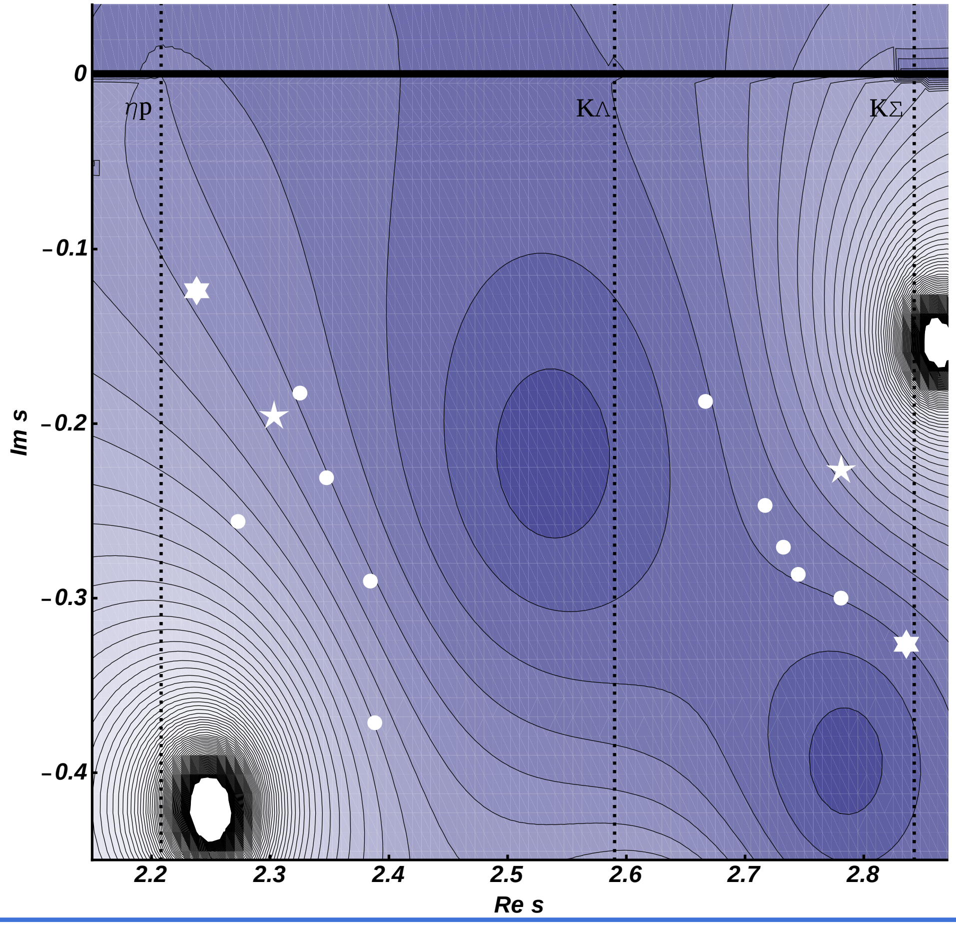
Results for the s-wave amplitudes



S11 partial wave amplitude from πN threshold to $2\,\mathrm{GeV}$.

Pole positions in GeV (best fit):

$$S_{11}(1535) \quad : \quad 1.506 - 0.140\, i$$
$$S_{11}(1650) \quad : \quad 1.692 - 0.046\, i$$



six-star: Nieves, Arriola (2001)
five-star: Döring et al. (2009)
dots: PDG (other sources)

Summary

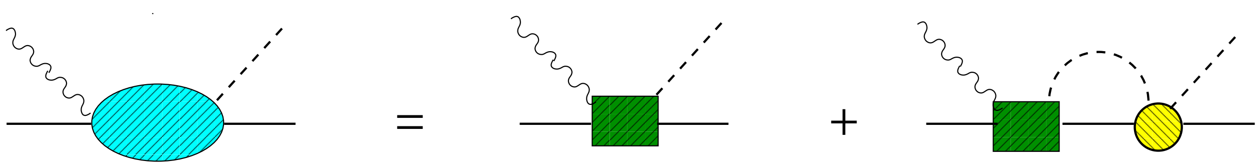
- We have analyzed pion-nucleon scattering in a $SU(3)$ coupled channel framework, employing the BSE to guarantee exact two-body unitarity.
- The driving term contains the Weinberg-Tomozawa term from $\mathcal{L}_{MB}^{(1)}$ as well as the contact terms from $\mathcal{L}_{MB}^{(2)}$
- Resonances $S_{11}(1535)$ and $S_{11}(1650)$ are generated dynamically in our approach.
  The structure associated with the $\Delta(1620)\, S_{31}$ could not be reproduced for a reasonable choice of fit parameters.
- More details can be found in

  P. C. Bruns, M. Mai, U.-G. Meißner,

  Chiral dynamics of the $\mathbf{S_{11}(1535)}$ and $\mathbf{S_{11}(1650)}$ resonances revisited
  arXiv: 1012.2233 [nucl-th],
  accepted for publication in Physics Letters B.

Outlook

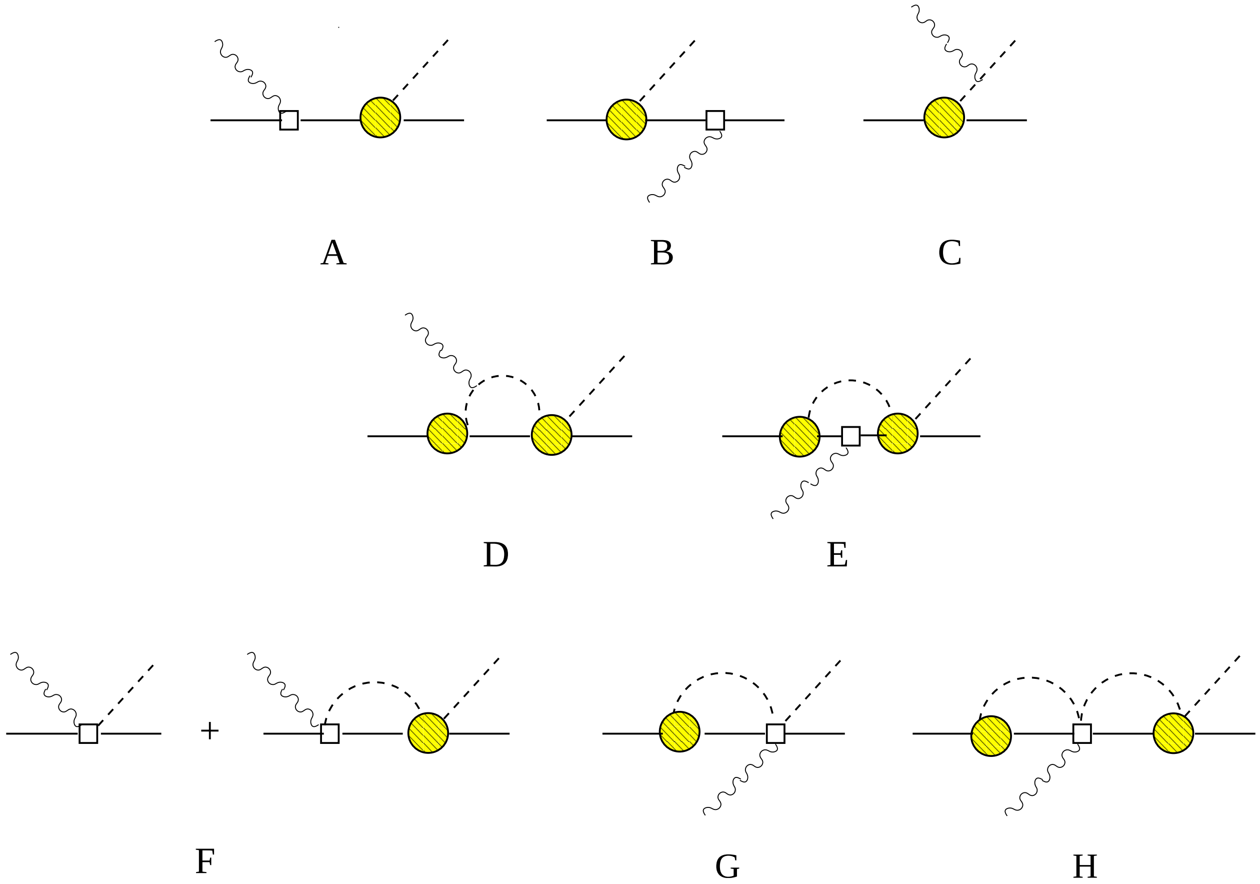
- Our aim is to use the meson-baryon scattering amplitude T as an extended vertex in an existing model for meson photoproduction on the proton Borasoy, Bruns, Meißner, Nißler, Eur. Phys. J. A **34** (2007) 161
- The full photoproduction amplitude corresponds to resummed classes of dimensionally regularized Feynman graphs
- Two-particle unitarity and gauge invariance are guaranteed by construction



$\mathcal{M} \qquad \mathcal{M}_0 \qquad \mathcal{M}_0 \quad \mathcal{T}$

Outlook

Full set of graphs entering the photoproduction amplitude:



Outlook

- Status: Photoproduction amplitudes have all been calculated
- There are two more LECs for photoproduction which will also be fitted to data
- Results will be published in the near future