What Top Mass is Measured at the LHC ?

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Outline

Which Top Mass is Measured at the LHC ?

- Why the question is relevant.
- What top mass is measured (= $m_t^{\rm Pythia}$)
- What is the relation to any mass theorists know ?
  - Factorization Theorem in e+e-
  - First rough answer
  - Plans to go on toward LHC
- Outlook and Conclusions

Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays "before hadronization" ($\Gamma_t \approx 1.5$ GeV)

$$m_t = 172.4 \pm 1.2 \text{ GeV}$$

FERMILAB-TM-2380-E
TEVEWWG/top 2007/01

~~$m_t = 172.6 \pm 1.4$ GeV~~

13th March 2007

~~$M_t = 170.9 \pm 1.8$ GeV/c^2~~

Combination of CDF and DØ Results on the Mass of the Top Quark

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 1 fb^{-1} of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 170.9 \pm 1.1(\text{stat}) \pm 1.5(\text{syst})$ GeV/c^2, which corresponds to a total uncertainty of 1.8 GeV/c^2. The top-quark mass is now known with a precision of 1.1%.

<1% precision !

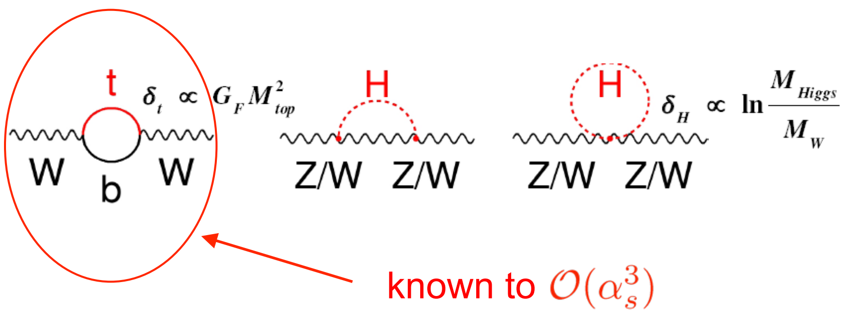
How shall we theorists judge the error ?

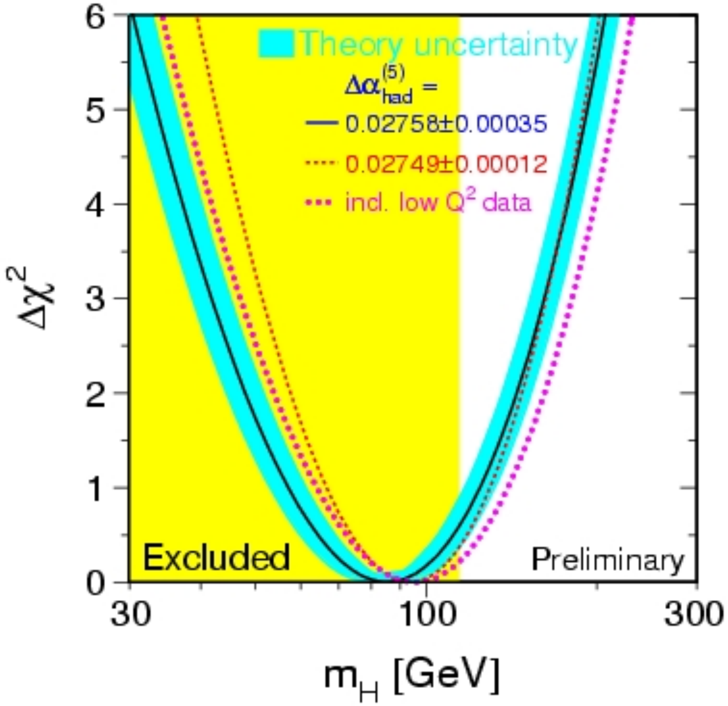
What is the theoretical error ?

What mass is it ?

Need for a precise Top mass

Fit to electroweak precision observables





$$\sin\theta_W \times \Big(1+\delta(m_t, m_H, \ldots)\Big)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

$$m_H = 76^{+33}_{-24}\ \text{GeV}$$

$$m_H < 182\ \text{GeV}\ \ (95\%\text{CL})$$

$$m_t = 170.9 \pm 1.8\ \text{GeV}$$

2 GeV error: 15% change in m_H

Need for a precise Top mass

Mass of Lightest MSSM Higgs Boson

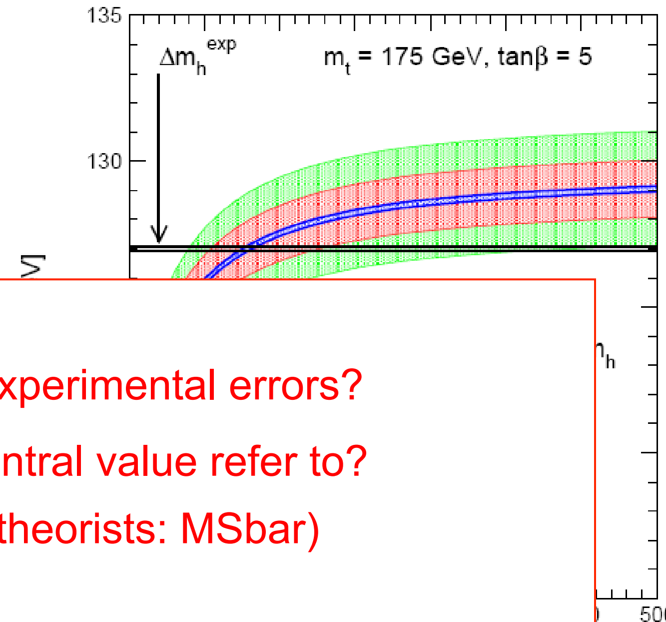
5 Higgs bosons:

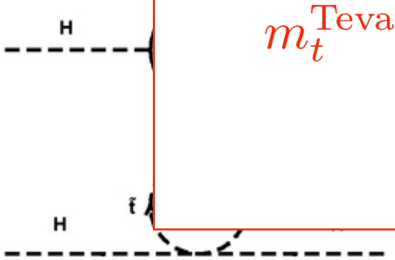
m_h (scalar, neutral)

m_H (scal

m_A (speu

$m_H^{\pm}$ (cha





- How to reduce theoretical and experimental errors?
- What mass scheme does the central value refer to?

(preferred mass scheme for NP-theorists: MSbar)

$$m_t^{\rm Tevatron} = m_t^{\rm pole}? \qquad m_t^{\rm 1S}?$$

$$\overline{m_t}(\mu)? \qquad m_t^{\rm PS}(\mu)? \qquad m_t^{\rm kin}(\mu)?$$

$\mathcal{O}(\alpha_s^2)$ corrections known

Top Quark Pole Mass

- Based on (unphysical) concept of top quark being a free parton

$$\not{p} - m_t - \Sigma(p, m_t)\big|_{p^2=m_t^2}$$

- No physical quantity (i.e. renormalization condition) exists that is tied to the pole mass scheme., also not the peak of the top invariant mass distribution.
- Pole mass renormalization condition introduces artificially large corrections.

$\overline{m}_t(\overline{m}_t)$ [GeV]	M_t^{pole} [GeV]		
	1-loop	2-loop	3-loop
160.00	167.44	169.05	169.56
165.00	172.64	174.28	174.80
170.00	177.84	179.52	180.05

$\alpha_s(M_z) = 0.119$

1.6 GeV

- Pole mass measurements are:
  - order-dependent
  - strongly correlated to other theory parameters

Top Quark Pole Mass

Top decay width

$$\Gamma_0 = \frac{G_F m_t^3 |V_{tb}|^2}{8\sqrt{2}\pi}$$

$$\Gamma_t(t \to bW) = \Gamma_0^{pole}[1 - 0.10\epsilon - 0.02\epsilon^2]$$

$$m_t^{\rm pole}$$

$$\Gamma(t \to bW) = \bar{\Gamma}_0[1 - 0.04\epsilon - 0.003\epsilon^2]$$

$$\overline{m}_t(\overline{m}_t)$$

Rho parameter

$$x_t \equiv 3\frac{G_F m_t^2}{8\sqrt{2}\pi^2}$$

$$\Delta\rho = x_t^{pole}[1 - 0.098\epsilon - 0.017\epsilon^2]$$

$$\Delta\rho = \bar{x}_t[1 - 0.007\epsilon - 0.007\epsilon^2]$$

Top MSbar mass preferred for electroweak precision fits.

Main Methods at Tevatron

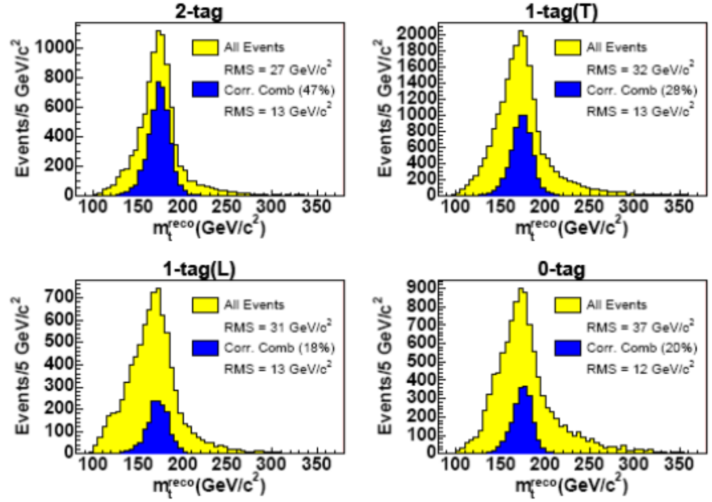
Template Method

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$
$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

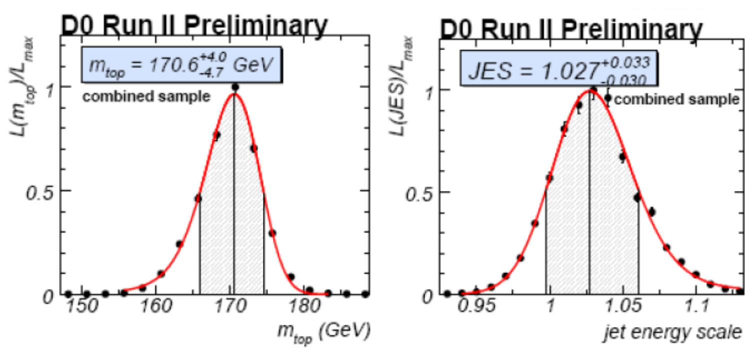
$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x|y)$$

parton distribution functions

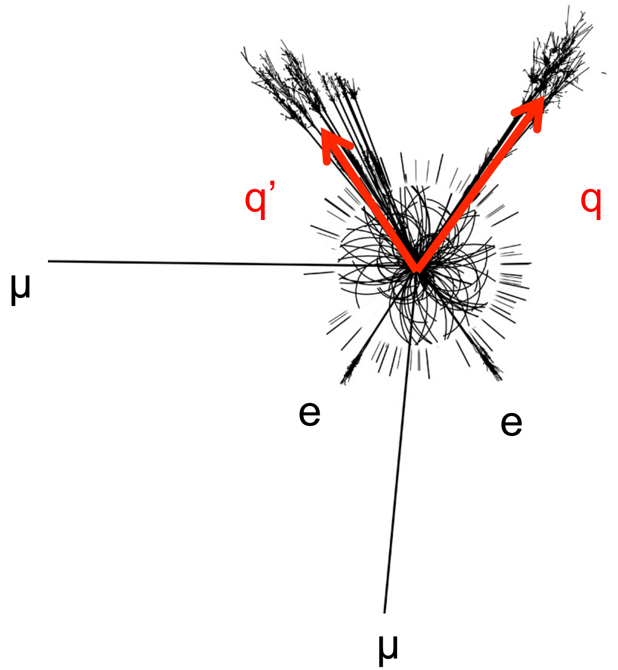
differential cross section (LO matrix element)

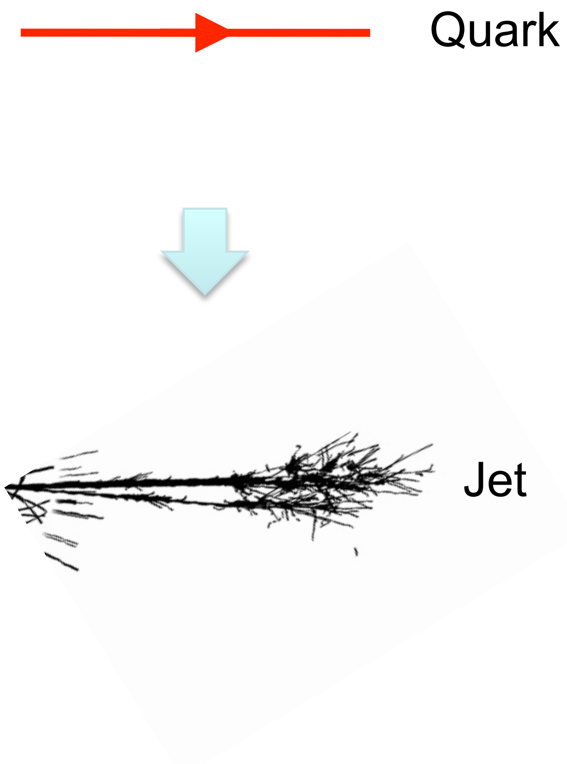
transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



Description of Jets



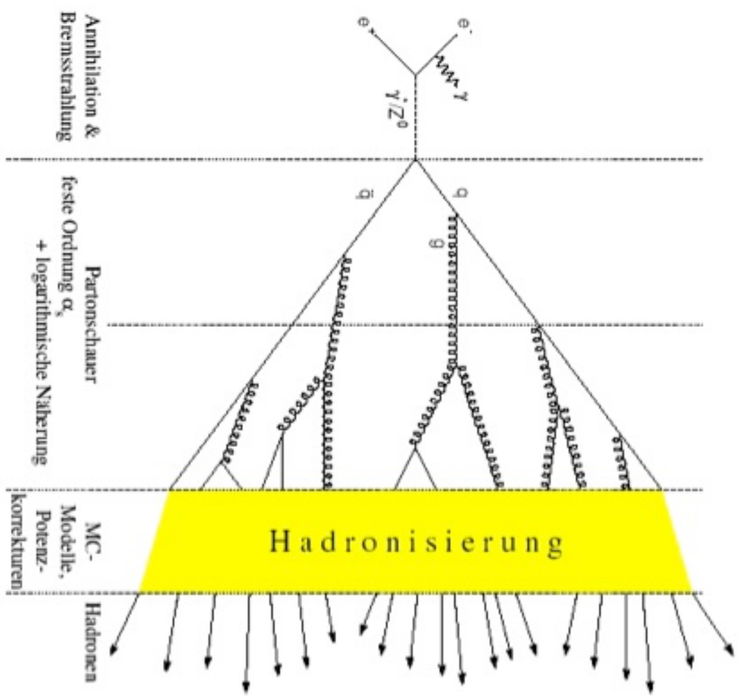


Description of Jets

Monte Carlo generators:

Universal instrument to describe hadronic final states.

- Hadronization models are “tuned” to experimental data.



- Parton-Shower: leading-log approximation
- Classic approximation
- No quantum interference
- Infrared regularization scheme in the parton showers is not specified.

Monte Carlo generators

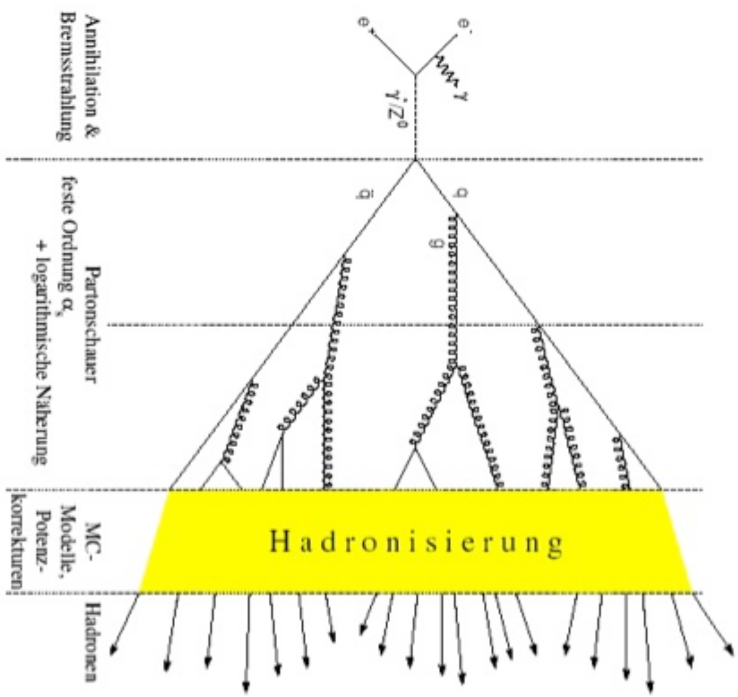
= QCD inspired model

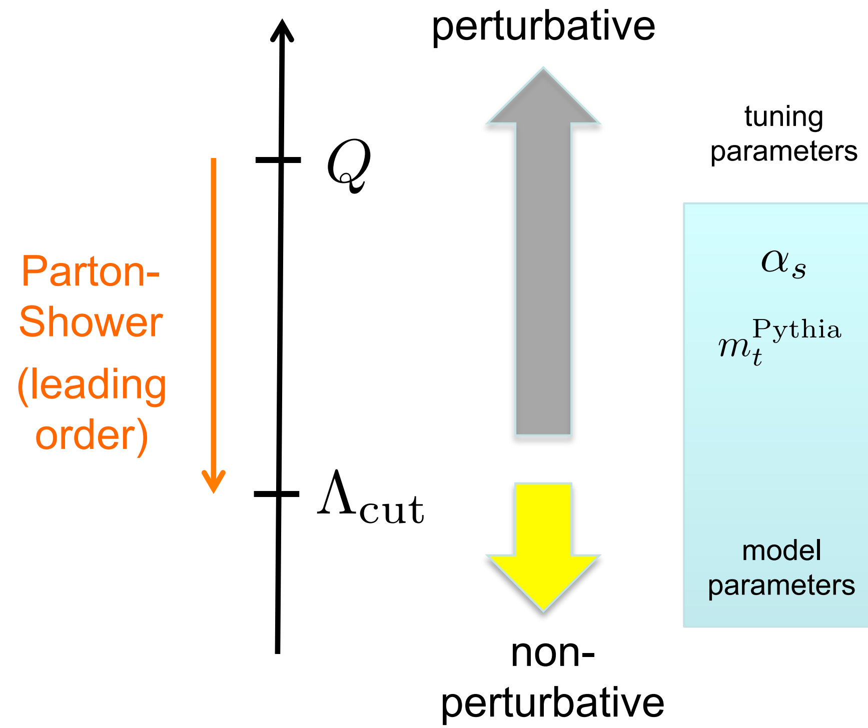
Description of Jets

Monte Carlo generators:

Universal instrument to describe hadronic final states.

- Hadronization model and α_s are “tuned” to experimental data.





Main Methods at Tevatron

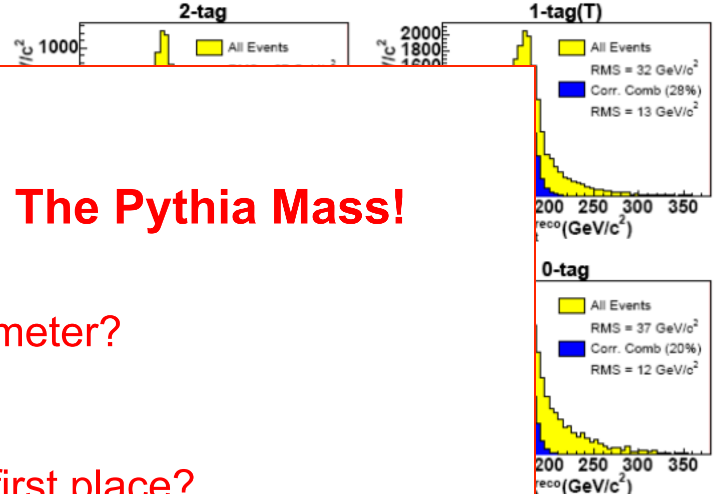
Template Method

- Principle: perform kinematic fit and reconstruct top mass event

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - \ldots}{\ldots} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2}$$

Usually pick

Lepton+jets (≥1 b-tag); Signal-only templates



What mass is measured?: The Pythia Mass!

- What is the Pythia mass parameter?
- It's not the pole mass !
- How reliable is the MC in the first place?
- How can we approach the issue?
- Should we be worried concerning top physics at LHC ?

Dynamic

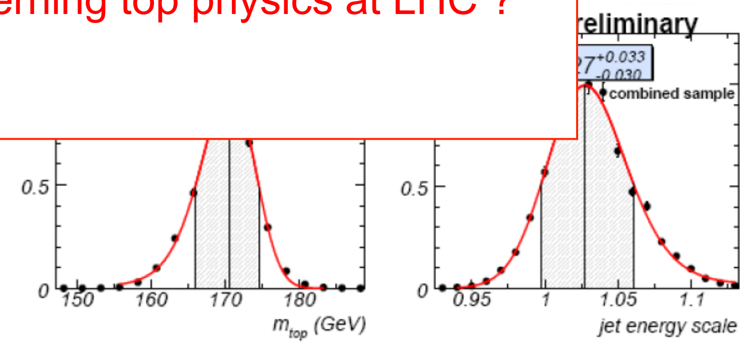
- Principle: c a function c objects in t Maximize s

parton distribution functions

$$P(x;m_t) = \frac{1}{\sigma}\int d^n\sigma(y;m_t)\,dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

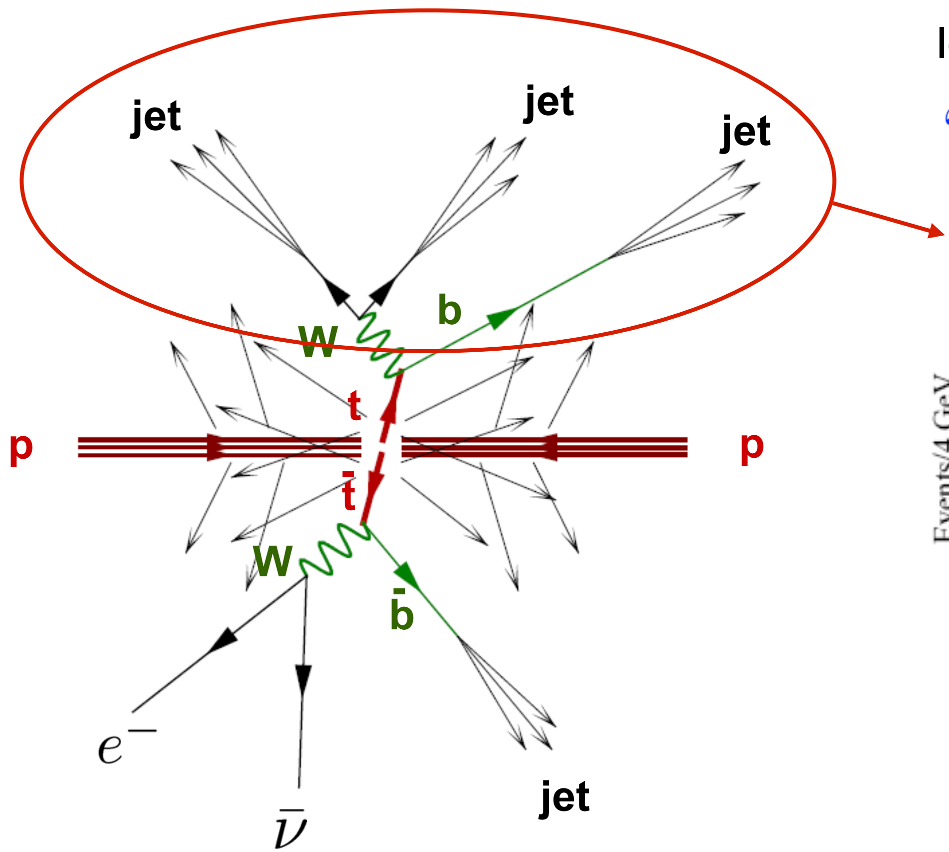
differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)



Description of Jets

LHC:

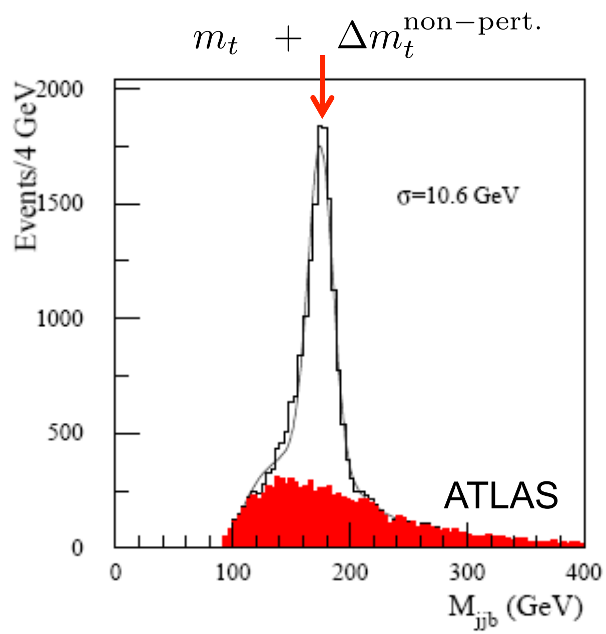


Principle of mass measurements:

Identification of the top decay products

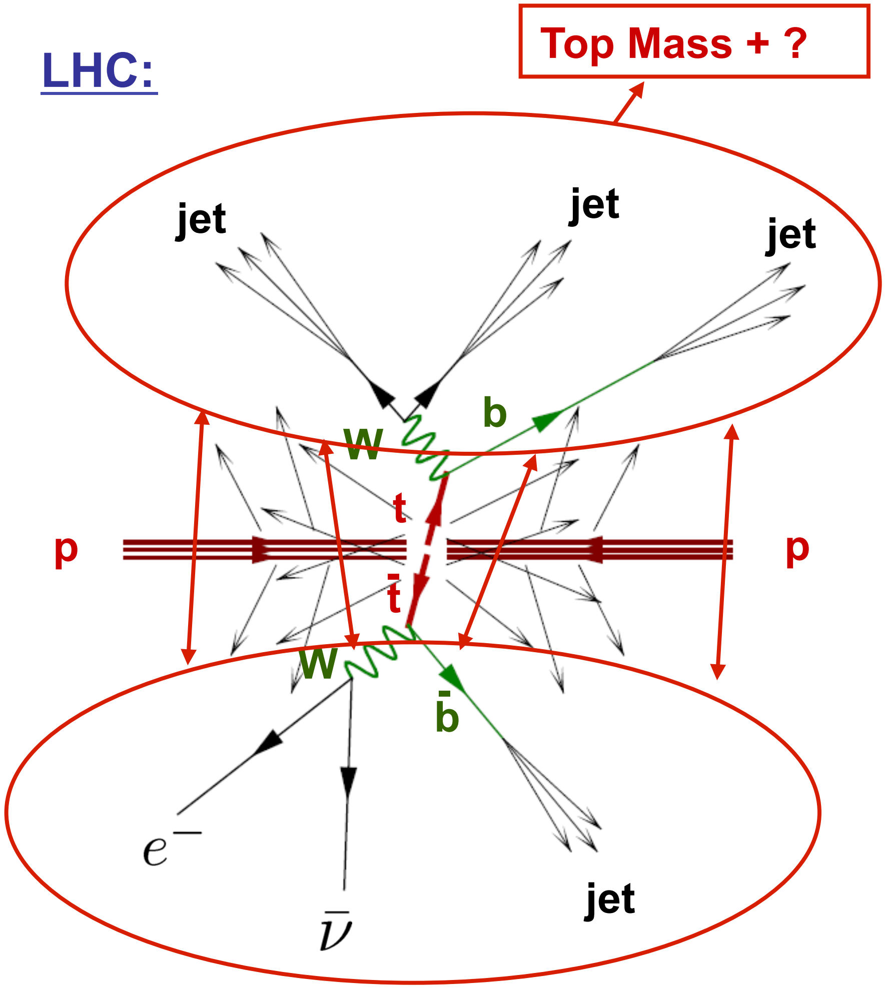
$$\text{“}\; m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;\text{”}$$

Invariant mass distribution



Description of Jets

LHC:



Principle of mass measurements:

Identification of the top decay products

$$\text{“}\; m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2 \;\text{”}$$

Problem is non-trivial !

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

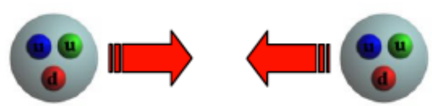
Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.

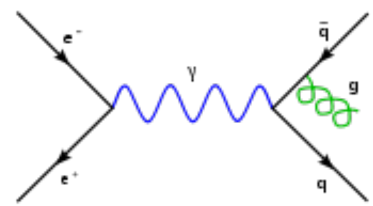
QCD Factorization

Drell-Yan: $pp \to \ell^+\ell^- + X$ (inclusive)

Collins, Soper,Sterman; Bodwin

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2\mathrm{d}Y} = \sum_{i,j}\int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\, f_i(x_1,\mu)\; f_j(x_2,\mu)\; H_{ij}^{\mathrm{incl}}(x_1,x_2,q^2,Y,\mu)$$





non-perturbative
parton distribution function

(process independent)

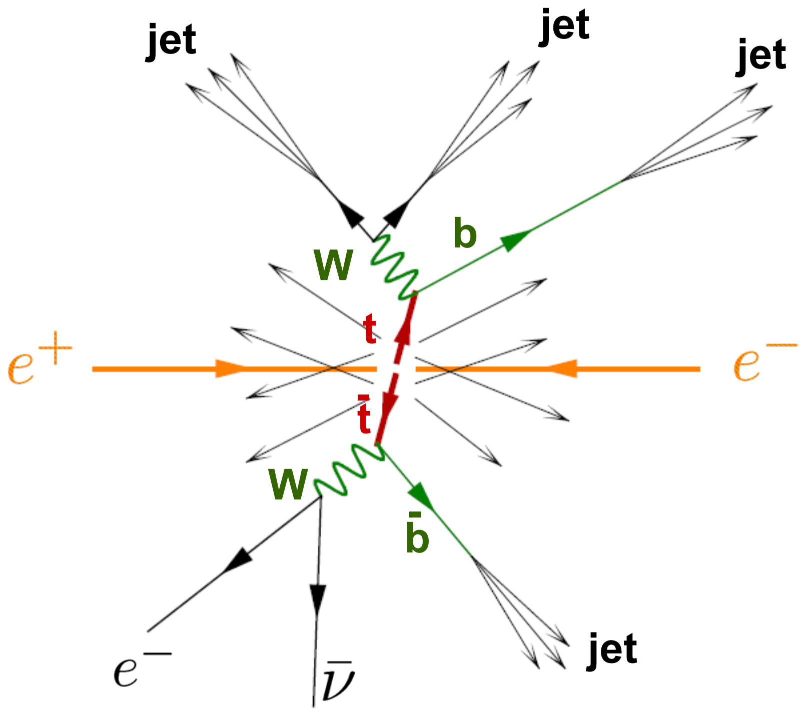
perturbative
hard cross section

(process dependent)

→ QCD factorization in the initial state

QCD Factorization

Top Invariant Mass Distribution:



$$e^+e^- \rightarrow t\bar{t}$$

$$Q \gg m_t \quad (p_T \gg m_t)$$

→ We need: QCD factorization in the final state

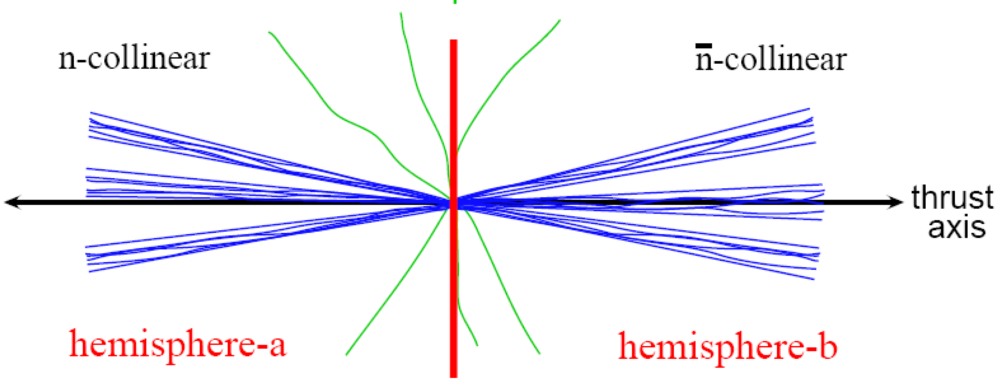
QCD Factorization

Top Invariant Mass Distribution:

Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008

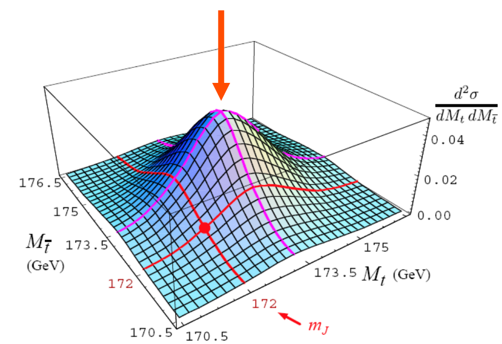
Definition of the observable



$$M_t^2 = \left(\sum_{i\,\epsilon\,a} p_i^\mu\right)^2 \qquad M_{\bar t}^2 = \left(\sum_{i\,\epsilon\,b} p_i^\mu\right)^2$$

resonance region:

$$M_{t,\bar t} - m_t \sim \Gamma$$



$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}}$$

Double differential hemisphere mass distribution

QCD Factorization

Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008

Phys.Rev.D77:114003,2008

Phys.Lett.B660:483-493,2008

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

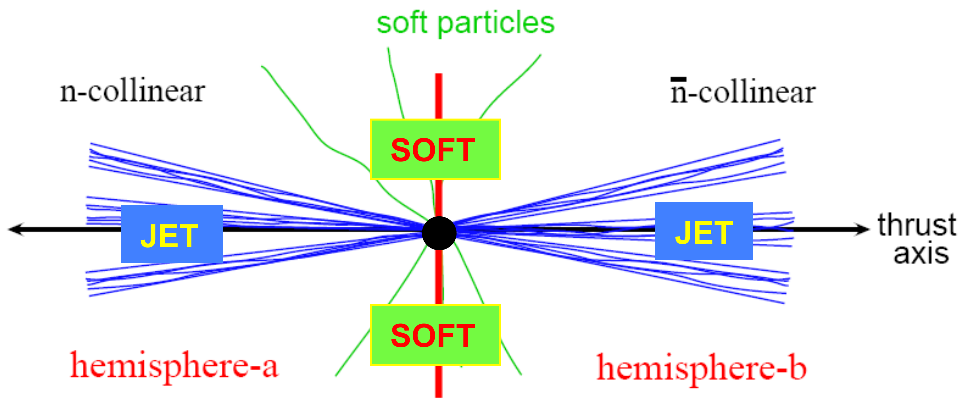
QCD

↓

Soft-Collinear-Effective-Theory

↓

Heavy-Quark-Effective-Theory + **Unstable-Particle-Effective-Theory**



Faktorization Formula

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m,\frac{Q}{m},\mu_m,\mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\right) S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

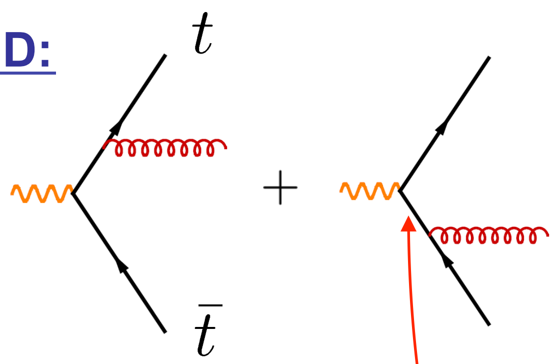
$$\hat s = \frac{M_t^2 - m_J^2}{m_J}$$

JET JET SOFT

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

QCD Factorization

full QCD:



3 phase space regions: $\lambda \sim m_t/Q$

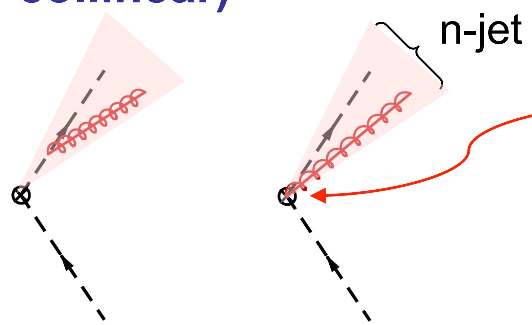
- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{\bar{t}}+k)^2 - m_t^2} \qquad (p_{\bar{t}}^2 \approx m_t^2\,,\ \ \bar{n}^2 = 0)$$

$$\downarrow$$

$$\frac{1}{Q\ \bar{n}.k}$$

Gluon collinear to the top:
(n-collinear)



$$W_n^\dagger(\infty, x) = \mathrm{P}\ \exp\left(ig \int_0^\infty ds\bar{n}\cdot A_+(ns+x)\right)$$

$h_{v_+}(x)$ → **gauge dependent**

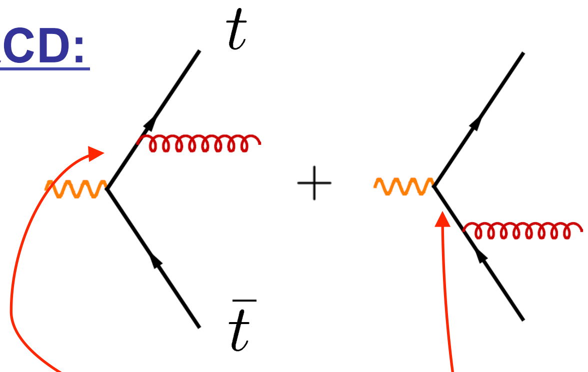
$W_n^\dagger(\infty, x)\, h_{v_+}(x)$ → **gauge independent**

$$B_+(\hat{s}, \Gamma_t, \mu) \ = \ \mathrm{Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\,\langle 0|\, T\,\{\bar{h}_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\ \}\,|0\rangle\right]$$

QCD Factorization

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

full QCD:



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- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

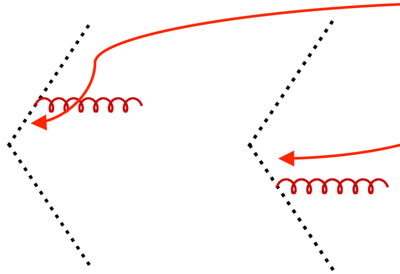
$$\frac{1}{(p_{t,\bar{t}}+k)^2-m_t^2}$$

$$(p^2_{t,\bar{t}} \approx m_t^2\,,\ \ n^2 = 0\,,\ \ \bar{n}^2 = 0)$$

Gluon soft:

$$\frac{1}{Q\ \ n.k}$$

$$\frac{1}{Q\ \ \bar{n}.k}$$



$$Y_n(x) = \overline{\mathrm{P}}\ \exp\Big(-ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

$$\overline{Y_{\bar{n}}}(x) = \overline{\mathrm{P}}\ \exp\Big(-ig\int_0^\infty ds\, \bar{n}\cdot \overline{A}_s(\bar{n}s+x)\Big)$$

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\,\langle 0|\,(\overline{Y}_n)^{cd}\,(Y_n)^{ce}(0)\,\delta(\ell^- - (\hat{P}_a^+)^\dagger)\delta(\ell^- - \hat{P}_b^-)\,(Y_n^\dagger)^{ef}(\overline{Y}_n^\dagger)^{df}(0)\,|0\rangle$$

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions: $$B_+(\hat s, \Gamma_t, \mu) = \mathrm{Im}\Big[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\, \langle 0|\, T\, \{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\, |0\rangle\Big]$$

- perturbative
- dependent on mass, width, color charge

$$B_\pm^{\rm Born}(\hat s, \Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c}\sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on color charge, kinematics

Independent of the mass !

NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172 \text{ GeV}$$

$$\Gamma = 1.43 \text{ GeV}$$

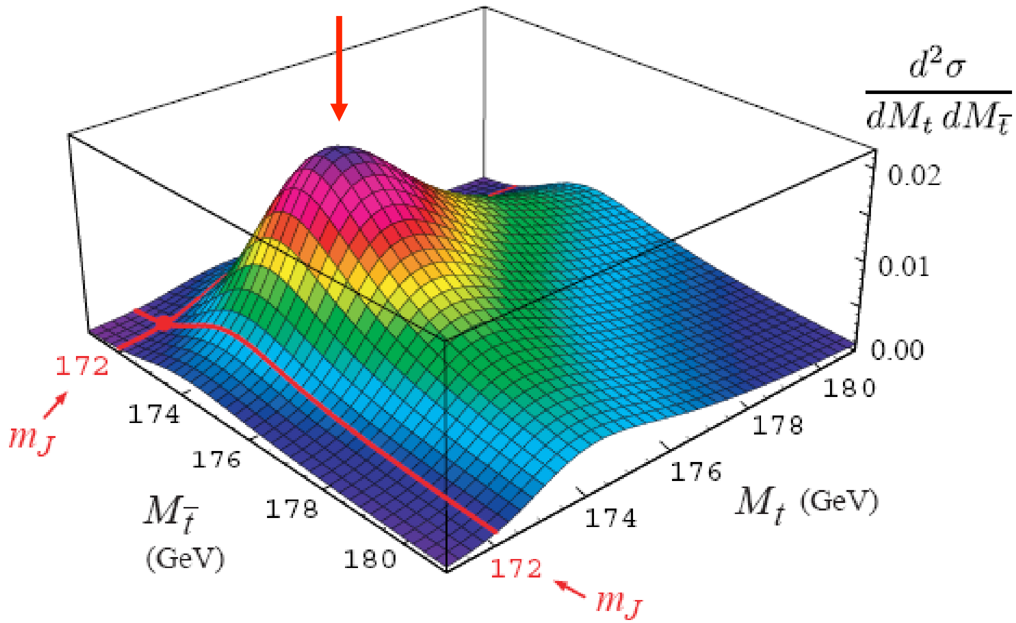
$$m_J(2\,\text{GeV}) = 172 \text{ GeV}$$

$$\mu_\Gamma = 5 \text{ GeV}$$

$$\mu_\Lambda = 1 \text{ GeV}$$

$$a = 2.5\,, \quad b = -0.4$$

$$\Lambda = 0.55 \text{ GeV}$$

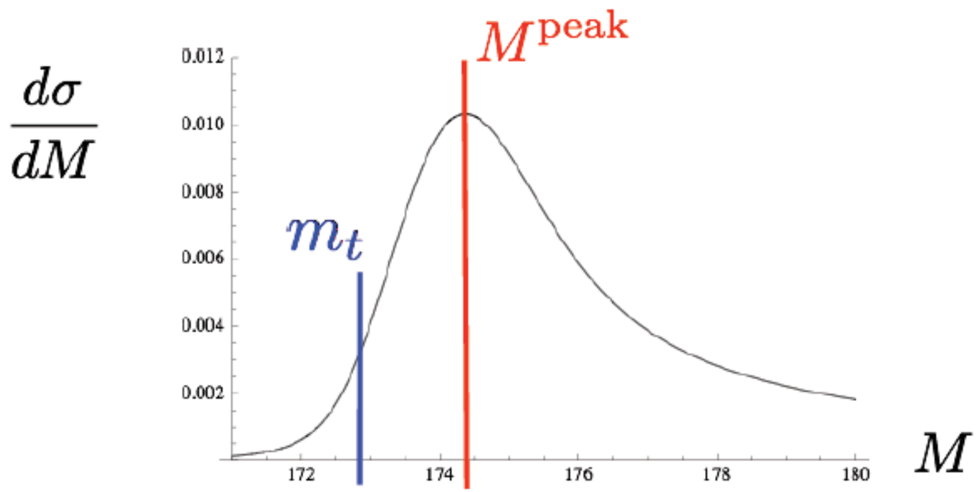


Non-perturbative effects shift the peak by +2.4 GeV and broaden the distribution.

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

QCD Factorization

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008



$$M^{\text{peak}} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q}{m_t}\,\Omega_1 + \mathcal{O}\left(\frac{m_t \Lambda_{\text{QCD}}}{Q}\right)$$

first moment of the soft function: $\Omega_1 = \int d\ell\, \ell\, S(\ell, \mu)$

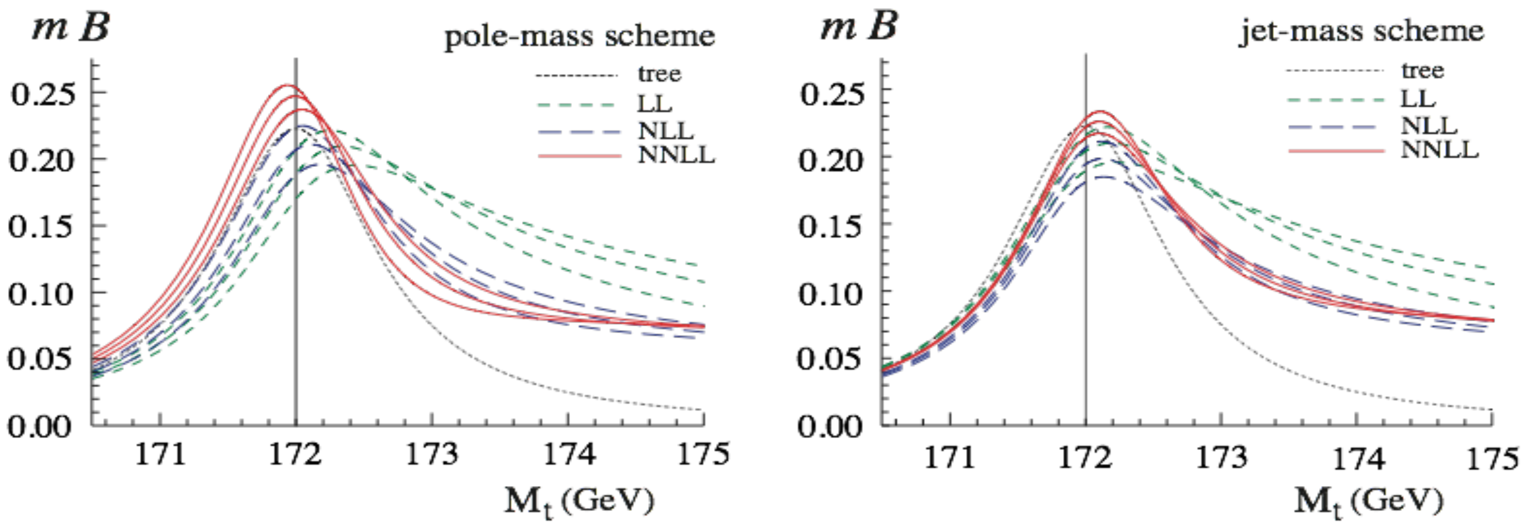
QCD Factorization

Higher Orders & Top Mass Scheme:

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008

$$\mathcal{B}_\pm(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\}$$

Jain, Scimemi, Stewart
PRD77, 094008(2008)

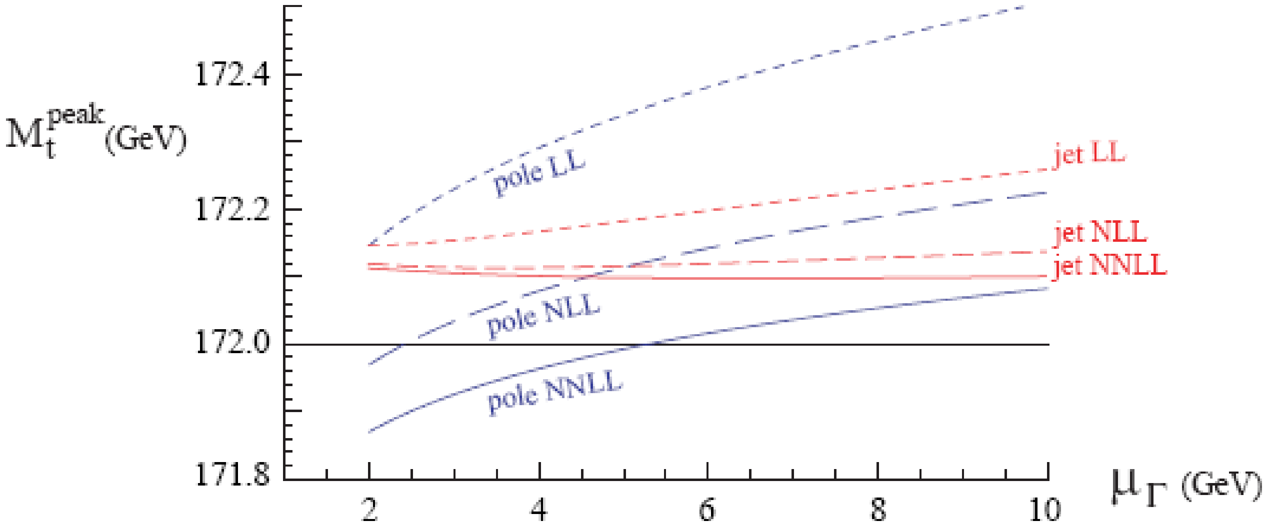


$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R \frac{\alpha_s(\mu)\, C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right]+\mathcal{O}(\alpha_s^2)$$

$$R \sim \Gamma_t$$

NLL Numerical Analysis

Scale-dependence of peak position



- Jet mass scheme: significanly better perturbative behavior.
- Renormalon problem of pole scheme already evident at NLL.

Theory Issues for $pp \to t\bar{t} + X$

- ★ definition of jet observables → Hadron event shapes

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}$$

Banfi, Salam, Zanderighi

- ★ initial state radiation
- ★ final state radiation
- • underlying events → Soft function ?
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

★ **Can be addressed in the framework of a LC.**

★ **Requires extensions of LC concepts and other known concepts**

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Final State Shower

- Start: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1\ \mathrm{GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

Let's assume that these aspects are treated correctly in the MC

MC Top Mass

Conclusion:

constant of order unity

$$m_t^{\mathrm{MC}}(R_{sc}) = m_t^{\mathrm{pole}} - R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right]$$

Determination of the MSbar mass:

$$m_t^{\mathrm{TeV}} = m_t^{\mathrm{MC}}(R_{sc}) = 172.6 \pm 0.8(stat) \pm 1.1(syst)$$

↓ 3-loop R-evolution equation

AHH,Jain, Scimemi, Stewart

PRL 101,151602(2008)

$$\overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3\,^{+0.6}_{-0.3}\ \mathrm{GeV} \qquad (c = 3^{+6}_{-2})$$

More systematic study needed for final answer!

Outlook & Conclusion

Conclusion:

→ Current top mass measurements from the Tevatron refer to the top mass parameter in Pythia $m_t^{\rm Pythia}$.

$m_t^J(2\ \mathrm{GeV})$

→ For a high energy Linear Collider we have a factorization theorem to do MC independent short-distance Lagrangian top mass measurements (jet mass)

→ The analogy between MC generators and factorization theorem indicates that the $m_t^{\rm Pythia}$ is a short-distance mass like the jet mass (and not the pole mass).

$m_t^{\rm Pythia}$	$m_t^J(2\ \mathrm{GeV})$		
	1-loop	2-loop	3-loop
160.00			
165.00			
170.00			

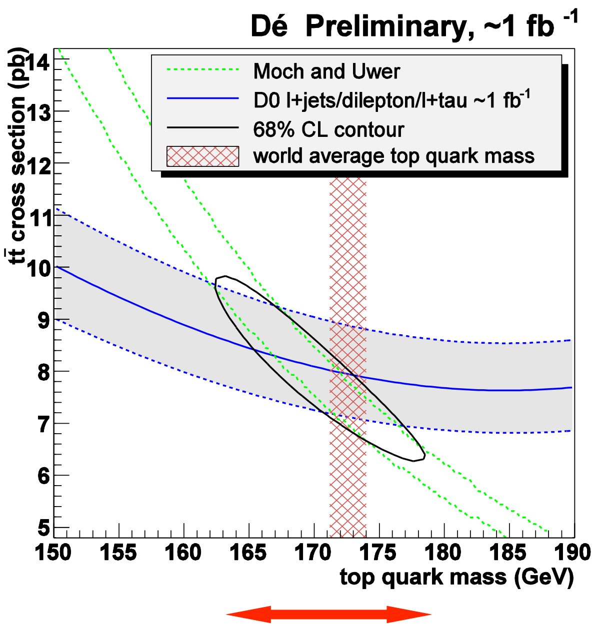
Plans:

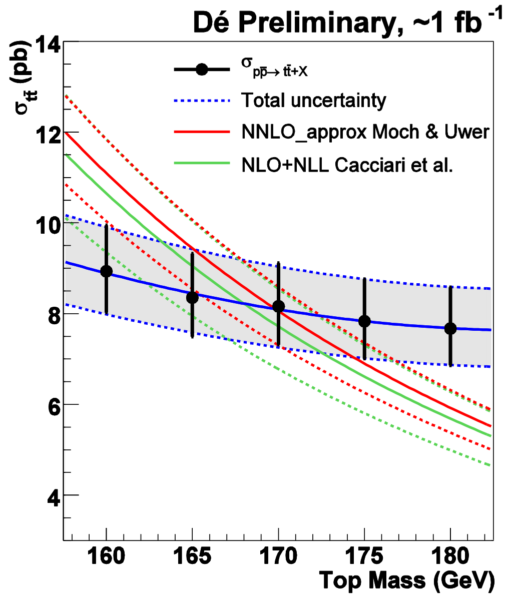
→ "Measure" the $m_t^{\rm Pythia}$ in terms of the Jet mass $m_t^J(2\ \mathrm{GeV})$ using thrust and other event shapes

→ Derivation of eventshape-like factorization theorems for Tevatron/LHC

→ "Measure" $m_t^{\rm Pythia}$ for LHC-Pythia

Top Mass from the Cross Section





Should be computed in the MSbar scheme !

- Theoretical cross section taken from theorists - computed in the pole scheme.
- More sensitivity to uncertainties that affect the normalization of the cross section.

- Experimental total cross section determined with MC, depends on MC top mass.
- MC top mass identified with the pole mass.
- Top mass dependence can be reduced by modifying the analysis.