Compton scattering: from deeply virtual to quasi-real

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- *Motivation*
- *Exact parameterization of (D)VCS amplitude*
- *Cross sections and ? Rosenbluth separation*
- *Some DVCS phenomenology*

A. Belitsky, DM, A. Kirchner hep-ph/0112108
A. Belitsky, DM, Y. Ji 1212.6674 [hep-ph]
K. Kumericki, DM, M. Morgan 1301.1230 [hep-ph]
E. Aschenauer, S. Fazio, K. Kumericki, DM (contains Rosenbluth separation, to submit)

Motivation

a simple and very convenient parameterization of the electromagnetic nucleon current

$$\langle p_2, s_2 | j_\rho(0) | p_1, s_1 \rangle = \overline{u}(p_2, s_2) \left[\gamma_\rho F_1(t) + i\sigma_{\rho\sigma} \frac{\Delta^\sigma}{2M} F_2(t) \right] u(p_1, s_1)$$

in terms of Dirac and Pauli form factors

form factors are fundamental quantities:

- revealing that the nucleon is a composite particle [Hofstadter 1956]
- subject of many experimental measurements (which are challenging)
  ? $-t \to 0$ (nucleon radius) , ? large $-t$ behavior, ? time-like behavior
- various theoretical approaches are employed to explain data
  lattice QCD, effective theories, Dyson-Schwinger approach, QCD sum rules, pQCD, wave function-modeling, GPD modeling, ...

~ 1600 papers on spires

Can you imagine a situation where ``everybody" has his own conventions to parameterize the electromagnetic nucleon current?

- Compton scattering is another fundamental and important process

- revealed the nature of light
- low energy theorem (Thomson limit) serves to define electric charge
- reveals that the nucleon is a rather rigid particle – small polarizabilities
- many (technological) applications

- virtual Compton scattering one ``measures'' generalized polarizabilities

- description of the proton in terms of mesonic degrees of freedom
- uses of dispersion relations

- deeply virtual Compton scattering one ``measures'' Compton form factors

- description of the proton in terms of generalized parton distributions
- uses of another set of dispersion relations, too

- deep inelastic scattering one measures DIS structure functions
  (absorptive part of forward Compton scattering amplitude)

- description of the proton in terms of parton distribution functions
- high energy QCD (small x_B)

~ 2300 papers on spires about Compton scattering, including
~ 200 about virtual Compton scattering
~ 300 about deeply virtual Compton scattering

How to parameterize (deeply) virtual Compton scattering amplitude or tensor?

counting complex valued helicity amplitudes

CS	*6 = 4 + 4 -2*
(D)VCS	*12 = 4 + 4 + 4*
VVCS /DDVCS	*18 = 4 + 4 + 4 + 4 + 2*

- ❑ forward kinematics standard convention (rest frame)
  - (DIS) hadronic tensor and virtual photoproduction cross section

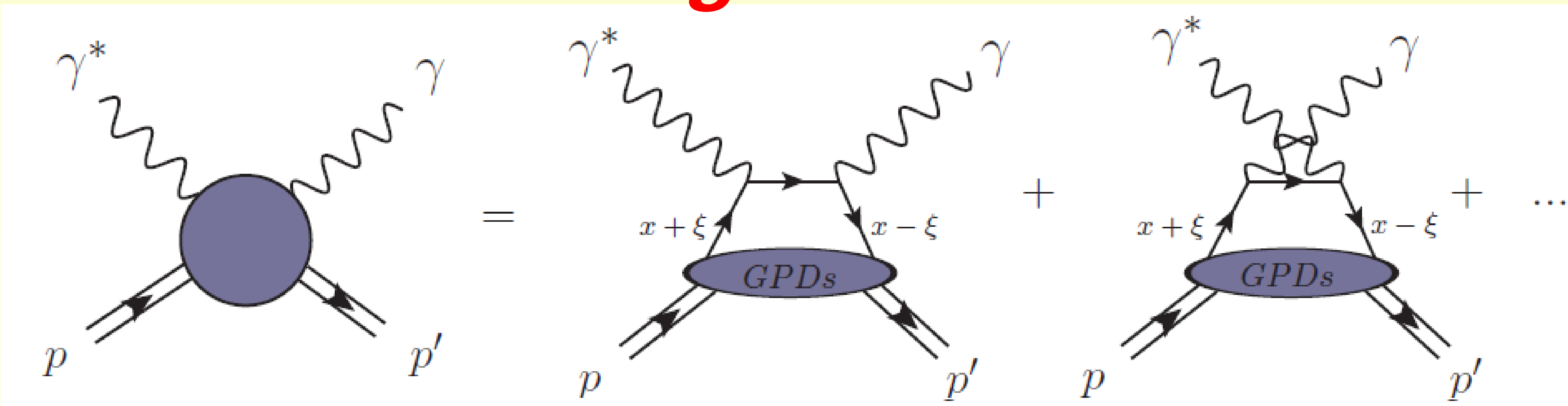
- ❑ VCS a kind of standard conventions were emerging during time
  - photon helicity amplitudes (center-of-mass frame) and Pauli-spinors
  - another set of VCS amplitudes for dispersion relations

- ❑ DVCS a kind of standard that arises from 1/Q expansion
  - one perturbatively calculates the hadronic tensor
  - nomenclature arises form that of generalized parton distributions
  - strictly spoken ``everybody'' uses its own convention

desired/needed:

- to have one standard parameterization
- to know the map among different parameterizations
- analytic form of leptoproduction cross section

Calculating DVCS tensor



$$T_{\mu\nu} = i \int d^4x \, e^{\frac{i}{2}(q_1+q_2)\cdot x} \langle p_2 | T\{j_\mu(x/2) j_\nu(-x/2)\} | p_1 \rangle$$

- collinear factorization approach (calculating Feynman diagrams on partonic level)
- operator product expansion (in terms of light-ray operators)

$$T j_\mu(x/2) j_\nu(-x/2) \stackrel{\mathrm{LO}}{=} \frac{S_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta \psi(-x/2) - \overline{\psi}(-x/2)\gamma^\beta\psi(x/2)\right]$$
$$+ \frac{i\epsilon_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta\gamma^5 \psi(-x/2) + \overline{\psi}(-x/2)\gamma^\beta\gamma^5\psi(x/2)\right]$$

- expansion in leading $1/x^2$ singularities is easily done by projection on the light cone $n_\mu \sim q_\mu + \ldots$ and $n_\mu^* \sim P_\mu + \ldots$

with $q_\mu = (q_{1\mu} + q_{2\mu})/2$ and $P_\mu = p_{1\mu} + p_{2\mu}$

$$T_{\mu\nu} \overset{\mathrm{LO}}{=} -g^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} - \frac{e_q^2}{\xi + x - i\epsilon} \right] q(x, \xi, t, \mathcal{Q}^2 | s_1, s_2)$$
$$-i\epsilon^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} + \frac{e_q^2}{\xi + x - i\epsilon} \right] \widetilde{q}(x, \xi, t, \mathcal{Q}^2 | s_1, s_2)$$

GPD nomenclature

$$q(\cdots | s_1, s_2) \; = \; \overline{u}(p_2, s_2) \left[n \cdot \gamma H(\cdots) + \frac{i n^\alpha \sigma_{\alpha\beta\Delta^\beta}}{2M} E(\cdots) \right] u(p_1, s_1)$$
$$\widetilde{q}(\cdots | s_1, s_2) \; = \; \overline{u}(p_2, s_2) \left[n \cdot \gamma\gamma^5 \widetilde{H}(\cdots) + \frac{n \cdot \Delta}{2M} \gamma^5 \widetilde{E}(\cdots) \right] u(p_1, s_1)$$

consequences of *1/Q* truncation and restriction to leading order in pQCD

- DVCS tensor structure depends on the choice of *n*
- scaling variable $\xi \sim x_B/(2-x_B)$ depends on the choice of *n*
- gauge invariance holds only to leading power accuracy
- DVCS tensor structure is not complete

to overcome these problems one can go

- to twist-3 accuracy, yields 4 other GPDs (LT photon helicity flips)
- to NLO, yields 4 gluon transversity GPDs (TT photon helicity flips)
- twist-4 accuracy pushes ambiguity to the $1/Q^4$ level **[Braun, Manashov 12]**
  but yields new parton correlation functions, however, no new structures

Our first proposal for DVCS tensor

embed perturbative results in a physical parameterization

$$\xi \quad \Rightarrow \quad \xi = \frac{x_{\rm B}\left(2\mathcal{Q}^2+t\right)/2}{2-x_{\rm B}+x_{\rm B}\frac{t}{\mathcal{Q}^2}} = \frac{2\mathcal{Q}^2+t}{s-u}$$

NOTE

$$q_1\cdot q_2 = \mathcal{Q}^2+t$$

$$g_{\mu\nu} \quad \Rightarrow \quad \widetilde{g}_{\mu\nu} = \mathcal{P}_{\mu\tau}g^{\tau\sigma}\mathcal{P}_{\sigma\nu},\cdots, \quad \text{with} \;\; \mathcal{P}_{\mu\nu} = g_{\mu\nu} - \frac{q_{1\mu}q_{2\nu}}{q_1\cdot q_2}$$

take your favored tensor structure (partially inspired from DIS tensor)

$$T_{\mu\nu} \;=\; -\widetilde{g}_{\sigma\tau}\frac{q\cdot V_1}{P\cdot q} + \left(\widetilde{P}_\mu\mathcal{P}_{\rho\nu}+\mathcal{P}_{\mu\rho}\widetilde{P}_\nu\right)\frac{V_{2\,\rho}}{P\cdot q} - i\widetilde{\epsilon}_{\sigma\tau q\rho}\frac{A_{1\,\rho}}{P\cdot q} + \widetilde{\tau}^{\perp\;\alpha\beta}_{\mu\nu}G^T_{\alpha\beta}$$

$$V_{2\,\rho} = \xi V_{1\,\rho} - \frac{\xi}{2}\frac{P_\rho}{P\cdot q}q\cdot V_1 + \frac{i}{2}\frac{\epsilon_{\rho\sigma\Delta q}}{P\cdot q}A_{1\,\sigma}$$

$$V_{1\,\rho} \;=\; P_\rho\frac{q\cdot h}{q\cdot P}\mathcal{H} + P_\rho\frac{q\cdot e}{q\cdot P}\mathcal{E} + \Delta^\perp_\rho\frac{q\cdot h}{q\cdot P}\mathcal{H}^3_+ + \Delta^\perp_\rho\frac{q\cdot e}{q\cdot P}\mathcal{E}^3_+ + \widetilde{\Delta}^\perp_\rho\frac{q\cdot\widetilde{h}}{q\cdot P}\widetilde{\mathcal{H}}^3_- + \widetilde{\Delta}^\perp_\rho\frac{q\cdot\widetilde{e}}{q\cdot P}\widetilde{\mathcal{E}}^3_-$$

$$A_{1\,\rho} \;=\; P_\rho\frac{q\cdot\widetilde{h}}{q\cdot P}\widetilde{\mathcal{H}} + P_\rho\frac{q\cdot\widetilde{e}}{q\cdot P}\widetilde{\mathcal{E}} + \Delta^\perp_\rho\frac{q\cdot\widetilde{h}}{q\cdot P}\widetilde{\mathcal{H}}^3_+ + \Delta^\perp_\rho\frac{q\cdot\widetilde{e}}{q\cdot P}\widetilde{\mathcal{E}}^3_+ + \widetilde{\Delta}^\perp_\rho\frac{q\cdot h}{q\cdot P}\mathcal{H}^3_- + \widetilde{\Delta}^\perp_\rho\frac{q\cdot e}{q\cdot P}\mathcal{E}^3_-$$

$$h_\rho \;=\; \bar{u}_2\gamma_\rho u_1\,, \quad e_\rho = \bar{u}_2 i\sigma_{\rho\sigma}\frac{\Delta^\sigma}{2M}u_1\,, \quad \widetilde{h}_\rho = \bar{u}_2\gamma_\rho\gamma_5 u_1\,, \quad \widetilde{e}_\rho = \frac{\Delta_\rho}{2M}\bar{u}_2\gamma_5 u_1$$

$$G^T_{\alpha\beta} \;=\; \frac{\Delta_\alpha}{2M}\bar{u}_2\left\{H_T\frac{q^\gamma}{P\cdot q}i\sigma_{\gamma\beta} + \widetilde{H}_T\frac{\Delta_\beta}{2M^2} + E_T\frac{1}{2M}\left(\frac{\gamma\cdot q}{P\cdot q}\Delta_\beta - \eta\,\gamma_\beta\right) - \widetilde{E}_T\frac{\gamma_\beta}{2M}\right\}u_1$$

Other proposals

- Prange(1958) in terms of Dirac-spinors
- Hearn, Leader (1962) Pauli-spinor representation

e.g., used for generalized polarizabilities proposed by Guichon et al. (1995)

$$\mathcal{N}^{-1}\, T^{VCS}(\lambda = 0) = a^l\, \vec{\varepsilon}\,'^{\star}.\widehat{q} + i\Big[b_1^l\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\times\widehat{q}\,'\ \vec{\sigma}.\vec{e}\,(1) + b_2^l\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\,\vec{\sigma}.\vec{e}\,(2) + b_3^l\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\times\widehat{q}\,'\ \vec{\sigma}.\vec{e}\,(3)\Big]$$

$$\begin{aligned}\mathcal{N}^{-1}\, T^{VCS}(\lambda = \pm 1) &= a^t\, \vec{\varepsilon}\,'^{\star}.\vec{\varepsilon} + a^{tl}\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\,\vec{\varepsilon}.\widehat{q}\,' \\ &+ i\big(b_1^t\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\,\vec{\varepsilon}.\widehat{q}\times\widehat{q}\,' + b_1^{tl}\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\times\widehat{q}\,'\ \vec{\varepsilon}.\widehat{q}\,'\big)\vec{\sigma}.\vec{e}\,(1) \\ &+ i\big(b_2^t\, \vec{\varepsilon}\,'^{\star}.\vec{\varepsilon} + b_2^{tl}\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\,\vec{\varepsilon}.\widehat{q}\,'\big)\vec{\sigma}.\vec{e}\,(2) \\ &+ i\big(b_3^t\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\,\vec{\varepsilon}.\widehat{q}\times\widehat{q}\,' + b_3^{tl}\, \vec{\varepsilon}\,'^{\star}.\widehat{q}\times\widehat{q}\,'\ \vec{\varepsilon}.\widehat{q}\,'\big)\vec{\sigma}.\vec{e}\,(3).\end{aligned}$$

used to express helicity amplitude*s* in terms of 10 (6) multipols

- Tarrach (1975) tensor structure, Dirac representation

free of kinematical singularities

- relating Prange to Tarrach parameterization, e.g., in Drechsel et al. (1997)
- to express Tarrach`s functions in terms of 10 (6) multipols
- used in dispersion relation approach to VCS Drechsel et al. (2002)
- used by us to express CFFs in terms of 10 (6) multipols

Tarrach
(1975)

? minimal

The so-obtained basis of gauge invariant tensors is expected to be minimal and pole free by construction and is the following one :

$$
\begin{aligned}
\mathcal{T}_1 &= k\cdot k'\, T_1 - T_3 \\
\mathcal{T}_2 &= k^2 k'^2\, T_1 + k\cdot k'\, T_2 - \frac{k^2+k'^2}{2}\, T_4 + \frac{k^2-k'^2}{2}\, T_5 \\
\mathcal{T}_3 &= (\ell\cdot K)^2\, T_1 + k\cdot k'\, T_6 - \ell\cdot K\, T_7 \\
\mathcal{T}_4 &= \ell\cdot K\,(k^2+k'^2)\, T_1 - \ell\cdot K\, T_4 - \frac{k^2+k'^2}{2}\, T_7 + \frac{k^2-k'^2}{2}\, T_8 + k\cdot k'\, T_9 \\
\mathcal{T}_5 &= -\ell\cdot K\,(k^2-k'^2)\, T_1 + \ell\cdot K\, T_5 + \frac{k^2-k'^2}{2}\, T_7 - \frac{k^2+k'^2}{2}\, T_8 + k\cdot k'\, T_{10} \\
\mathcal{T}_6 &= \ell\cdot K\, T_2 - \frac{k^2+k'^2}{4}\, T_9 - \frac{k^2-k'^2}{4}\, T_{10} - M\, T_{12} + M\,\frac{k^2+k'^2}{4}\, T_{23} - M\,\frac{k^2-k'^2}{4}\, T_{24} \\
&\quad + \frac{k^2-k'^2}{8}\, T_{29} - \frac{k^2+k'^2}{8}\, T_{30} - \frac{k^2 k'^2}{4}\, T_{33} \\
\mathcal{T}_7 &= 8\, T_{16} - 4\,\ell\cdot K\, T_{21} + \ell\cdot K\, T_{34} \\
\mathcal{T}_8 &= T_{19} + \frac{k^2-k'^2}{2}\, T_{22} - \ell\cdot K\, T_{23} + \frac{k^2+k'^2}{8}\, T_{34} \\
\mathcal{T}_9 &= T_{20} - \frac{k^2+k'^2}{2}\, T_{22} + \ell\cdot K\, T_{24} - \frac{k^2-k'^2}{8}\, T_{34} \\
\mathcal{T}_{10} &= -8\, k\cdot k'\, T_6 + 4\,\ell\cdot K\, T_7 + 4M\, k\cdot k'\, T_{21} - 4M\,\ell\cdot K\, T_{25} - 2\,\ell\cdot K\, T_{32} - 2\,k\cdot k'\,\ell\cdot K\, T_{33} + M\, k\cdot k'\, T_{34} \\
\mathcal{T}_{11} &= T_{18} - k\cdot k'\, T_{22} + \ell\cdot K\, T_{26} \\
\mathcal{T}_{12} &= \ell\cdot K\, T_4 - \frac{k^2-k'^2}{2}\, T_8 - k\cdot k'\, T_9 - M\, T_{14} + M\, k\cdot k'\, T_{23} - M\,\frac{k^2-k'^2}{2}\, T_{26} - \frac{k^2+k'^2}{4}\, T_{32} - k\cdot k'\,\frac{k^2+k'^2}{4}\, T_{33} \\
\mathcal{T}_{13} &= \ell\cdot K\, T_5 - \frac{k^2+k'^2}{2}\, T_8 + k\cdot k'\, T_{10} - M\, T_{15} + M\, k\cdot k'\, T_{24} - M\,\frac{k^2+k'^2}{2}\, T_{26} - \frac{k^2-k'^2}{4}\, T_{32} - k\cdot k'\,\frac{k^2-k'^2}{4}\, T_{33} \\
\mathcal{T}_{14} &= 2\,\ell\cdot K\, T_8 - 2M\, k\cdot k'\, T_{22} + 2M\,\ell\cdot K\, T_{26} - k\cdot k'\, T_{27} + \ell\cdot K\, T_{31} \\
\mathcal{T}_{15} &= -(k^2-k'^2)\, T_7 + (k^2+k'^2)\, T_8 - 2\,k\cdot k'\, T_{10} - 2M\, k\cdot k'\, T_{24} + M\,(k^2-k'^2)\, T_{25} + M\,(k^2+k'^2)\, T_{26} \\
&\quad - k\cdot k'\, T_{29} + \frac{k^2+k'^2}{2}\, T_{31} + \frac{k^2-k'^2}{2}\, T_{32} \\
\mathcal{T}_{16} &= -(k^2+k'^2)\, T_7 + (k^2-k'^2)\, T_8 + 2\,k\cdot k'\, T_9 - 2M\, k\cdot k'\, T_{23} + M\,(k^2+k'^2)\, T_{25} + M\,(k^2-k'^2)\, T_{26} \\
&\quad - k\cdot k'\, T_{30} + \frac{k^2-k'^2}{2}\, T_{31} + \frac{k^2+k'^2}{2}\, T_{32} \\
\mathcal{T}_{17} &= -4\,\ell\cdot K\, T_1 + 2\, T_7 + 4M\, T_{11} - 2M\, T_{25} + T_{32} + k\cdot k'\, T_{33} \\
\mathcal{T}_{18} &= 4\, T_{17} - 4\,\ell\cdot K\, T_{25} + k\cdot k'\, T_{34}
\end{aligned}
\qquad (12)
$$

Our second proposal

(making live simple)

parameterize 3 independent helicity amplitudes in terms of 4 CFFs

(proton at rest)

$$
\begin{aligned}
\mathcal{T}^{\rm VCS}_{ab}(\phi) &= (-1)^{a-1}\varepsilon_2^{\mu *}(b) T_{\mu\nu}\varepsilon_1^{\nu}(a) \\
\mathcal{T}^{\rm VCS}_{ab} &= \mathcal{V}(\mathcal{F}_{ab}) - b\,\mathcal{A}(\mathcal{F}_{ab}) \quad \text{for} \quad a \in \{0,+,-\},\ b \in \{+,-\} \\
\mathcal{V}(\mathcal{F}_{ab}) &= \bar{u}_2 \left(\not{m}\mathcal{H}_{ab} + i\sigma_{\alpha\beta}\frac{m^\alpha \Delta^\beta}{2M}\,\mathcal{E}_{ab} \right) u_1 \\
\mathcal{A}(\mathcal{F}_{ab}) &= \bar{u}_2 \left(\not{m}\gamma_5\,\widetilde{\mathcal{H}}_{ab} + \gamma_5 \frac{m\cdot\Delta}{2M}\,\widetilde{\mathcal{E}}_{ab} \right) u_1
\end{aligned}
$$

- What is the best choice of m^μ (corresponds to light cone vector) ?

  free nucleon Dirac equation is used to reduce choices

  (polarization vectors are given in terms of physical momenta)

  $m^\mu = q^\mu / P.q$ is free of kinematical singularities and respects Bose symmetry

  (except if s=u which appears in the low energy limit)

- kinematical singularities can be cured by switching to ``electric'' CFFs

$$
\mathcal{G}_{ab} = \mathcal{H}_{ab} + \frac{t}{4M^2}\mathcal{E}_{ab} \ \text{ and } \ \widetilde{\mathcal{G}}_{ab} = \widetilde{\mathcal{H}}_{ab} + \frac{t}{4M^2}\widetilde{\mathcal{E}}_{ab} \ \text{ for } \ a \neq b
$$

Constructing a Compton tensor

requirements:

- manifest current conservation and Bose symmetry
- a close match with conventions used in deeply virtual Compton kinematics
- singularity-free kinematical dependence

±1→±1: $$g^{\perp}_{\mu\nu} \to \widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_{1\mu}p_\nu}{p\cdot q} - \frac{q_{2\nu}p_\mu}{p\cdot q} + \frac{q_1\cdot q_2}{p\cdot q}\frac{p_\mu p_\nu}{p\cdot q}$$

$$\varepsilon^{\perp}_{\mu\nu} \to \widetilde{\varepsilon}_{\mu\nu} = \frac{1}{p\cdot q}\left[\varepsilon_{\mu\nu pq} + \frac{p_\mu}{2\,p\cdot q}\varepsilon_{\Delta\nu pq} - \varepsilon_{\mu\Delta pq}\frac{p_\nu}{2\,p\cdot q} + \varepsilon_{\mu\nu\Delta q}\frac{p\cdot p}{2\,p\cdot q}\right]$$

± 1→∓1: $$\tau^{\perp}_{\mu\nu;\rho\sigma}\frac{\Delta^\rho T^\sigma}{M^2} \to \left(g_\mu{}^\alpha - \frac{p_\mu\, q_2^\alpha}{p\cdot q}\right)\left(g_\nu{}^\beta - \frac{p_\nu\, q_1^\beta}{p\cdot q}\right)\tau^{\perp}_{\alpha\beta;\rho\sigma}\frac{\Delta^\rho T^\sigma}{M^2}$$

0 →±1: $$\left(q_{2\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(g_{\nu\rho} - \frac{p_\nu\, q_1^\rho}{p\cdot q}\right) \quad \text{and} \quad \left(q_{1\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\left(g_{\mu\rho} - \frac{p_\mu\, q_2^\rho}{p\cdot q}\right)$$

0 →0: $$\left(q_{2\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(q_{1\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)$$

$$T_{\mu\nu} = -\widetilde{g}_{\mu\nu}\frac{q\cdot V_{\rm T}}{p\cdot q} + i\widetilde{\varepsilon}_{\mu\nu}\frac{q\cdot A_{\rm T}}{p\cdot q} + \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\frac{q\cdot V_{\rm L}}{p\cdot q}$$
$$+ \left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\left(g_{\mu\rho} - \frac{p_\mu\, q_{2\,\rho}}{p\cdot q}\right)\left[\frac{V^\rho_{\rm LT}}{p\cdot q} + \frac{i\epsilon^\rho{}_{qp\sigma}}{p\cdot q}\frac{A^\sigma_{\rm LT}}{p\cdot q}\right]$$
$$+ \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(g_{\nu\rho} - \frac{p_\nu\, q_{1\,\rho}}{p\cdot q}\right)\left[\frac{V^\rho_{\rm TL}}{p\cdot q} + \frac{i\epsilon^\rho{}_{qp\sigma}}{p\cdot q}\frac{A^\sigma_{\rm TL}}{p\cdot q}\right]$$
$$+ \left(g_\mu{}^\rho - \frac{p_\mu\, q_2^\rho}{p\cdot q}\right)\left(g_\nu{}^\sigma - \frac{p_\nu\, q_1^\sigma}{p\cdot q}\right)\left[\frac{\Delta_\rho\Delta_\sigma + \widetilde{\Delta}^\perp_\rho\widetilde{\Delta}^\perp_\sigma}{2M^2}\frac{q\cdot V_{\rm TT}}{p\cdot q} + \frac{\Delta_\rho\widetilde{\Delta}^\perp_\sigma + \widetilde{\Delta}^\perp_\rho\Delta_\sigma}{2M^2}\frac{q\cdot A_{\rm TT}}{p\cdot q}\right]$$

with $\Delta^\perp_\sigma = \Delta_\sigma - \frac{\Delta\cdot q}{q\cdot P}P_\sigma$ and $\widetilde{\Delta}^\perp_\sigma = \varepsilon_{\sigma\Delta pq}/p\cdot q$

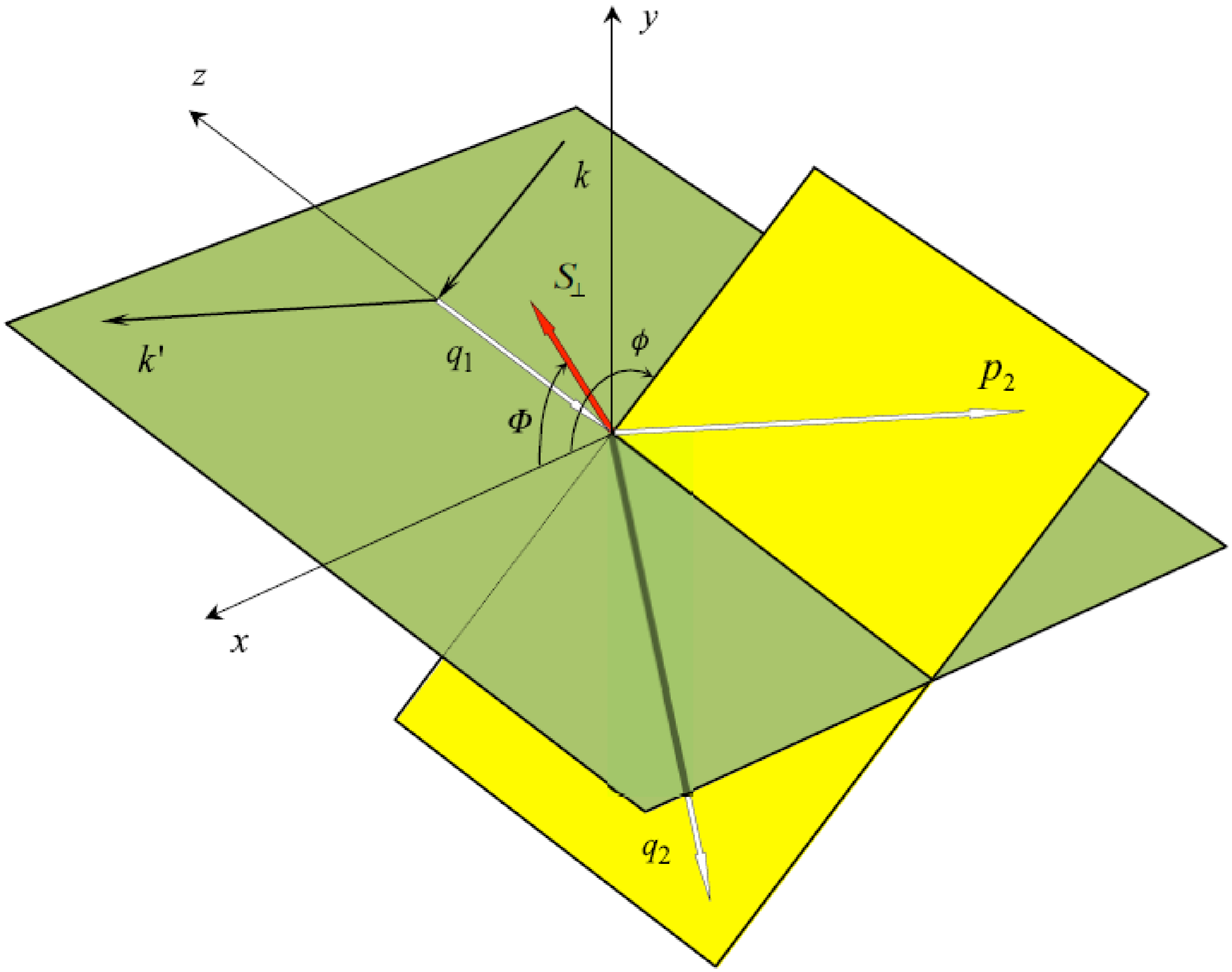
relations of these CFFs to helicity dependent CFFs are easily calculated:

$$\mathcal{F}_{+b} = \left[\frac{1+b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}} + \frac{(1-x_{\rm B})x_{\rm B}^2(4M^2-t)\left(1+\frac{t}{\mathcal{Q}^2}\right)}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\right]\mathcal{F}_{\rm T}$$
$$+\frac{1-b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}}\frac{\widetilde{K}^2}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm TT} + \frac{2x_{\rm B}\widetilde{K}^2}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm LT}$$

$$\mathcal{F}_{0+} = \frac{\sqrt{2}\,\widetilde{K}}{\sqrt{1+\epsilon^2}\mathcal{Q}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\left\{\left[1+\frac{2x_{\rm B}^2\left(4M^2-t\right)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm LT}\right.$$
$$\left. + x_{\rm B}\left[1+\frac{2x_{\rm B}(4M^2-t)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm T} + x_{\rm B}\left[2-\frac{4M^2-t}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm TT}\right\}$$

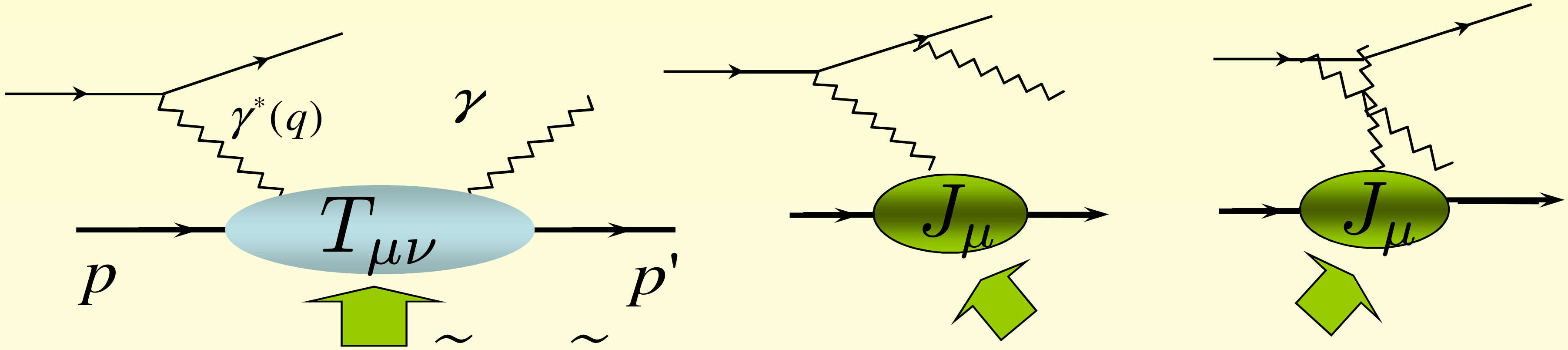
Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

$$\frac{d\sigma}{dx_{\rm B}d\mathcal{Q}^2dtd\phi d\varphi} = \frac{\alpha^3 x_{\rm B} y^2}{16\,\pi^2\,\mathcal{Q}^4}\frac{1}{\sqrt{1+\epsilon^2}}\left|\frac{\mathcal{T}}{e^3}\right|^2, \quad \epsilon \equiv \frac{2Mx_{\rm B}}{\mathcal{Q}}$$



$$\begin{aligned} x_{\rm B} &= \frac{\mathcal{Q}^2}{2p_1\cdot q_1} \approx \frac{2\xi}{1+\xi} \\ y &= \frac{p_1\cdot q_1}{p_1\cdot k} \\ t &= \Delta^2 = (p_2-p_1)^2 \\ \mathcal{Q}^2 &= -q_1^2, \\ \phi &= \phi_N - \phi_e \\ \varphi &= \Phi - \phi_N \end{aligned}$$

interference of *DVCS* and *Bethe-Heitler* processes



12 CFFs $\mathcal{H}_{ab}, \mathcal{E}_{ab}, \widetilde{\mathcal{H}}_{ab}, \widetilde{\mathcal{E}}_{ab}$

elastic form factors F_1, F_2

❑ VCS amplitude square

$$|\mathcal{T}^{\rm VCS}|^2 = \frac{1}{\mathcal{Q}^2} \sum_{a=-,0,+} \sum_{b=-,0,+} \mathcal{L}_{ab}(\lambda,\phi) \mathcal{W}_{ab}$$

$$\mathcal{W}_{ab} = \mathcal{T}^{\rm VCS}_{a+} \left(\mathcal{T}^{\rm VCS}_{b+}\right)^* + \mathcal{T}^{\rm VCS}_{a-} \left(\mathcal{T}^{\rm VCS}_{b-}\right)^*$$

common leptonic helicity amplitudes

$$\mathcal{L}_{++}(\lambda) = \frac{1}{y^2(1+\epsilon^2)}\left(2-2y+y^2+\frac{\epsilon^2}{2}y^2\right) - \frac{2-y}{\sqrt{1+\epsilon^2}y}\lambda$$

$$\mathcal{L}_{00} = \frac{4}{y^2(1+\epsilon^2)}\left(1-y-\frac{\epsilon^2}{4}y^2\right)$$

$$\mathcal{L}_{0+}(\lambda,\phi) = \frac{2-y-\lambda y\sqrt{1+\epsilon^2}}{y^2(1+\epsilon^2)}\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}\, e^{-i\phi}$$

$$\mathcal{L}_{+-}(\phi) = \frac{2}{y^2(1+\epsilon^2)}\left(1-y-\frac{\epsilon^2}{4}y^2\right) e^{i2\phi}$$

CFFs are contained in 4 bilinear combinations

$$\sum_{S'} \Big[\mathcal{V}(\mathcal{F}) + \mathcal{A}(\mathcal{F})\Big]\Big[\mathcal{V}^\dagger(\mathcal{F}^*) + \mathcal{A}^\dagger(\mathcal{F}^*)\Big] = \Bigg[\mathcal{C}^{\rm VCS}_{\rm unp} + \Lambda\cos(\theta)\frac{1}{\sqrt{1+\epsilon^2}}\mathcal{C}^{\rm VCS}_{\rm LP}$$

$$+\Lambda\sin(\theta)\sin(\varphi)\frac{i\widetilde{K}}{2M}\mathcal{C}^{\rm VCS}_{\rm TP-} + \Lambda\sin(\theta)\cos(\varphi)\frac{\widetilde{K}}{2M\sqrt{1+\epsilon^2}}\mathcal{C}^{\rm VCS}_{\rm TP+}\Bigg](\mathcal{F},\mathcal{F}^*)$$

harmonic expansion of VCS square

$$|\mathcal{T}^{\rm VCS}(\phi,\varphi)|^2 = \frac{e^6}{y^2 \mathcal{Q}^2}\left\{ c_0^{\rm VCS}(\varphi) + \sum_{n=1}^{2}\left[c_n^{\rm VCS}(\varphi)\,\cos(n\phi) + s_n^{\rm VCS}(\varphi)\,\sin(n\phi)\right]\right\}$$

harmonics for unpolarized target

$$c_{0,\rm unp}^{\rm VCS} = 2\frac{2-2y+y^2+\frac{\epsilon^2}{2}y^2}{1+\epsilon^2}\,\mathcal{C}_{\rm unp}^{\rm VCS}(\mathcal{F}_{++},\mathcal{F}_{++}^{*}|\mathcal{F}_{-+},\mathcal{F}_{-+}^{*}) + 8\frac{1-y-\frac{\epsilon^2}{4}y^2}{1+\epsilon^2}\,\mathcal{C}_{\rm unp}^{\rm VCS}(\mathcal{F}_{0+},\mathcal{F}_{0+}^{*}),$$

$$\left\{\begin{array}{c} c_{1,\rm unp}^{\rm VCS} \\ s_{1,\rm unp}^{\rm VCS}\end{array}\right\} = \frac{4\sqrt{2}\sqrt{1-y-\frac{\epsilon^2}{4}y^2}}{1+\epsilon^2}\left\{\begin{array}{c} 2-y \\ -\lambda y\sqrt{1+\epsilon^2}\end{array}\right\}\left\{\begin{array}{c}\Re{\rm e} \\ \Im{\rm m}\end{array}\right\}\mathcal{C}_{\rm unp}^{\rm VCS}\left(\mathcal{F}_{0+}|\mathcal{F}_{++}^{*},\mathcal{F}_{-+}^{*}\right),$$

$$c_{2,\rm unp}^{\rm VCS} = 8\frac{1-y-\frac{\epsilon^2}{4}y^2}{1+\epsilon^2}\,\Re{\rm e}\,\mathcal{C}_{\rm unp}^{\rm VCS}\left(\mathcal{F}_{-+},\mathcal{F}_{++}^{*}\right)$$

in terms of bilinear CFF combinations

$$\mathcal{C}_{\rm unp}^{\rm VCS} = \frac{4(1-x_{\rm B})\left(1+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}{\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\left[\mathcal{H}\mathcal{H}^* + \widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^*\right] + \frac{\left(2+\frac{t}{\mathcal{Q}^2}\right)\epsilon^2}{\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\widetilde{\mathcal{H}}\widetilde{\mathcal{H}}^* - \frac{t}{4M^2}\mathcal{E}\mathcal{E}^*$$

$$-\frac{x_{\rm B}^2}{\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\left\{\left(1+\frac{t}{\mathcal{Q}^2}\right)^2\left[\mathcal{H}\mathcal{E}^* + \mathcal{E}\mathcal{H}^* + \mathcal{E}\mathcal{E}^*\right] + \widetilde{\mathcal{H}}\widetilde{\mathcal{E}}^* + \widetilde{\mathcal{E}}\widetilde{\mathcal{H}}^* + \frac{t}{4M^2}\widetilde{\mathcal{E}}\widetilde{\mathcal{E}}^*\right\}$$

+ 3 other sets of harmonics for longitudinally and transversally polarized proton with 3 different bilinear combinations

❑ interference term

$$\mathcal{I} = \frac{\pm e^6}{t\,\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{a=-,0,+} \sum_{b=-,+} \sum_{S'} \left\{ \mathcal{L}^{\rho}_{ab}(\lambda,\phi)\mathcal{T}_{ab}J^{\dagger}_{\rho} + \left(\mathcal{L}^{\rho}_{ab}(\lambda,\phi)\mathcal{T}_{ab}J^{\dagger}_{\rho}\right)^{*} \right\}$$

additional ϕ-dependence stems from (scaled) BH propagators
(up to second even harmonics)

leptonic part is straightforward to treat (a bit cumbersome)

hadronic part yields now 4 linear combinations of CFFs + their addenda

$$\begin{aligned}
\sum_{S'} \mathcal{V}(\mathcal{F})J^{\dagger}_{\rho} \;=\;& p_\rho \left[\mathcal{C}^{\mathcal{I}}_{\rm unp}(\mathcal{F}) - \mathcal{C}^{\mathcal{I},A}_{\rm unp}\right](\mathcal{F}) + 2q_\rho \frac{t}{\mathcal{Q}^2}\mathcal{C}^{\mathcal{I},V}_{\rm unp}(\mathcal{F}) \\
& - p_\rho \frac{\Lambda \sin(\theta)\sin(\varphi)M}{i\widetilde{K}} \left[\mathcal{C}^{\mathcal{I}}_{\rm TP-} - \mathcal{C}^{\mathcal{I},A}_{\rm TP-}\right](\mathcal{F}) \; - 2q_\rho \frac{t}{\mathcal{Q}^2} \frac{\Lambda \sin(\theta)\sin(\varphi)M}{i\widetilde{K}} \mathcal{C}^{\mathcal{I},V}_{\rm TP-}(\mathcal{F}) \\
& + \frac{2i\varepsilon_{pq\Delta\rho}}{\mathcal{Q}^2} \left[\frac{\Lambda\cos(\theta)}{\sqrt{1+\epsilon^2}}\mathcal{C}^{\mathcal{I},V}_{\rm LP} + \frac{\Lambda\sin(\theta)\cos(\varphi)M}{\sqrt{1+\epsilon^2}\widetilde{K}}\,\mathcal{C}^{\mathcal{I},V}_{\rm TP+}\right](\mathcal{F}) \\
\sum_{S'} \mathcal{A}(\mathcal{F})J^{\dagger}_{\rho} \;=\;& p_\rho \frac{\Lambda\cos(\theta)}{\sqrt{1+\epsilon^2}} \left[\mathcal{C}^{\mathcal{I}}_{\rm LP} - \mathcal{C}^{\mathcal{I},V}_{\rm LP}\right](\mathcal{F}) + 2q_\rho \frac{t}{\mathcal{Q}^2}\, \frac{\Lambda\cos(\theta)}{\sqrt{1+\epsilon^2}}\,\mathcal{C}^{\mathcal{I},A}_{\rm LP}(\mathcal{F}) \\
& + p_\rho \frac{\Lambda\sin(\theta)\cos(\varphi)M}{\sqrt{1+\epsilon^2}\widetilde{K}} \left[\mathcal{C}^{\mathcal{I}}_{\rm TP+} - \mathcal{C}^{\mathcal{I},V}_{\rm TP+}\right](\mathcal{F}) + 2q_\rho \frac{t}{\mathcal{Q}^2} \frac{\Lambda\sin(\theta)\cos(\varphi)M}{\sqrt{1+\epsilon^2}\widetilde{K}}\mathcal{C}^{\mathcal{I},A}_{\rm TP+}(\mathcal{F}) \\
& + \frac{2i\varepsilon_{pq\Delta\rho}}{\mathcal{Q}^2} \left[\mathcal{C}^{\mathcal{I},A}_{\rm unp} - \frac{\Lambda\sin(\theta)\sin(\varphi)M}{i\widetilde{K}}\,\mathcal{C}^{\mathcal{I},A}_{\rm TP-}\right](\mathcal{F}).
\end{aligned}$$

harmonic expansion of interference term

$$\mathcal{I}(\phi,\varphi) = \frac{\pm e^6}{x_{\mathrm{B}} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left[\sum_{n=0}^{3} c^{\mathcal{I}}_{n,\mathrm{S}}(\varphi)\,\cos(n\phi) + \sum_{n=1}^{3} s^{\mathcal{I}}_{n,\mathrm{S}}(\varphi)\,\sin(n\phi)\right]$$

$$c^{\mathcal{I}}_{0,\mathrm{unp}} = C_{++}(0)\,\Re\mathrm{e}\,\mathcal{C}^{\mathcal{I}}_{++,\mathrm{unp}}\,(0|\mathcal{F}_{++}) + \{{\scriptstyle ++} \to {\scriptstyle 0+}\} + \{{\scriptstyle ++} \to {\scriptstyle -+}\}$$

$$\left\{ \begin{array}{c} c^{\mathcal{I}}_1 \\ s^{\mathcal{I}}_1 \end{array} \right\}_{\mathrm{unp}} = \left\{ \begin{array}{c} C_{++}(1) \\ \lambda\, S_{++}(1) \end{array} \right\} \left\{ \begin{array}{c} \Re\mathrm{e} \\ \Im\mathrm{m} \end{array} \right\} \left\{ \begin{array}{c} \mathcal{C}^{\mathcal{I}}_{++}\,(1|\mathcal{F}_{++}) \\ \mathcal{S}^{\mathcal{I}}_{++}\,(1|\mathcal{F}_{++}) \end{array} \right\}_{\mathrm{unp}} + \{{\scriptstyle ++} \to {\scriptstyle 0+}\} + \{{\scriptstyle ++} \to {\scriptstyle -+}\}$$

$$\left\{ \begin{array}{c} c^{\mathcal{I}}_2 \\ s^{\mathcal{I}}_2 \end{array} \right\}_{\mathrm{unp}} = \left\{ \begin{array}{c} C_{0+}(2) \\ \lambda\, S_{0+}(2) \end{array} \right\} \left\{ \begin{array}{c} \Re\mathrm{e} \\ \Im\mathrm{m} \end{array} \right\} \left\{ \begin{array}{c} \mathcal{C}^{\mathcal{I}}_{0+}\,(2|\mathcal{F}_{0+}) \\ \mathcal{S}^{\mathcal{I}}_{0+}\,(2|\mathcal{F}_{0+}) \end{array} \right\}_{\mathrm{unp}} + \{{\scriptstyle 0+} \to {\scriptstyle ++}\} + \{{\scriptstyle 0+} \to {\scriptstyle -+}\}$$

$$c^{\mathcal{I}}_{3,\mathrm{unp}} = C_{-+}(3)\,\Re\mathrm{e}\,\mathcal{C}^{\mathcal{I}}_{-+,\mathrm{unp}}\,(3|\mathcal{F}_{-+}) + \{{\scriptstyle -+} \to {\scriptstyle ++}\} + \{{\scriptstyle -+} \to {\scriptstyle 0+}\}$$

$$\mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}) = F_1\mathcal{H} - \frac{t}{4M^2}F_2\mathcal{E} + \frac{x_{\mathrm{B}}}{2 - x_{\mathrm{B}} + \frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}}(F_1 + F_2)\widetilde{\mathcal{H}}$$

$$\mathcal{C}^{\mathcal{I},V}_{\mathrm{unp}}(\mathcal{F}) = \frac{x_{\mathrm{B}}}{2 - x_{\mathrm{B}} + \frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}}(F_1 + F_2)(\mathcal{H} + \mathcal{E})$$

$$\mathcal{C}^{\mathcal{I},A}_{\mathrm{unp}}(\mathcal{F}) = \frac{x_{\mathrm{B}}}{2 - x_{\mathrm{B}} + \frac{x_{\mathrm{B}}t}{\mathcal{Q}^2}}(F_1 + F_2)\widetilde{\mathcal{H}}$$

+ 3 sets for polarized CFF combinations + 12 pages of leptonic coefficients

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) + s_n^{\rm BH} \sin(\phi) \right\}$$

exactly known
(LO, QED)

$$|\mathcal{T}_{\rm VCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm VCS} + \sum_{n=1}^{2} \left[c_n^{\rm VCS} \cos(n\phi) + s_n^{\rm VCS} \sin(n\phi) \right] \right\}$$

harmonics
1:1
helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 t \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm INT} + \sum_{n=1}^{3} \left[c_n^{\rm INT} \cos(n\phi) + s_n^{\rm INT} \sin(n\phi) \right] \right\}$$

harmonics
helicity ampl.

- both kinds of leptons allows to access the interference term and BH² +DVCS²
- in principle, one can extract all 12 CFFs (24 functions)

sector		harmonics in $\mathcal{I}$				extraction	P of	$\Delta_\perp^l$ behavior	
twist	$\mathcal{C}$'s	unp	LP	TP$_x$	TP$_y$	of CFFs	$\mathcal{Q}^{-P}$	unp, LP	TP
two	$\Re\mathrm{e}\mathcal{C}(\mathcal{F}),\ \Delta\mathcal{C}(\mathcal{F})$	c_1, c_0	c_1, c_0	c_1, c_0	s_1, -	over compl.	1,2	1,0	0,1
	$\Im\mathrm{m}\mathcal{C}(\mathcal{F}),\ \Delta\mathcal{C}(\mathcal{F})$	s_1, -	s_1, -	s_1, -	c_1, c_0	over compl.	1,2	1,0	0,1
three	$\Re\mathrm{e}\mathcal{C}(\mathcal{F}^{\rm eff})$	c_2	c_2	c_2	s_2	complete	2	2	1
	$\Im\mathrm{m}\mathcal{C}(\mathcal{F}^{\rm eff})$	s_2	s_2	s_2	c_2	complete	2	2	1
two	$\Re\mathrm{e}\mathcal{C}_T(\mathcal{F}_T)$	c_3	-	-	-	$1\times\Re\mathrm{e}$ of 4	1	3	2
	$\Im\mathrm{m}\mathcal{C}_T(\mathcal{F}_T)$	-	s_3	s_3	c_3	$3\times\Im\mathrm{m}$ of 4	1	3	2

Harmonic analysis and Rosenbluth separation

naively, one would think that in the charge odd sector one should take

$$\left\{ \begin{array}{c} c_n^{\rm INT} \\ s_n^{\rm INT} \end{array} \right\} \propto \frac{1}{\pi} \int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi|y)\mathcal{P}_2(\phi|y) \left\{ \begin{array}{c} \cos(n\phi) \\ \sin(n\phi) \end{array} \right\} \left[\frac{d\sigma^+}{d\phi \cdots} - \frac{d\sigma^-}{d\phi \cdots} \right]$$

however, Rosenbluth separation looks cumbersome

? how looks the *y*-dependence if one takes common Fourier analysis
(important in near future for JLAB@12 experiments)

$$\left\{ \begin{array}{c} c_n^{\rm BH}(y) + c_n^{\rm INT}(y) + c_n^{\rm VCS}(y) \\ s_n^{\rm BH}(y) + s_n^{\rm INT}(y) + s_n^{\rm VCS}(y) \end{array} \right\} \propto \frac{1}{\pi} \int_{-\pi}^{\pi} d\phi \left\{ \begin{array}{c} \cos(n\phi) \\ \sin(n\phi) \end{array} \right\} \frac{d\sigma^-}{d\phi \cdots}$$

n=0 case
(Hand convention)

transverse photon asymmetry

ratio of longitudinal to transverse polarized photon fluxes

$$\frac{d\sigma^{\rm TOT}}{dt} = \frac{y^2 \left[\frac{d\sigma_{\rm T}^{\rm BH}}{dt} + \varepsilon(y) \frac{d\sigma_{\rm L}^{\rm BH}}{dt} \right]}{\left(1 - y \frac{(1-x_{\rm B})t}{\mathcal{Q}^2+t}\right)\left(\frac{\mathcal{Q}^2+t}{\mathcal{Q}^2+x_{\rm B}t} - y\right)} \pm \frac{y\left(1-\frac{y}{2}\right)\sqrt{1+\epsilon^2}}{1-y+\frac{y^2}{2}+\frac{\epsilon^2 y^2}{4}} \frac{d\sigma_{\rm T}^{\rm INT}}{dt} + \frac{d\sigma_{\rm T}^{\rm DVCS}}{dt} + \varepsilon(y) \frac{d\sigma_{\rm L}^{\rm VCS}}{dt}\,.$$

stems from BH propagators   *BH-propagators dies out*

Real photon limit and low energy limit

setting $x_{\mathrm{B}} = \dfrac{\mathcal{Q}^2}{s + \mathcal{Q}^2 - M^2}$ with $s = (q_1 + p_1)^2$

taking real photon limit $Q^2 \to 0$ of VCS cross section for a point particle yields Klein-Nishina formula (here generalized for polarized proton)

$$\frac{d^2\sigma}{d\cos(\theta_{\gamma\gamma})d\varphi} = \frac{R^2}{2}\left(\frac{\omega'}{\omega}\right)^2 \Big[\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2(\theta_{\gamma\gamma}) + \lambda\Lambda\left(\frac{\omega}{\omega'} - \frac{\omega'}{\omega}\right)\cos(\theta_{\gamma\gamma})\cos(\theta) - \lambda\Lambda\left(1 - \frac{\omega'}{\omega}\right)\sin(\theta_{\gamma\gamma})\sin(\theta)\cos(\varphi)\Big]$$

peculiarities of low energy limit in VCS (center-of-mass frame)

$$q_1 = (\sqrt{\omega'^2 + M^2} + \omega' - \sqrt{\bar{q}^2 + M^2}, 0, 0, \bar{q}) \quad q_2 = (\omega', \omega'\sin\vartheta, 0, \omega'\cos\vartheta)$$

$$p_1 = (\sqrt{\bar{q}^2 + M^2}, 0, 0, -\bar{q}) \quad p_2 = (\sqrt{\omega'^2 + M^2}, -\omega'\sin\vartheta, 0, -\omega'\cos\vartheta)$$

$$\lim_{\omega'\to 0}\mathcal{F}_{ab} = \left[\frac{1}{\omega'}\mathcal{F}_{ab}^{\mathrm{Born},-1} + \mathcal{F}_{ab}^{\mathrm{Born},0} + \omega'\mathcal{F}_{ab}^{\mathrm{Born},1}\right] + \omega'\mathcal{F}_{ab}^{\mathrm{non-Born},1}(\mathrm{6multipols}) + \omega'^2$$

NOTE

subtraction of singularities is done in experiment by Monte-Carlo simulations

electromagnetic form factors of Born term depend on *ω'*

DVCS data and perspectives

existing data

including longitudinal and transverse polarized proton data

new data

HERMES (recoil detector data)

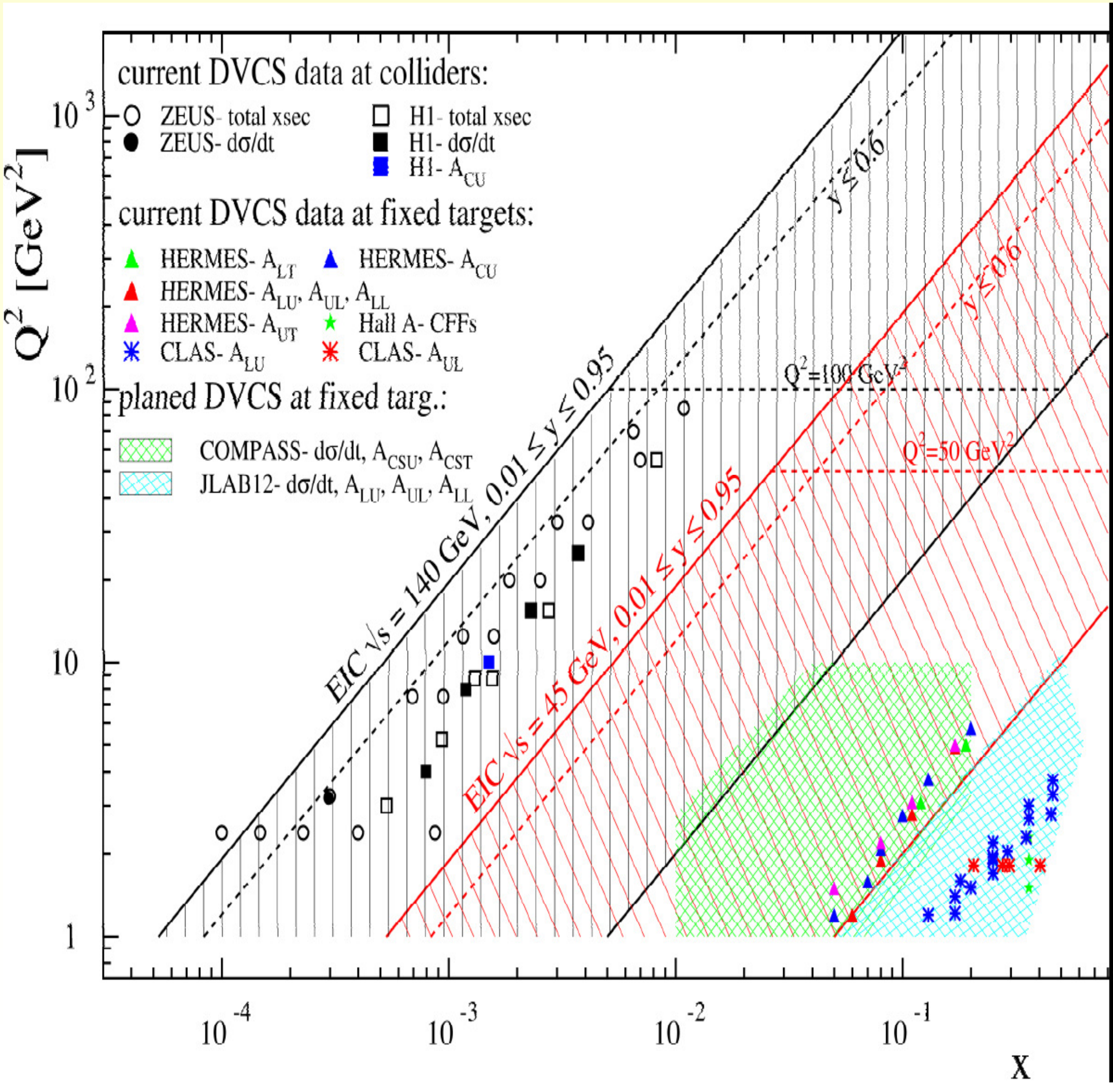
JLAB (longitudinal TSA, cross sections)

planned

COMPASS II, JLAB 12

proposed

EIC



DVCS HERMES data to CFFs

- **?** 1:1 map of charge odd asymmetries (interference term) to CFFs

toy example DVCS off a scalar target

- for the first step we use twist two dominance hypothesis (neglecting twist-three and transversity associated CFFs)

• linearized set of equations (approximately valid)

$$A_{\mathrm{LU,I}}^{\sin(1\phi)} \approx N c_{\mathfrak{Im}}^{-1} \mathcal{H}^{\mathfrak{Im}} \quad \text{and} \quad A_{\mathrm{C}}^{\cos(1\phi)} \approx N c_{\mathfrak{Re}}^{-1} \mathcal{H}^{\mathfrak{Re}}$$

• normalization *N* is bilinear in CFFs

$$0 \lesssim N(\boldsymbol{A}) \approx \frac{1}{1+\frac{k}{4}|\mathcal{H}|^2} \approx \frac{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma_{\mathrm{BH}}(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\left[d\sigma_{\mathrm{BH}}(\phi) + d\sigma_{\mathrm{DVCS}}(\phi)\right]} \lesssim 1$$

• cubic equation for *N* with two non-trivial solutions

$$N(\boldsymbol{A}) \approx \frac{1}{2}\left(1 \pm \sqrt{1 - k\, c_{\mathfrak{Im}}^2 \left(A_{\mathrm{LU,I}}^{\sin(1\phi)}\right)^2 - k\, c_{\mathfrak{Re}}^2 \left(A_{\mathrm{C}}^{\cos(1\phi)}\right)^2}\right)$$

+ BH regime
- DVCS regime

• standard error propagation

(**NOTE:** that the philosophy of CFF extraction has been questioned)

- mathematical generalization to nucleon case is straightforward
- HERMES provided an *almost* complete measurement

- having a look to the twist-two sector

$$\boldsymbol{\mathcal{F}}^{\Im\mathfrak{m}} = \Im\mathfrak{m}\begin{pmatrix}\mathcal{H}\\ \widetilde{\mathcal{H}}\\ \mathcal{E}\\ \hat{\mathcal{E}}\end{pmatrix} \quad \text{and} \quad \boldsymbol{\mathcal{F}}^{\Re\mathfrak{e}} = \Re\mathfrak{e}\begin{pmatrix}\mathcal{H}\\ \widetilde{\mathcal{H}}\\ \mathcal{E}\\ \hat{\mathcal{E}}\end{pmatrix}, \quad \text{where } \hat{\mathcal{E}} = \frac{x_{\rm B}}{2-x_{\rm B}}\widetilde{\mathcal{E}}$$

- rotate data $A^{\sin(1\phi)}_{\rm UL,+} \to\approx A^{\sin(1\phi)}_{\rm UL,I}$, $A^{\cos(1\phi)}_{\rm LL,+} \to\approx A^{\cos(1\phi)}_{\rm LL,I}$, $A^{\cos(0\phi)}_{\rm LL,+} \to\approx A^{\cos(0\phi)}_{\rm LL,I} + A^{\cos(0\phi)}_{\rm LL,DVCS}$

$$\boldsymbol{A}^{\sin} = \begin{pmatrix} A^{\sin(1\phi)}_{\rm LU,I}\\ A^{\sin(1\phi)}_{\rm UL,I}\\ A^{\sin(\varphi)\cos(1\phi)}_{\rm UT,I}\\ A^{\cos(\varphi)\sin(1\phi)}_{\rm UT,I}\end{pmatrix} \quad \text{and} \quad \boldsymbol{A}^{\cos} = \begin{pmatrix} A^{\cos(1\phi)}_{\rm C}\\ A^{\cos(1\phi)}_{\rm LL,I}\\ A^{\sin(\varphi)\cos(0\phi)}_{\rm UT,DVCS}\\ A^{\cos(0\phi)}_{\rm LL,I} + A^{\cos(0\phi)}_{\rm LL,DVCS}\end{pmatrix}$$

6 x linear constraints

2 x quadratic constraints

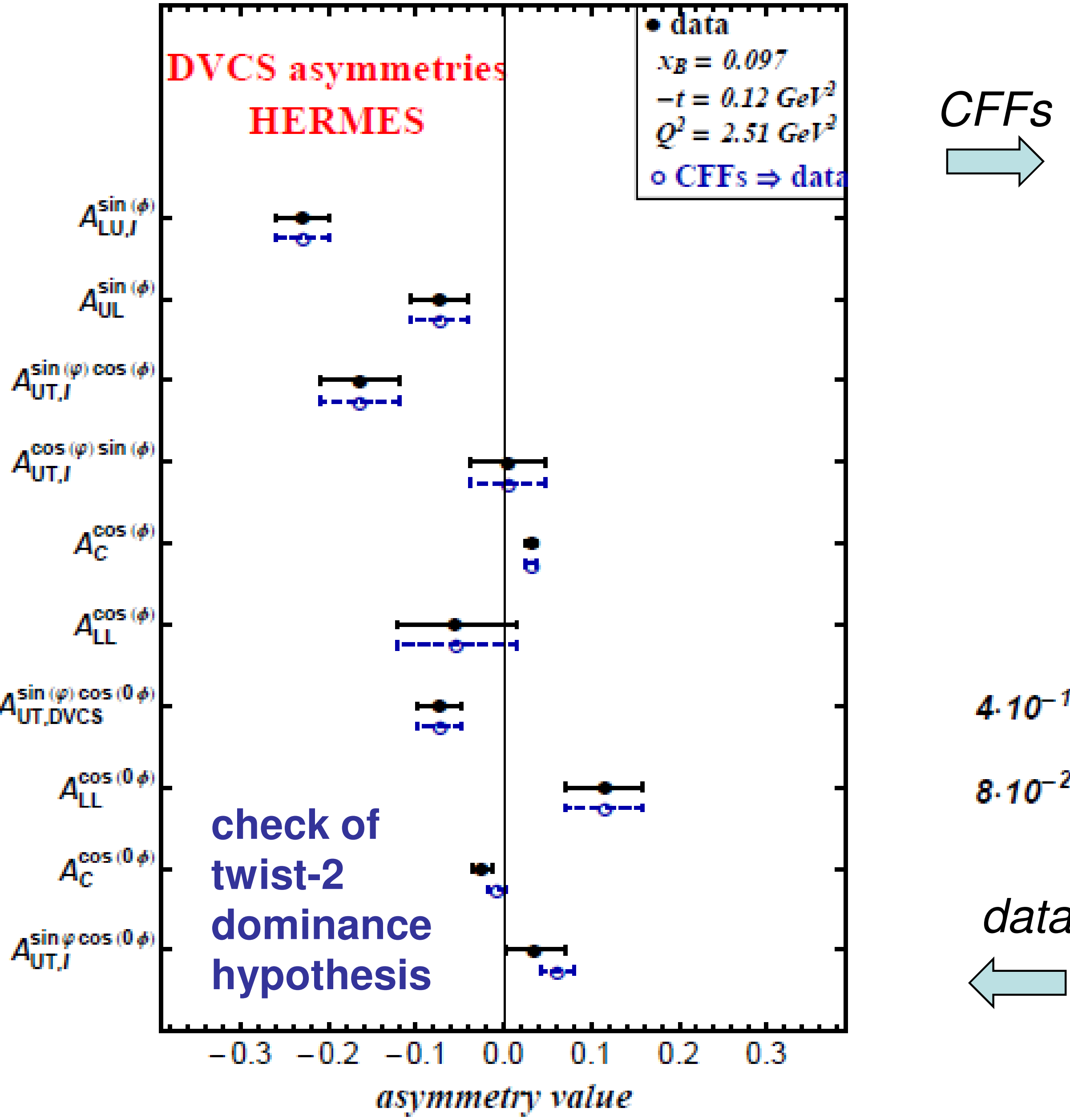
- non-linear solution may be written as

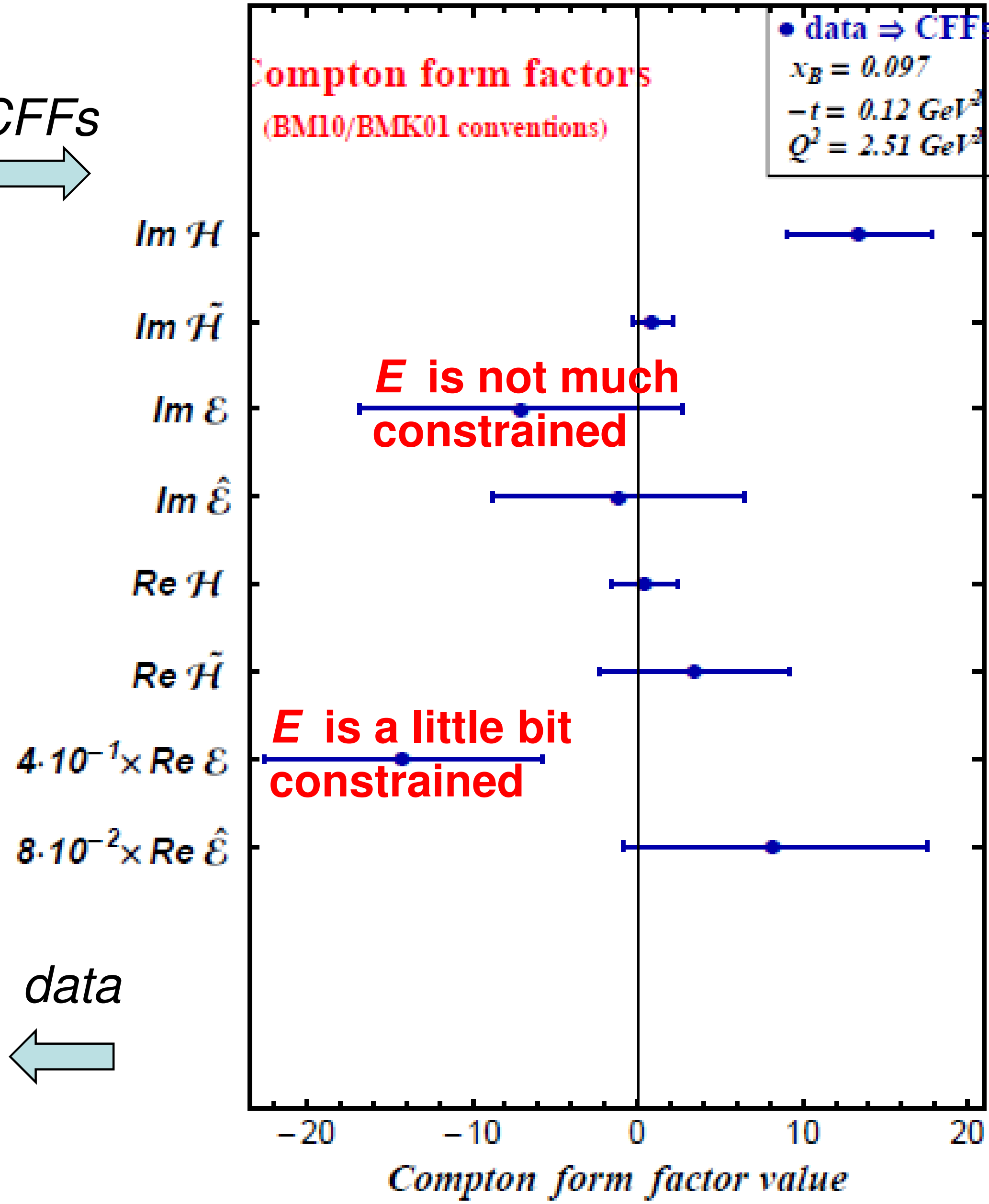
$$\begin{pmatrix}\Im\mathfrak{m}\,\mathcal{F}\\ \Re\mathfrak{e}\,\mathcal{F}\end{pmatrix} = \frac{1}{N(\boldsymbol{A})}\begin{pmatrix}\mathbf{c}_{\Im\mathfrak{m}} & \mathbf{0}_{4\times4}\\ \mathbf{0}_{4\times4} & \mathbf{c}_{\Re\mathfrak{e}}(\boldsymbol{A}|N(\boldsymbol{A}))\end{pmatrix}\cdot\begin{pmatrix}\boldsymbol{A}^{\sin}\\ \boldsymbol{A}^{\cos}\end{pmatrix}$$

imaginary parts needed to evaluate real parts

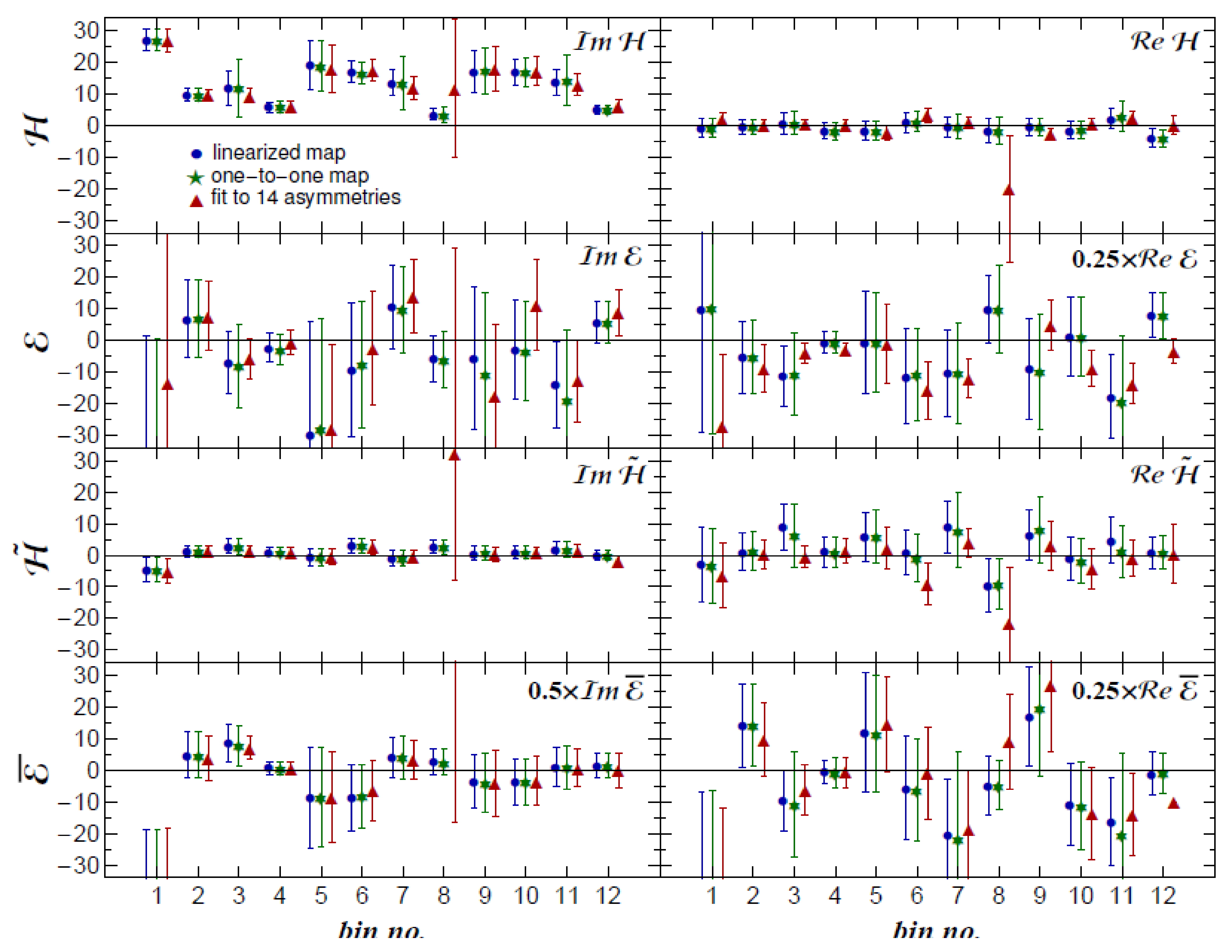
$$\mathrm{cov}(\boldsymbol{\mathcal{F}}) = \left[\frac{\partial\boldsymbol{\mathcal{F}}}{\partial\boldsymbol{A}}\right]\cdot\mathrm{cov}\left(\boldsymbol{A}\right)\cdot\left[\frac{\partial\boldsymbol{\mathcal{F}}}{\partial\boldsymbol{A}}\right]^{\mathsf{T}}$$

DVCS to CFF map for $\frac{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma_{\mathrm{BH}}(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\left[d\sigma_{\mathrm{BH}}(\phi) + d\sigma_{\mathrm{DVCS}}(\phi)\right]} \approx 0.84$



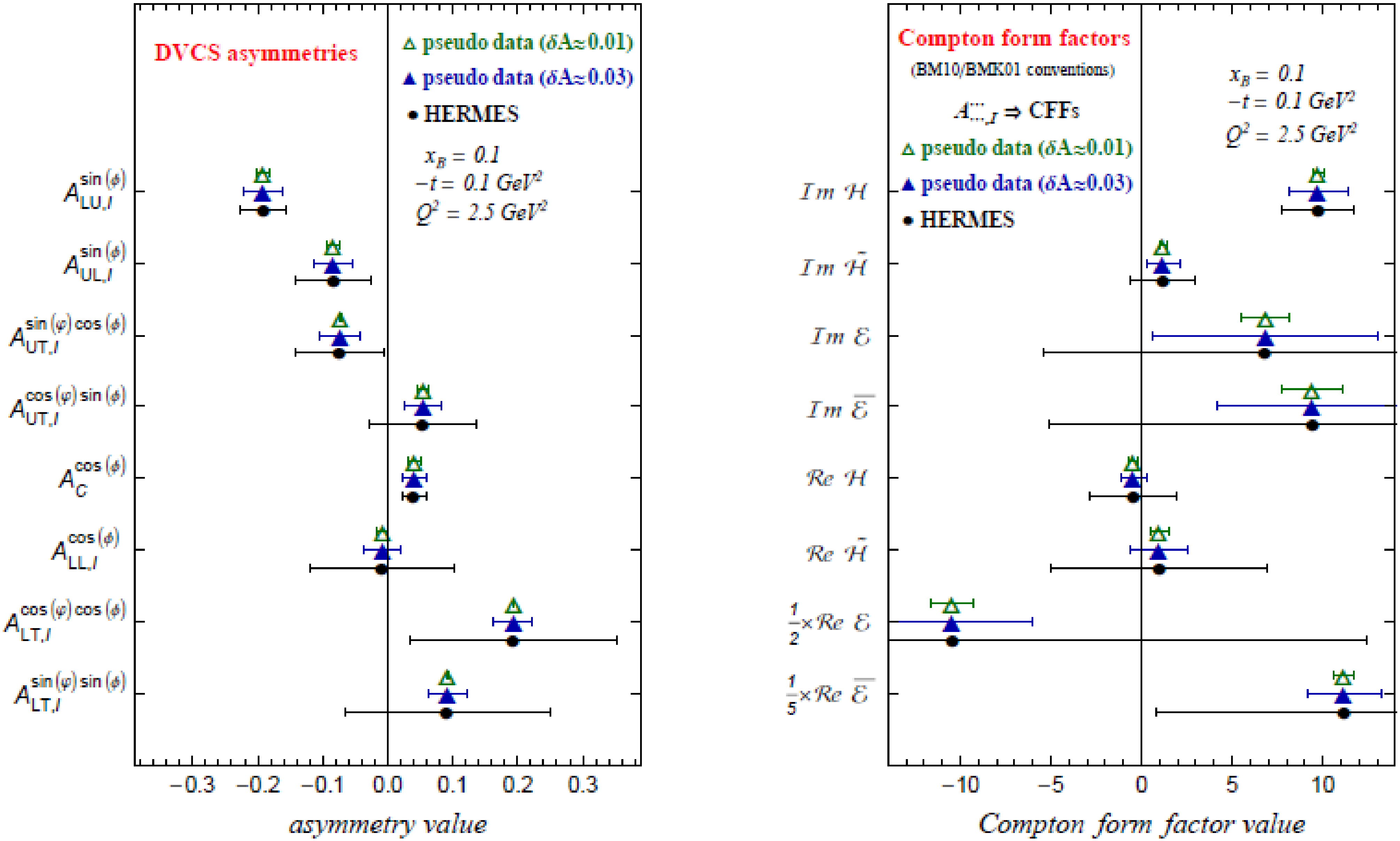


NOTE: three combinations of CFFs are (very) well constrained

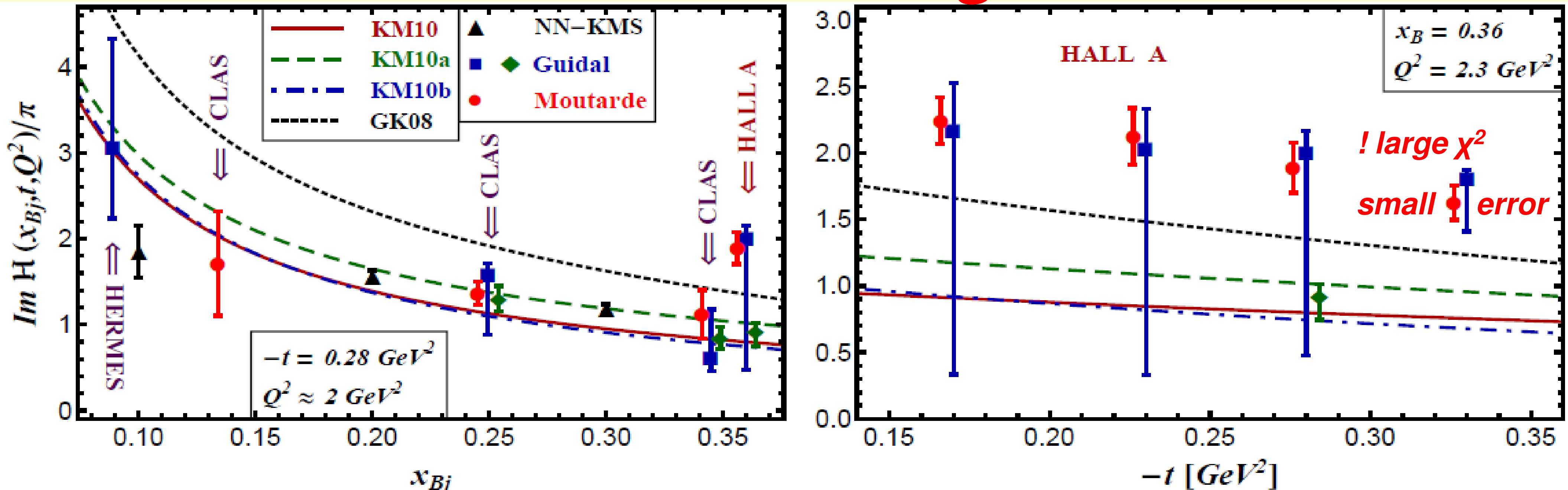


Im H
Re H
Im E
0.25×Re E
Im H̃
Re H̃
0.5×Im Ē
0.25×Re Ē
linearized map
one−to−one map
fit to 14 asymmetries
bin no.
bin no.

Projections for a HERMES like experiment with higher statistics and dedicated detector



KM10... versus CFF fits & large-x “model” fit



GUIDAL 7 parameter CFF fit to all harmonics with twist-two dominance hypothesis
propagated errors + “theoretical“ error estimate

GUIDAL same + longitudinal TSA

Moutarde H dominance hypothesis within a smeared polynomial expansion
propagated errors + “theoretical“ error estimate

KMS neural network within H dominance hypothesis

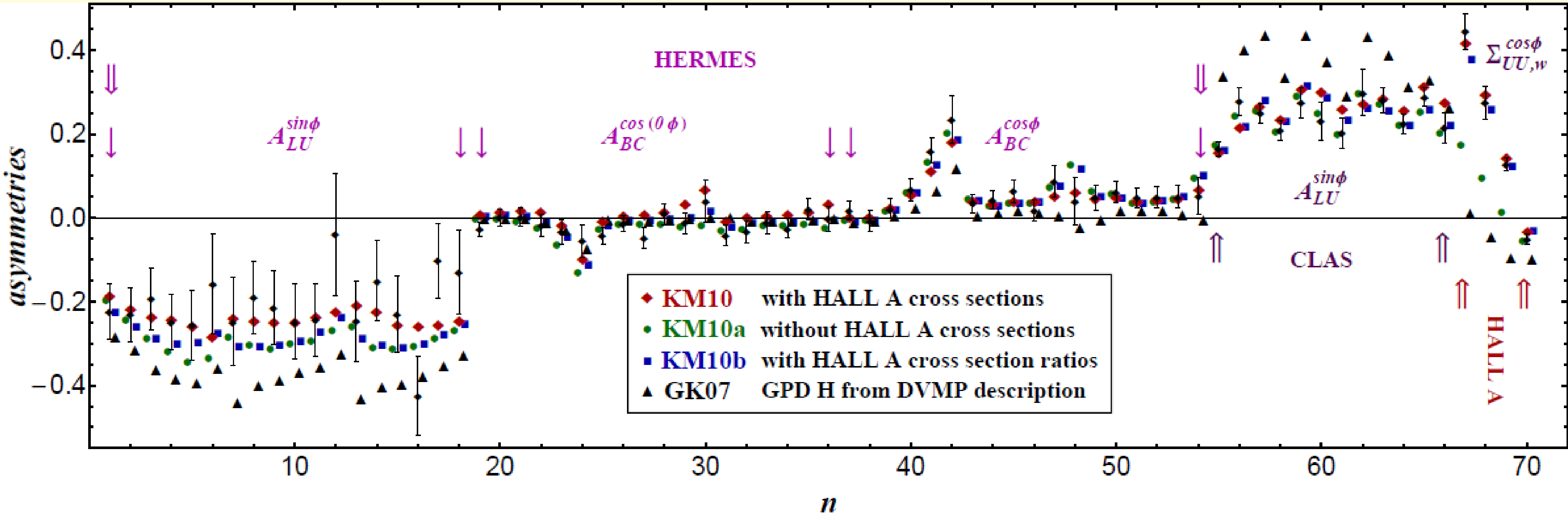
KM10 (KM10a) [KM10b] curves with (without) [ratios of] HALL A data

GK07 model based on RDDA pinned down by DVMP data via handbag approach

- reasonable agreement for HERMES and CLAS kinematics
- large x_B-region and real part remains unsettled

Global fits with hybrid models KM10...

- 150-200 unpolarized proton data from CLAS, HALL A, HERMES, H1 & ZEUS (projected on twist-two dominated observables)
- fitted with hybrid models (`dispersion relation' + flexible sea quark/gluon models) (good fits with χ^2/d.o.f. $\approx$ 1)



- also good description of small x_B data from H1 and ZEUS
- KM10... cross sections are published as code http://calculon.phy.hr/gpd/
- GK07 model from DVMP hand-bag description overshoots A_{LU} **[Goloskokov, Kroll (07)]** (typical for RDDA prediction like VGG of BMK01 models)

Summary and outlook

- we derived in terms of helicity dependent CFFs

✓ complete cross section expressions for
photon leptoproduction of polarized proton in leading order of α_{em}

✓ low energy limit relations to generalized polarizabilities

✓ Born term was calculated, too

- a proposal for a Compton tensor parameterization

➢ formulae set can be employed in various manner

- to understand how Rosenbluth separation works
  [maybe useful for JLAB (and perhaps far future EIC) measurements]
- unifying dispersion relation approaches
  sum rules for GPDs in terms of generalized polarizabilities
- will be used in DVCS and maybe VCS phenomenology
- a common VCS and DVCS phenomenological description is needed
  for numerical evaluation of radiative electromagnetic corrections
- numerical cross checks of KM GPD fitting code to GPD prediction codes