Evgeny Epelbaum, RUB

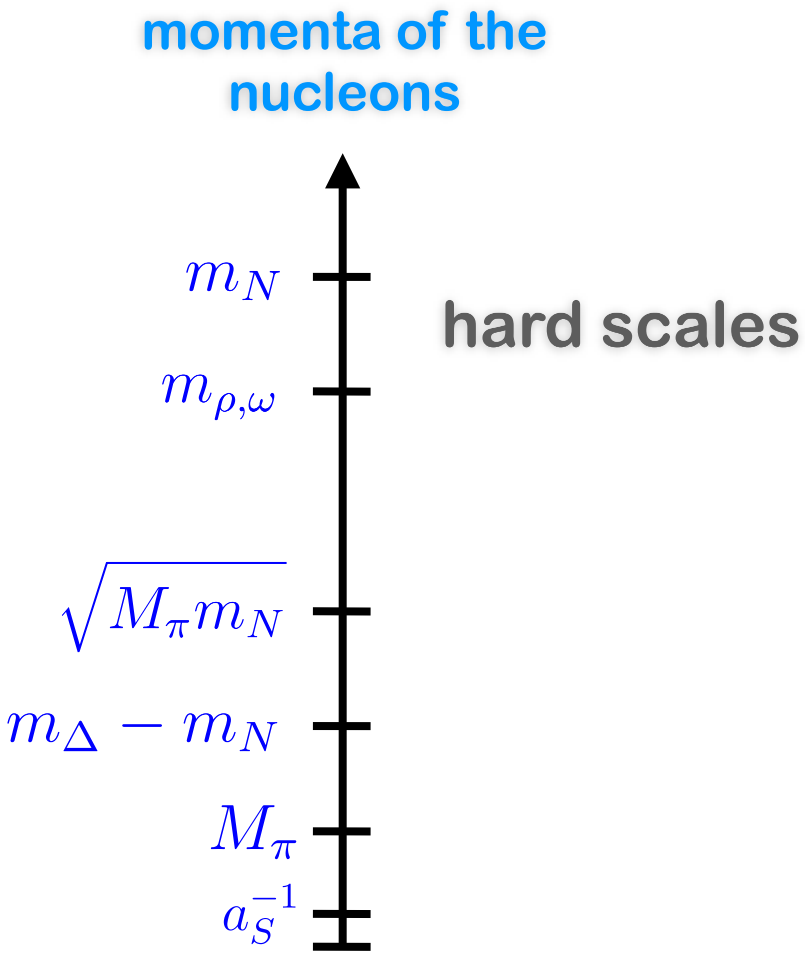
EINN2011, Paphos, Oct. 31 - Nov. 4, 2011

New vistas in chiral effective field theory for few-body systems

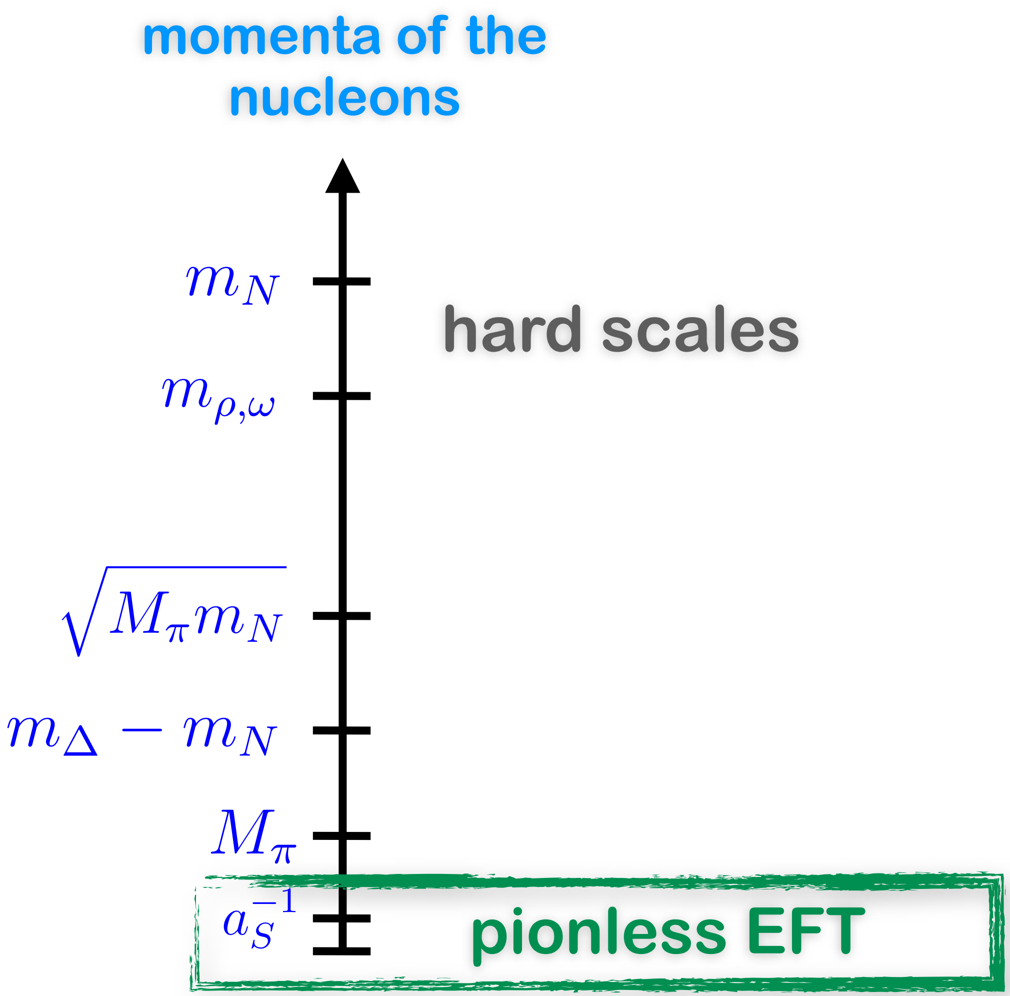
Outline

- Introduction
- Nuclear forces: where do we stand?
- Few N‘s and external probes
- Few nucleons on the lattice
- Summary & outlook

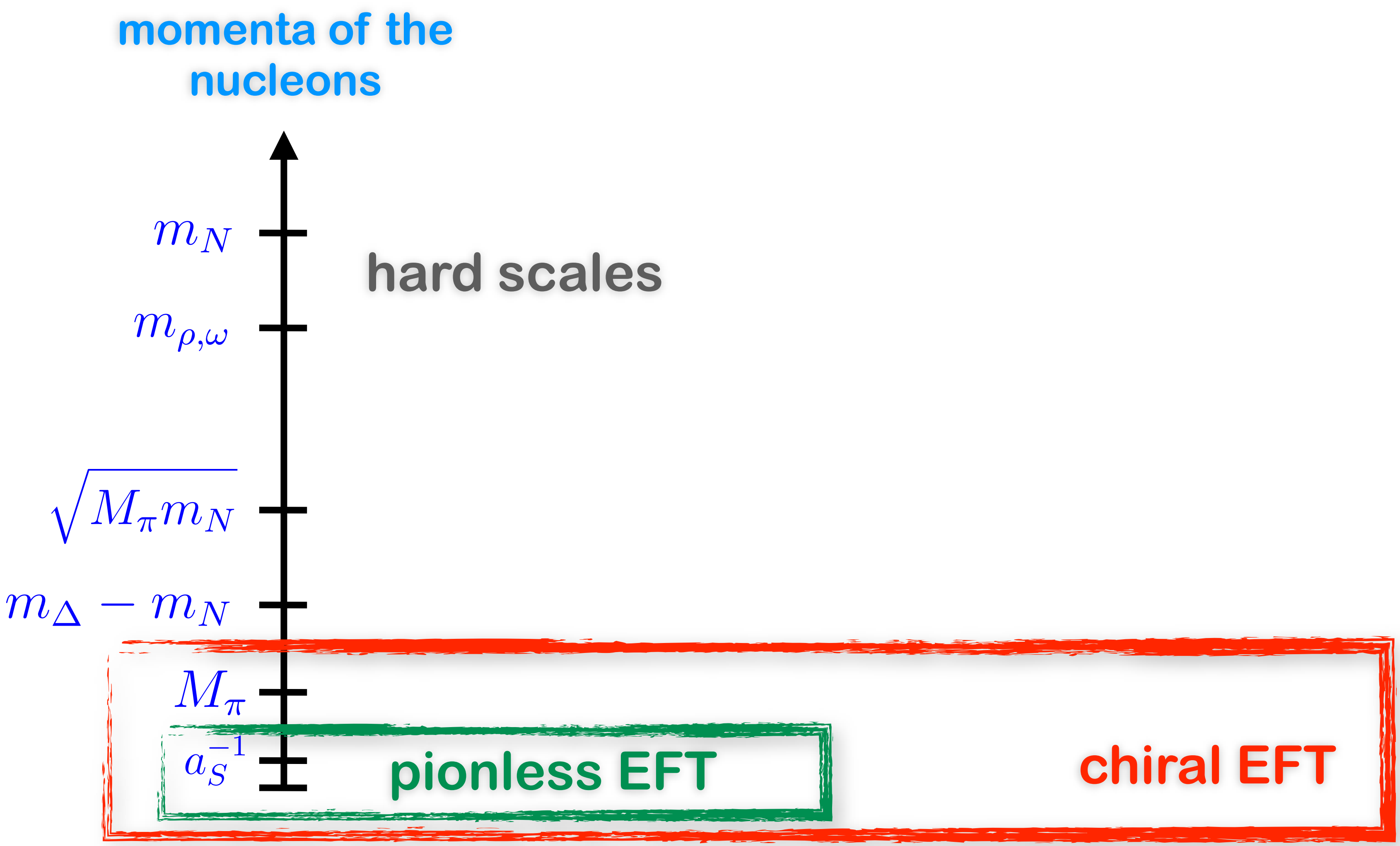
Hierarchy of scales in nuclear physics



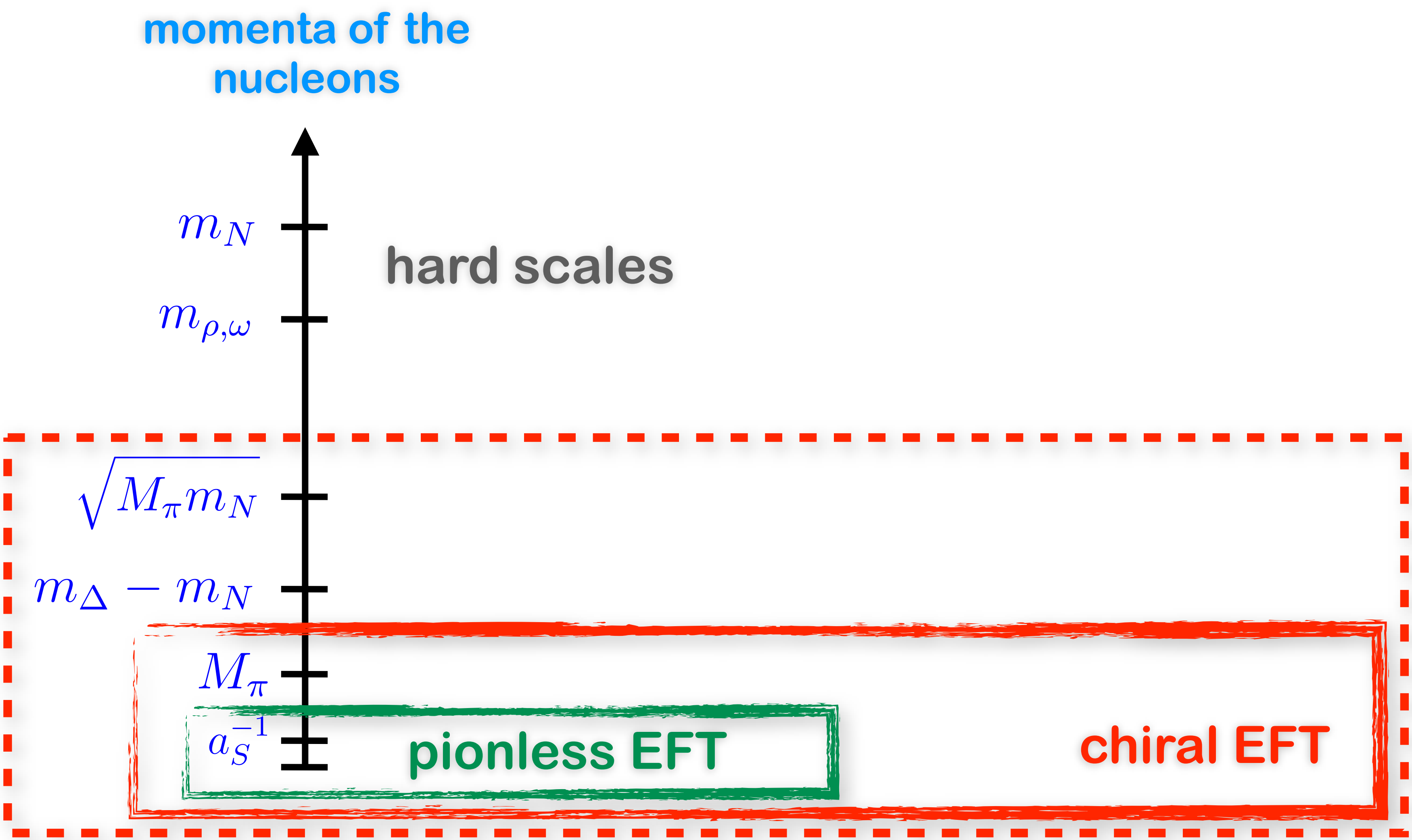
Hierarchy of scales in nuclear physics



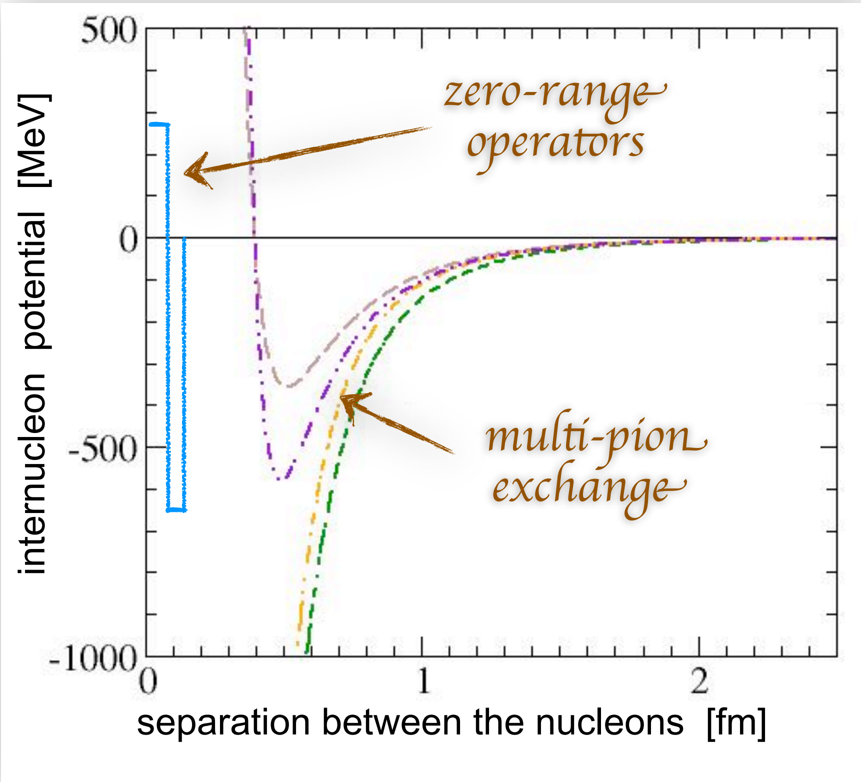
Hierarchy of scales in nuclear physics



Hierarchy of scales in nuclear physics



Chiral EFT for nuclear forces



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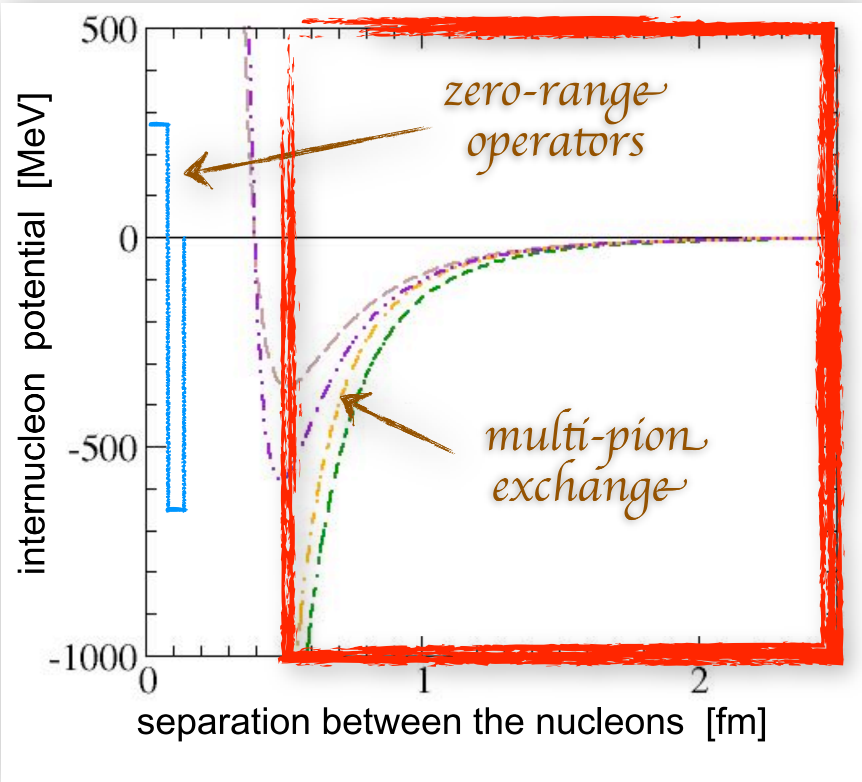
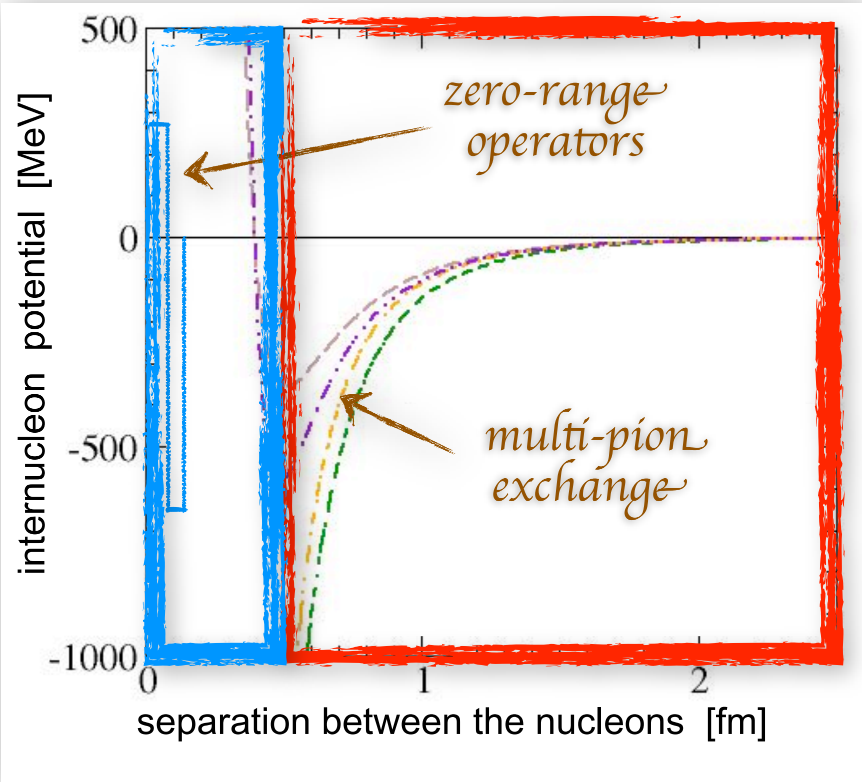


- Nonlocal (long-range) potentials obtained in chiral perturbation theory

Chiral EFT for nuclear forces

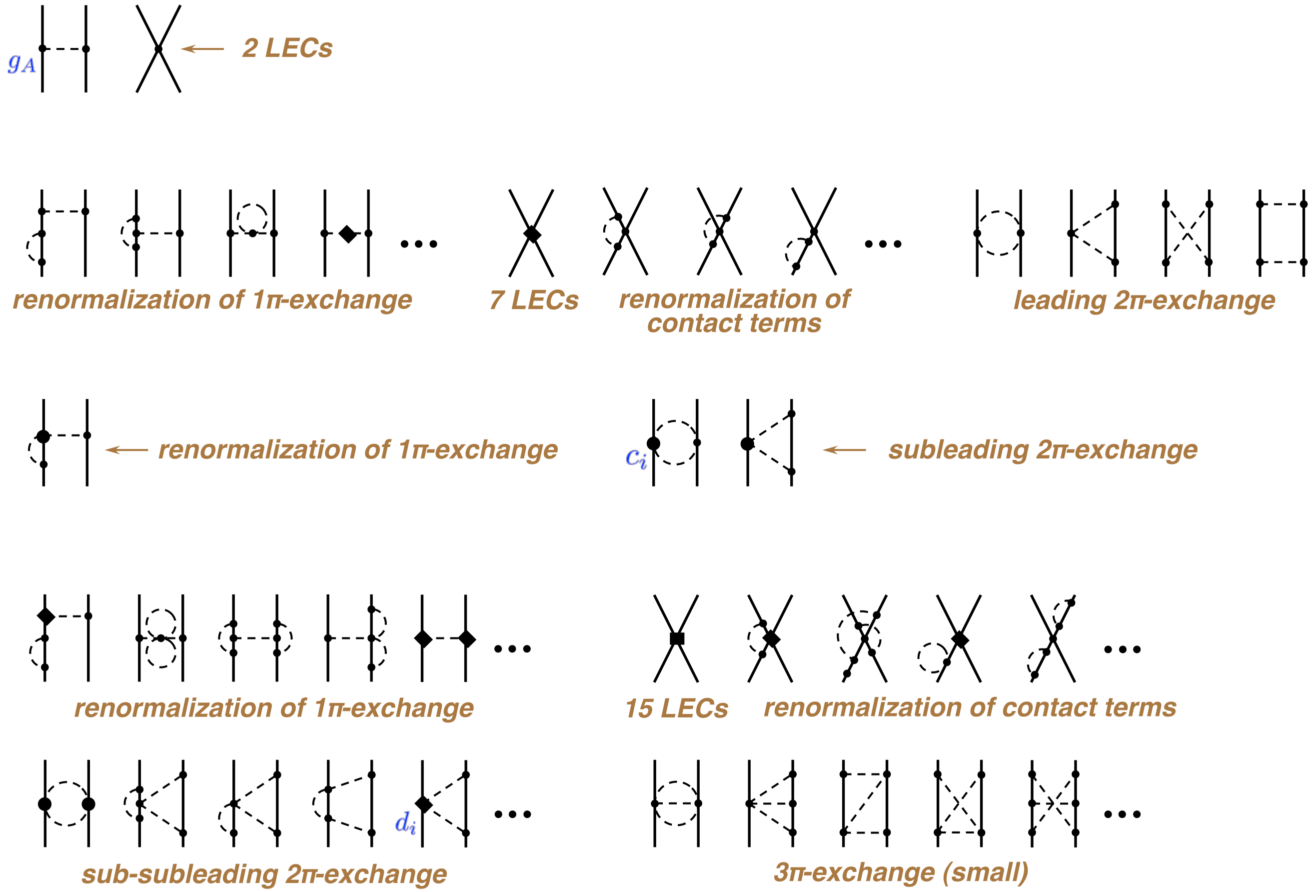


- Nonlocal (long-range) potentials obtained in chiral perturbation theory
- Parametrized in a most general way

Chiral expansion of the NN force

Ordonez et al. ’94; Friar & Coon ’94; Kaiser et al. ’97; E.E. et al. ’98,‘03; Kaiser ’99-’01; Higa, Robilotta ’03; …

- LO (Q^0):
- NLO (Q^2):
- N^2LO (Q^3):
- N^3LO (Q^4):



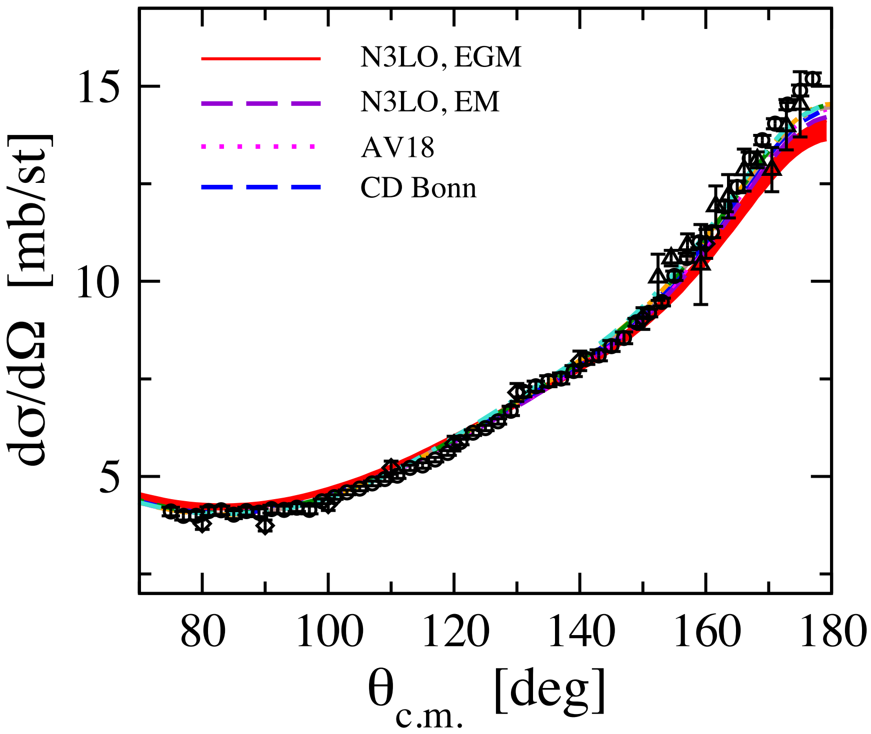
\+ isospin-breaking corrections…

van Kolck et al. ’93,’96; Friar et al. ’99,’03,’04; Niskanen ’02; Kaiser ’06; E.E. et al. ’04,’05,’07; …

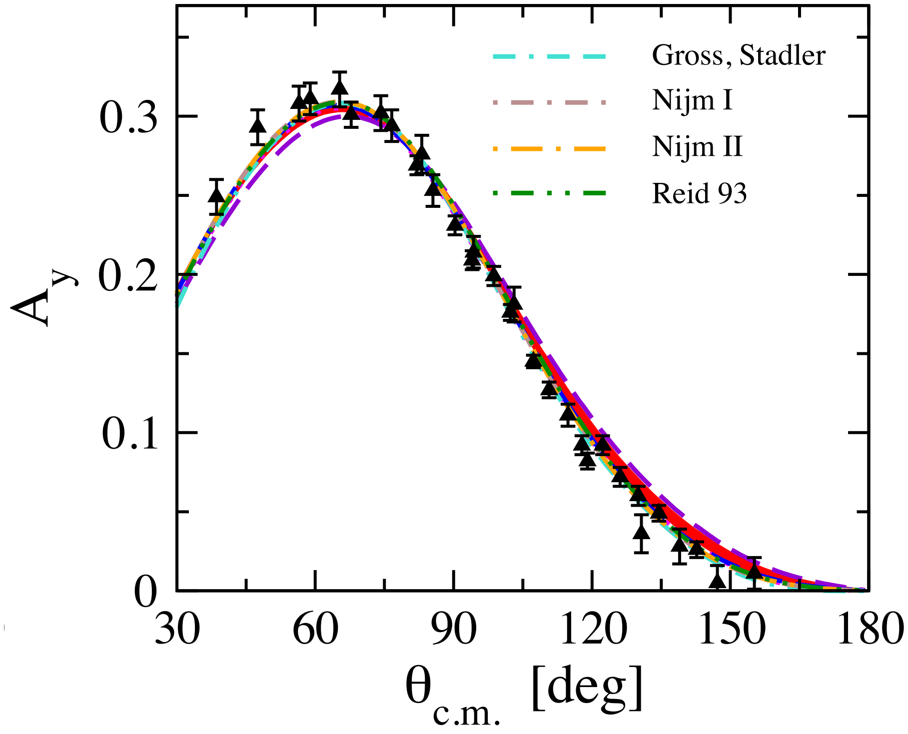
Two nucleons at N^3LO

Entem, Machleidt '04; E.E., Glöckle, Meißner '05

np differential cross section at 96 MeV



Nucleon A_y at 67.5 MeV



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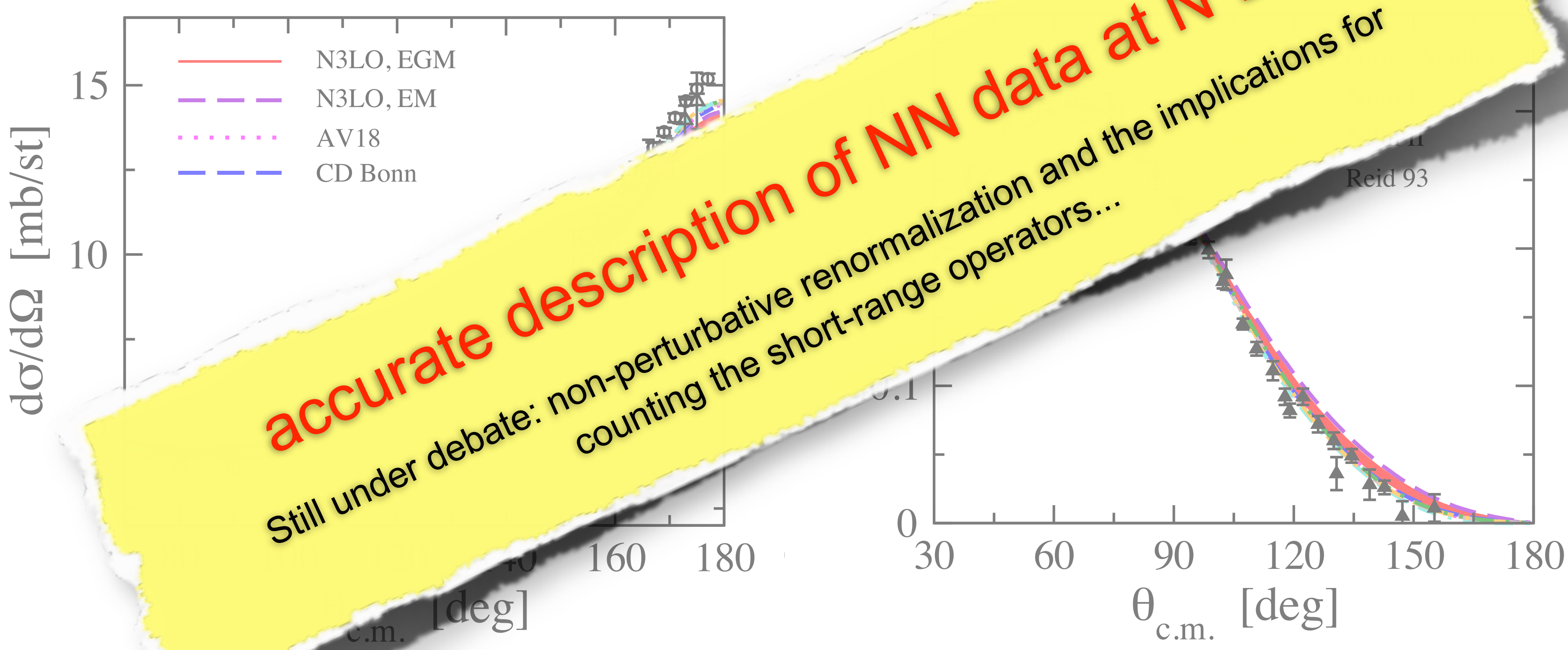
Nu...



accurate description of NN data at N^3LO

Still under debate: non-perturbative renormalization and the implications for counting the short-range operators...

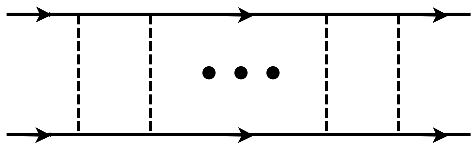
Renormalization, power counting and all that...

D. B. Kaplan, M. J. Savage, and M. B. Wise, Nucl. Phys. **B478** (1996) 629; Phys. Lett. B **424** (1998) 390; Nucl. Phys. **B534** (1998) 329,
S. Fleming, T. Mehen, and I. W. Stewart, Nucl. Phys. **A677** (2000) 313; Phys. Rev. C **61** (2000) 044005.
D. R. Phillips, S. R. Beane, and T. D. Cohen, Ann. Phys. (N.Y.) **263** (1998) 255.
T. Frederico, V. S. Timoteo, and L. Tomio, Nucl. Phys. **A653** (1999) 209.
M. C. Birse, Phys. Rev. C **74** (2006) 014003; Phys. Rev. C **76** (2007) 034002.
S. R. Beane, P. F. Bedaque, M. J. Savage, and U. van Kolck, Nucl. Phys. **A700** (2002) 377.
M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **72** (2005) 054002.
A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. C **72** (2005) 054006.
M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **74** (2006) 054001.
M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **74** (2006) 064004; Erratum: Phys. Rev. C **75** (2007) 059905.
E. Epelbaum and U.-G. Meißner, *On the renormalization of the one-pion exchange potential and the consistency of Weinberg's power counting*, arXiv:nucl-th/0609037.
M. Pavon Valderrama and E. Ruiz Arriola, Ann. Phys. (N.Y.) **323** (2008) 1037.
D. R. Entem, E. Ruiz Arriola, M. Pavón Valderrama, and R. Machleidt, Phys. Rev. C **77** (2008) 044006.
C.-J. Yang, Ch. Elster, and D. R. Phillips, Phys. Rev. C **77** (2008) 014002; **80** (2009) 034002, 044002.
B. Long and U. van Kolck, Ann. Phys. (N.Y) **323** (2008) 1304.
S. R. Beane, D. B. Kaplan, and A. Vuorinen, *Perturbative nuclear physics*, arXiv:0812.3938 [nucl-th].
M. Pavon Valderrama, A. Nogga, E. Ruiz Arriola, and D. R. Phillips, Eur. Phys. J. A **36** (2008) 315.
M. P. Valderrama, *Perturbative Renormalizability of Chiral Two Pion Exchange in Nucleon-Nucleon Scattering*, arXiv:0912.0699 [nucl-th].
R. Machleidt, P. Liu, D. R. Entem, and E. Ruiz Arriola, Phys. Rev. C **81** (2010) 024001.
E. Epelbaum and J. Gegelia, Eur. Phys. J. **A41** (2009) 341.
G. P. Lepage, *How to Renormalize the Schrödinger Equation*, nucl-th/9706029.

⋮

Renormalization, power counting and all that...

- There is strong evidence that iterations of the OPEP are non-perturbative at $p \sim M_\pi$ in (some) spin-triplet channels Fleming, Mehen, Stewart, Cohen, Hansen, Gegelia …



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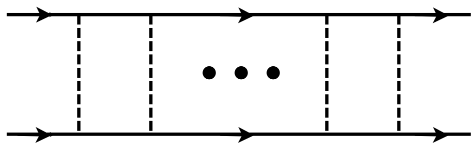
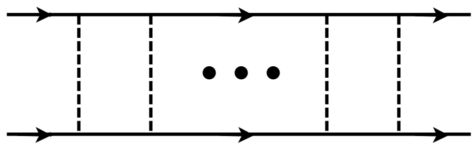


- No approximation to the OPEP is known that would (i) capture the non-perturbative physics, (ii) be (analytically) resummable and (iii) (explicitly) renormalizable
E.g. the Kaplan-Savage-Wise ansatz fulfills (ii), (iii) but not (i)...

Renormalization, power counting and all that...



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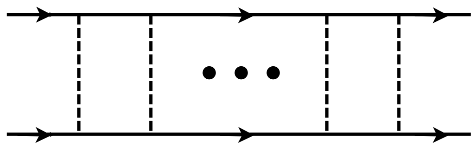

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- A tricky issue:
first renormalize ($\Lambda \to \infty$) and then resum $\neq$ first resum and then „renormalize"
(first resum and then „renormalize": violates LETs, not compatible with EFT EE, Gegelia)

Renormalization, power counting and all that...

- There is strong evidence that iterations o
are non-perturbative at p ~ M_π in (some
channels Fleming, Mehen, Stewart, Cohen, Ha

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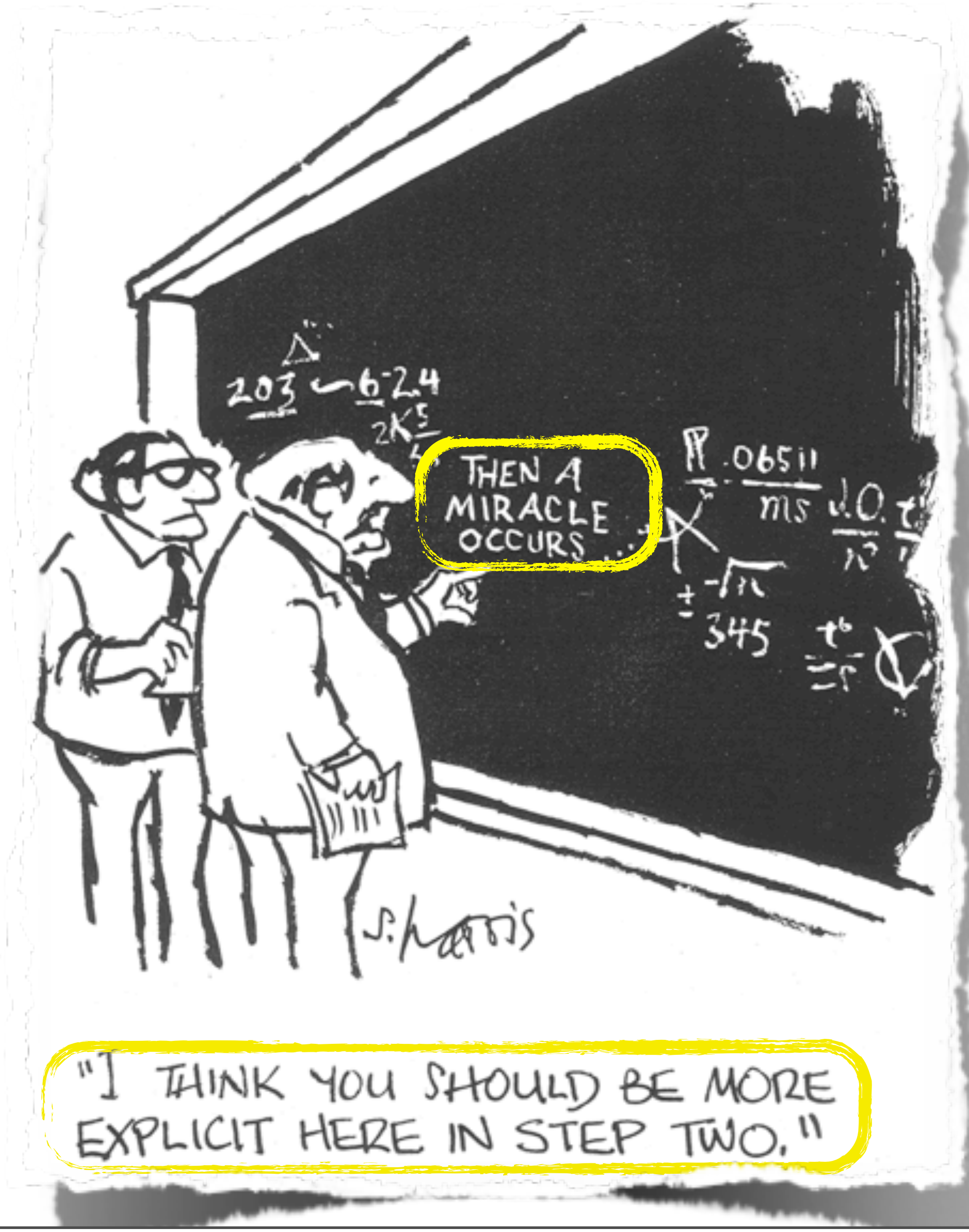

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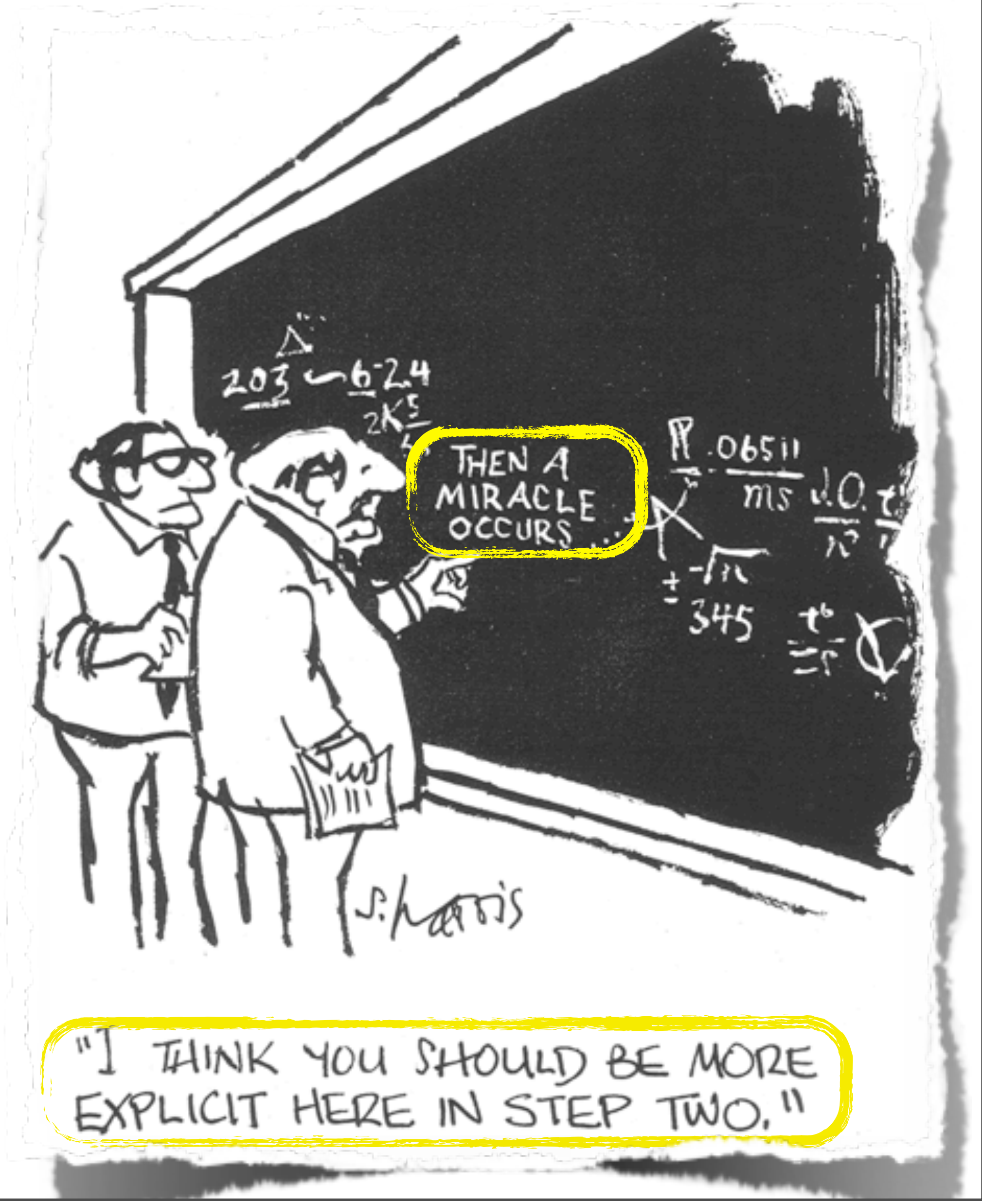
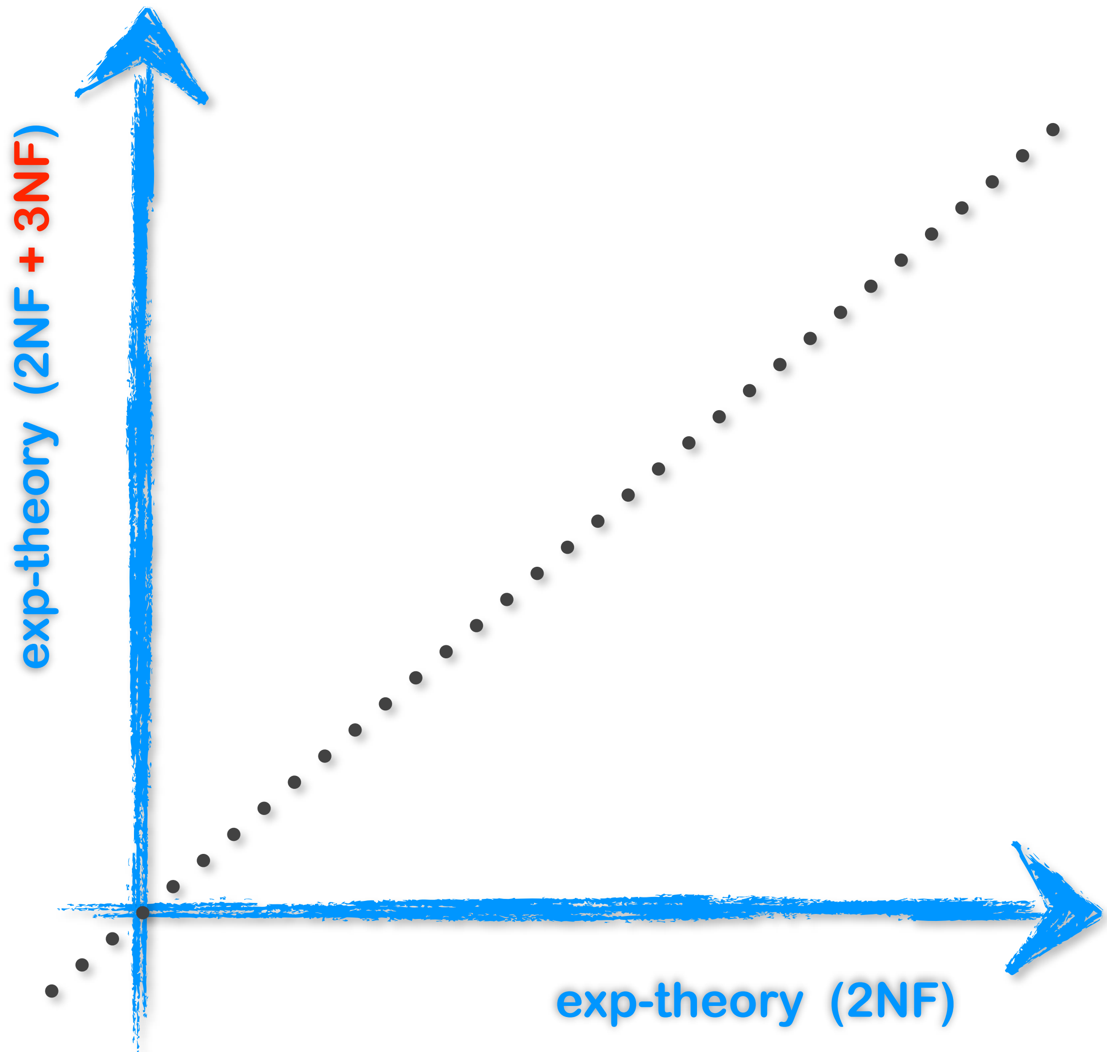
- More work needed to better understand
power counting for NN amplitude.
Insights from RG analysis (Birse)?



Three-nucleon force: Motivation



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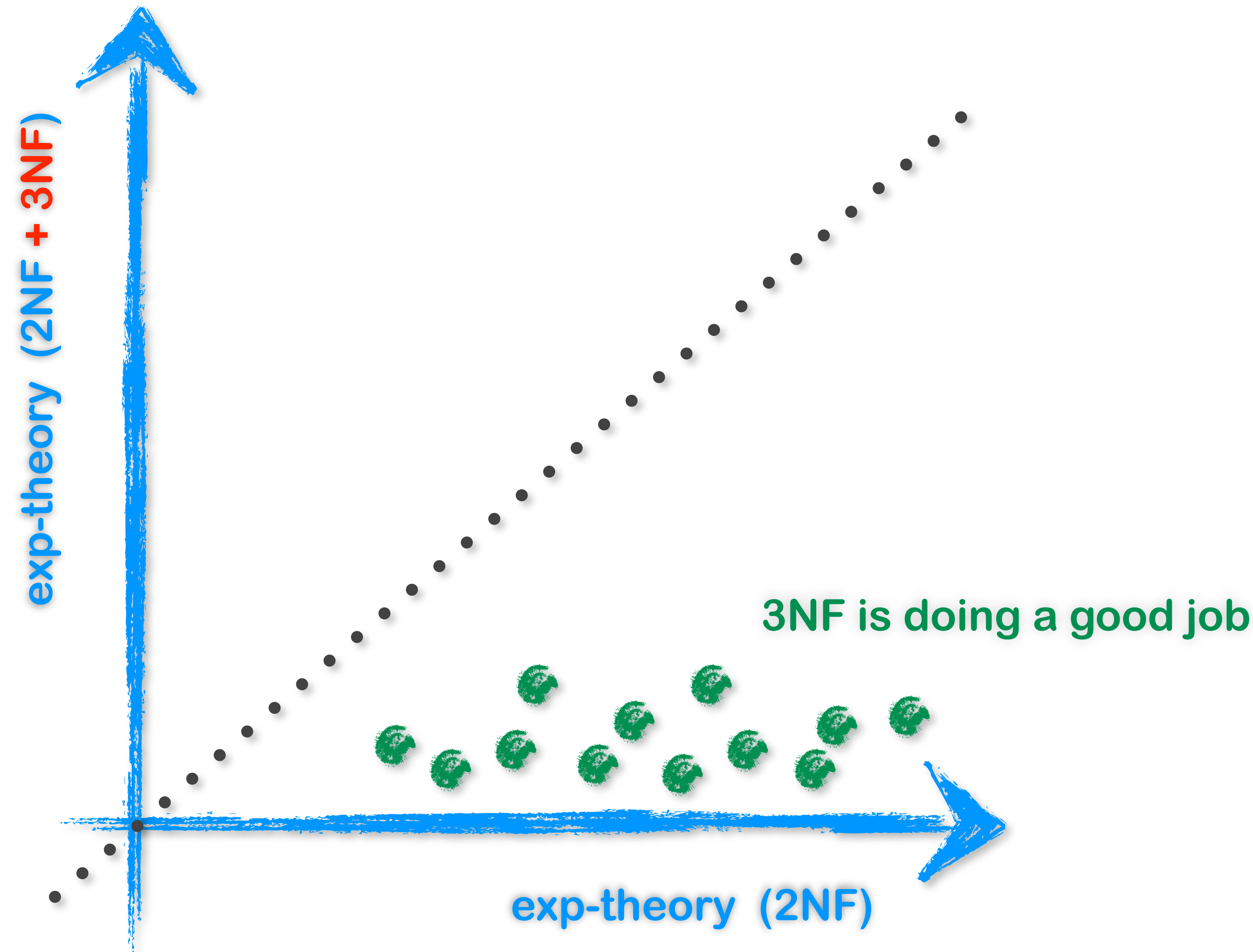

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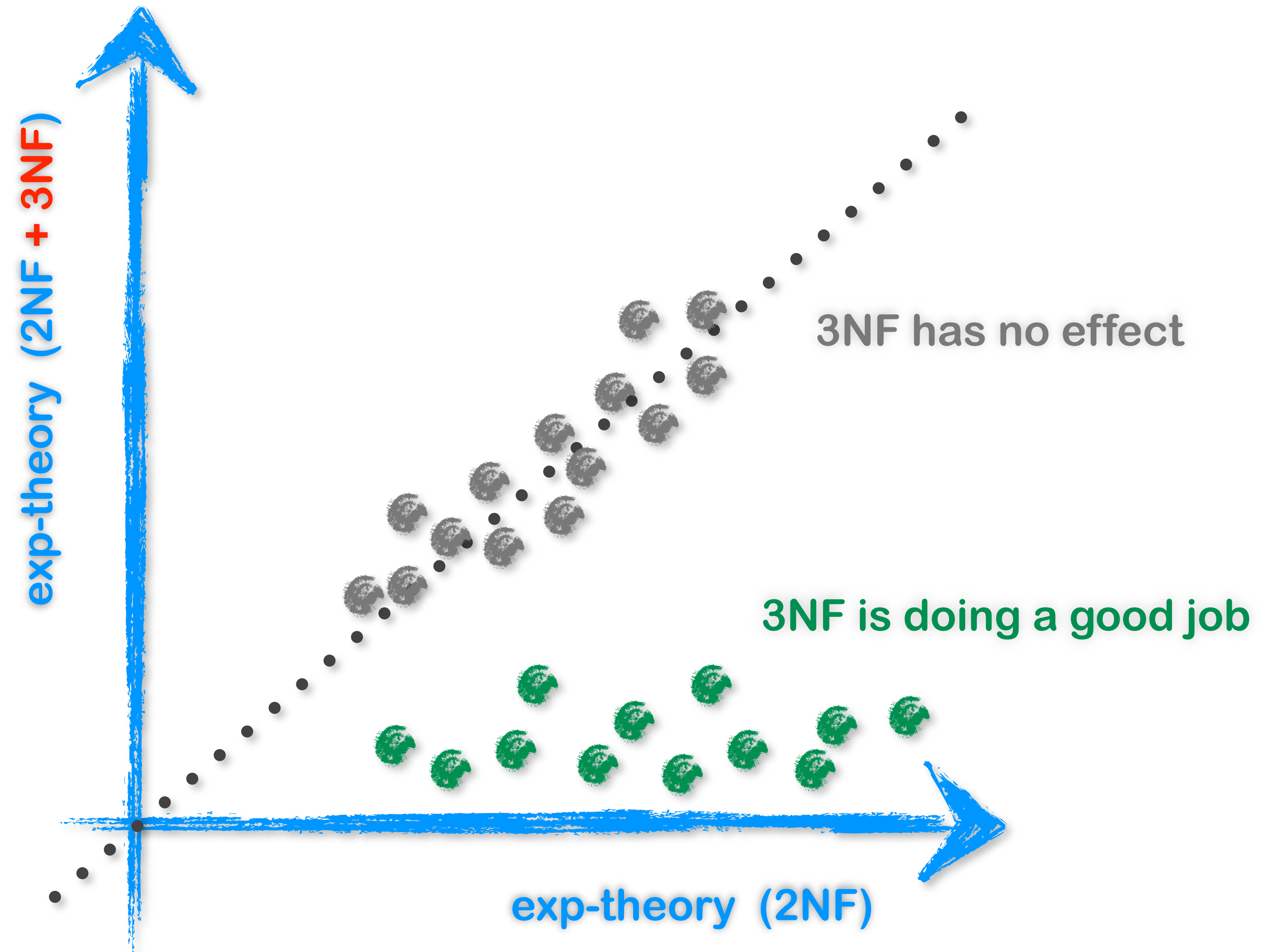

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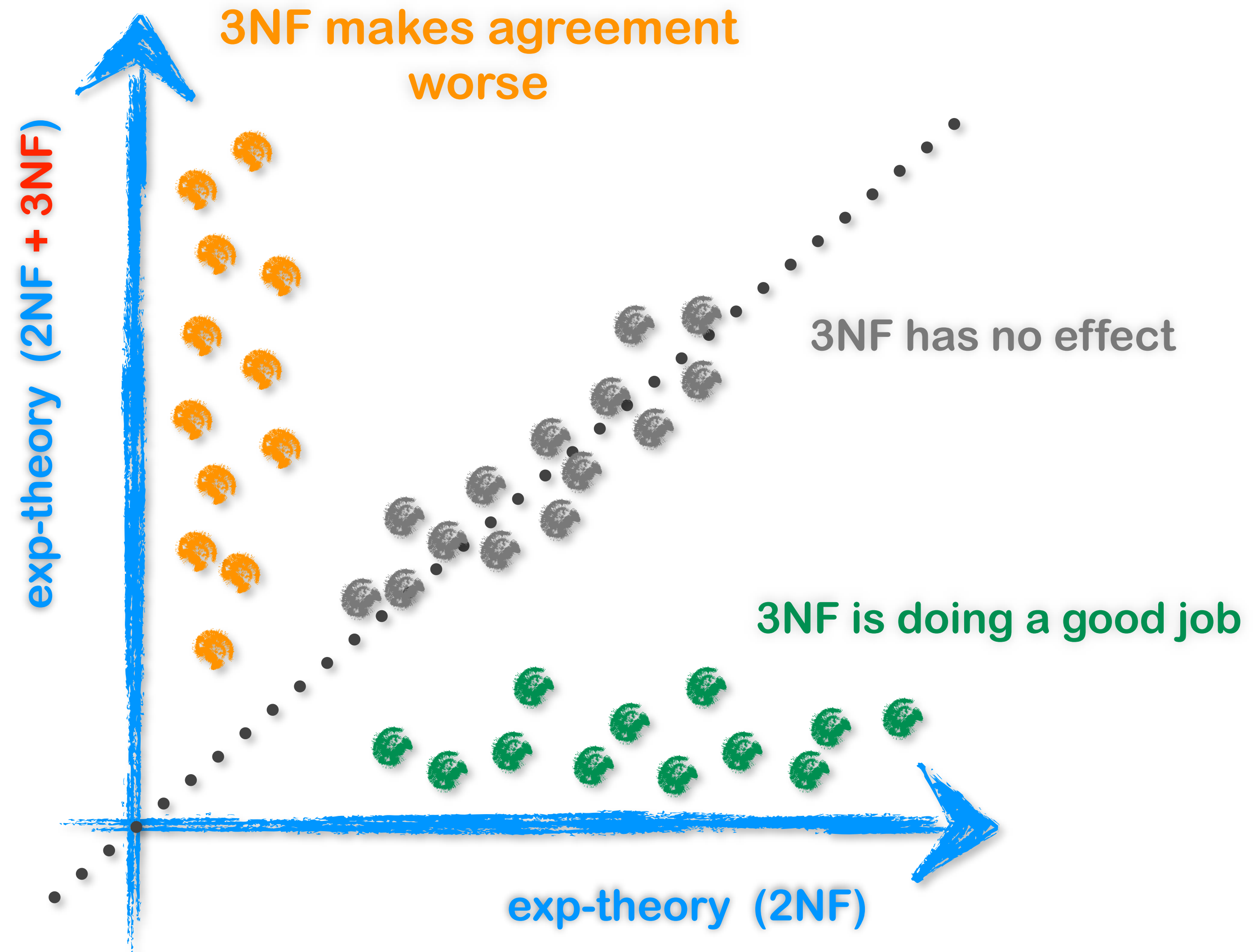
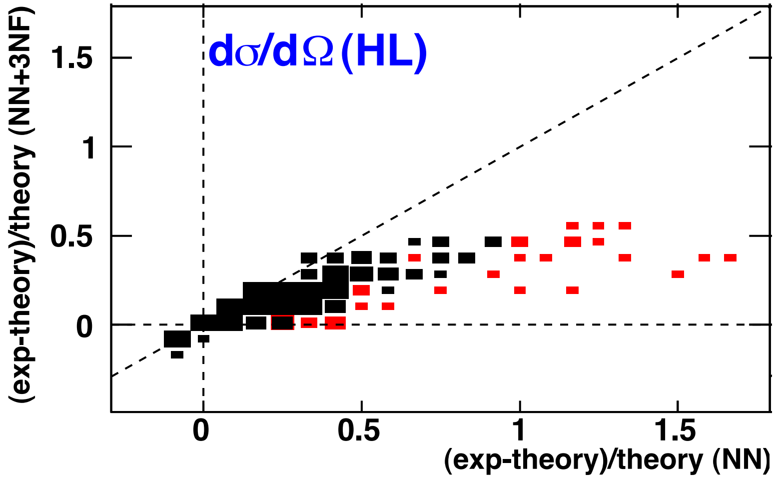
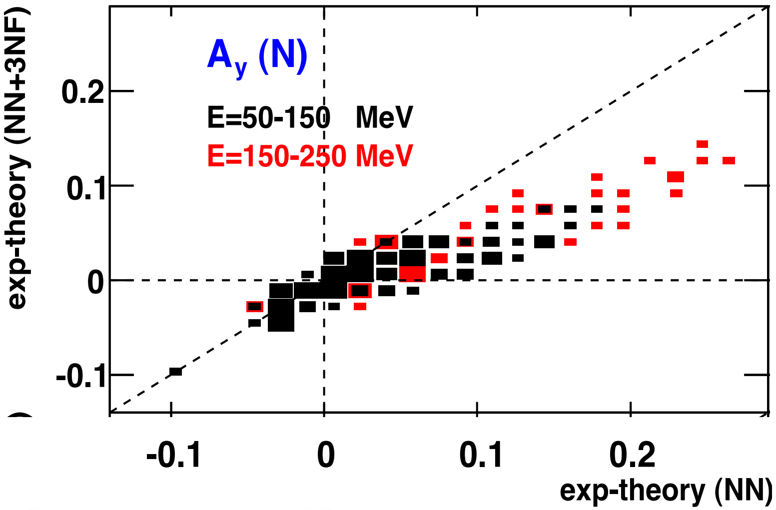


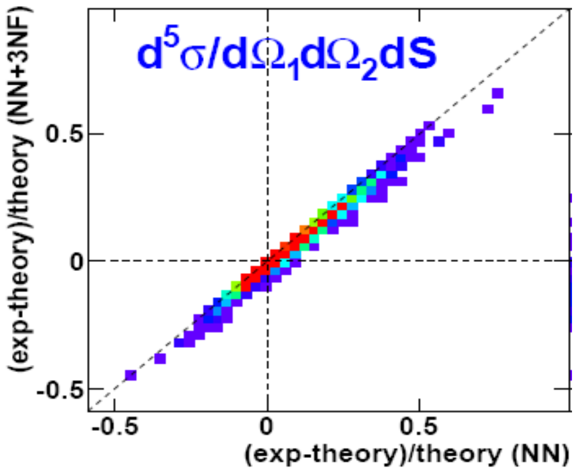
Three-nucleon force: Motivation

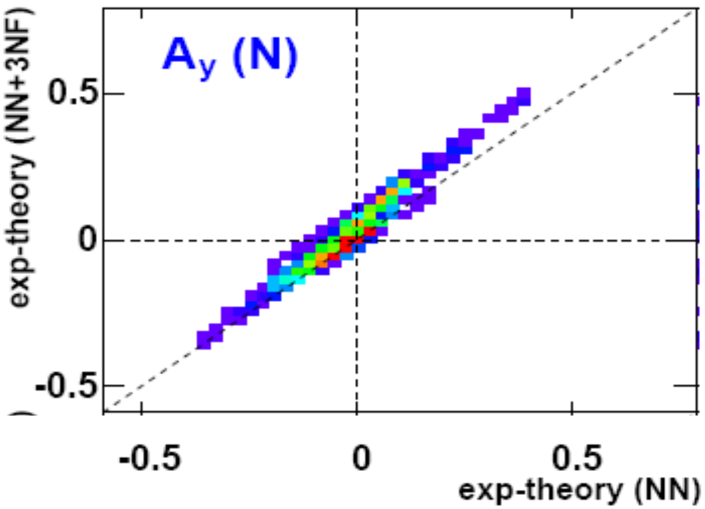
Elastic nucleon-deuteron scattering





Deuteron breakup reaction

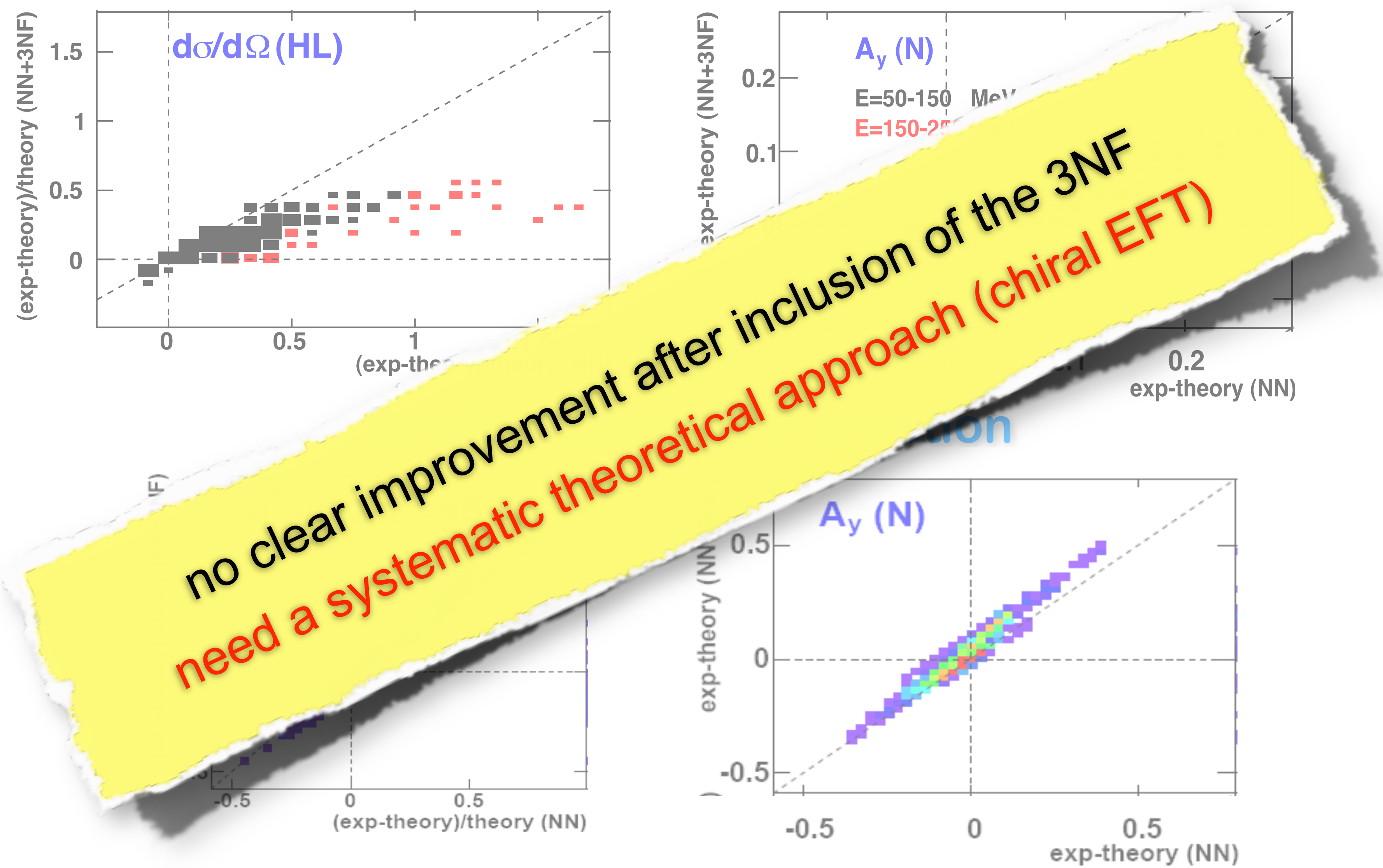




from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, arXiv:0811.1338, to appear in Rev. Mod. Phys.

Three-nucleon force: Motivation

Elastic nucleon-deuteron scattering



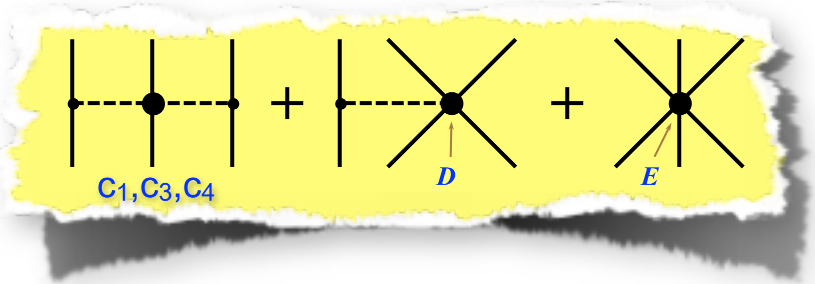
no clear improvement after inclusion of the 3NF

need a systematic theoretical approach (chiral EFT)

from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, arXiv:0811.1338, to appear in Rev. Mod. Phys.

Chiral 3NF at N²LO

3NF first appears ar N²LO:



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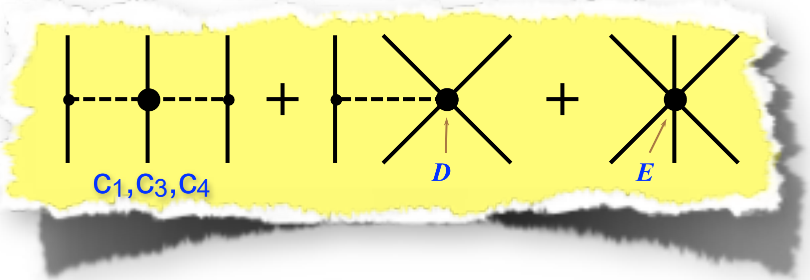
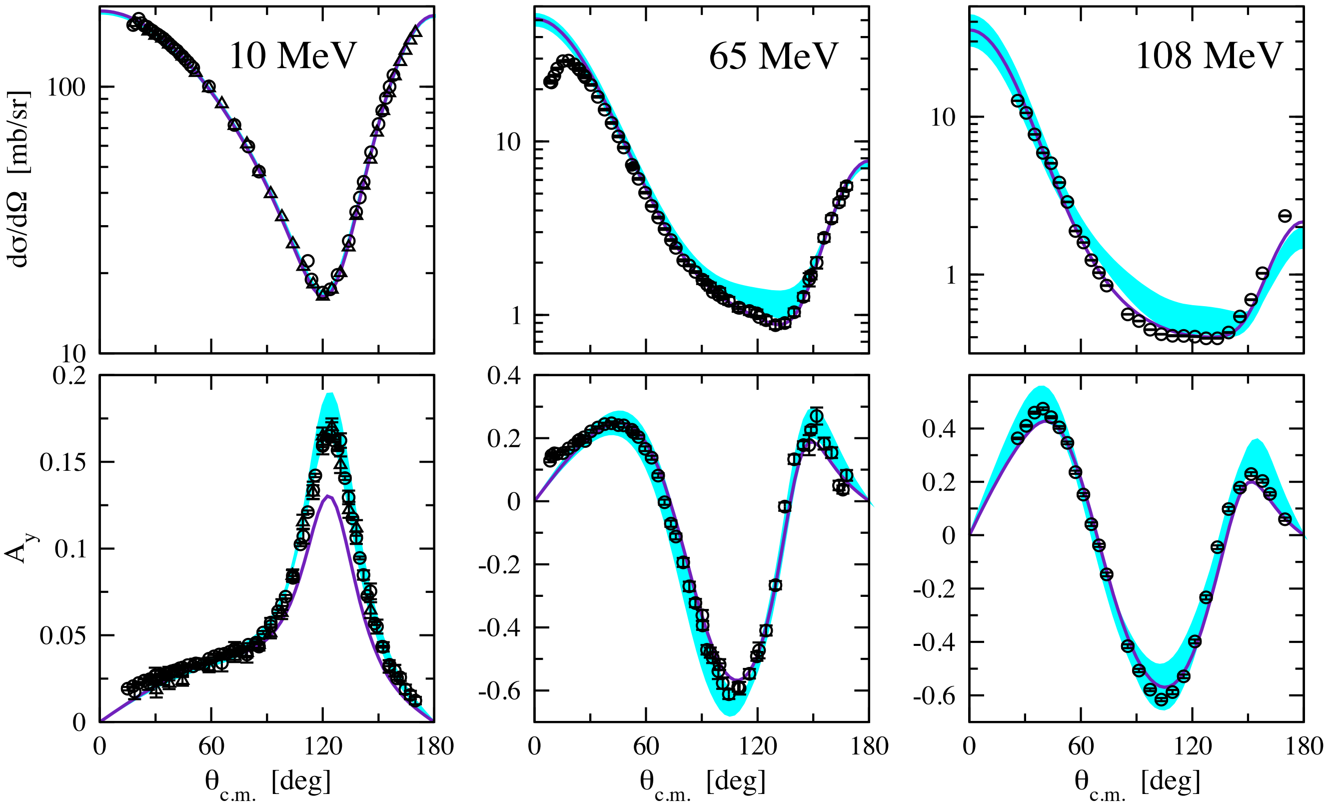


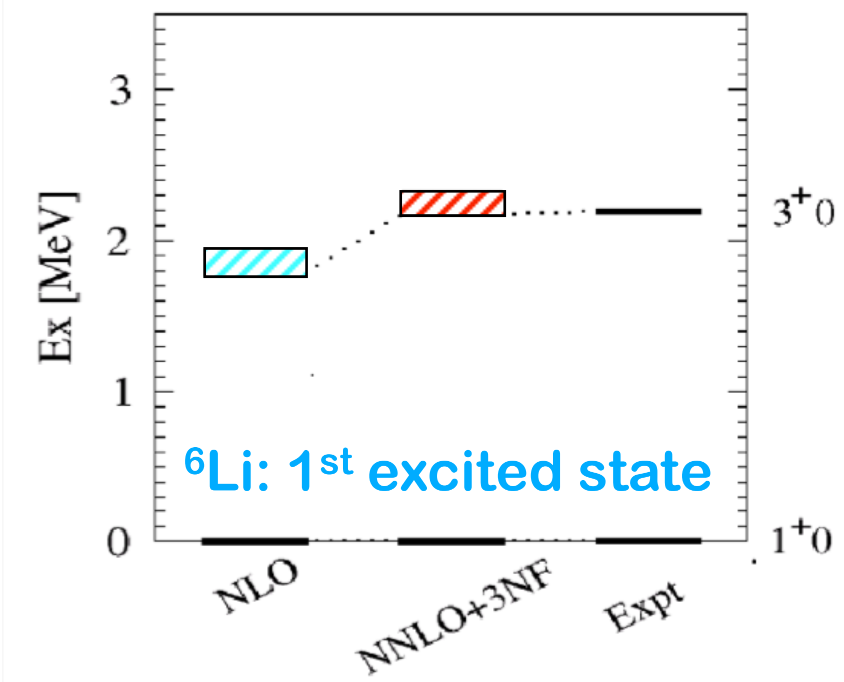
Elastic nucleon-deuteron scattering at N²LO

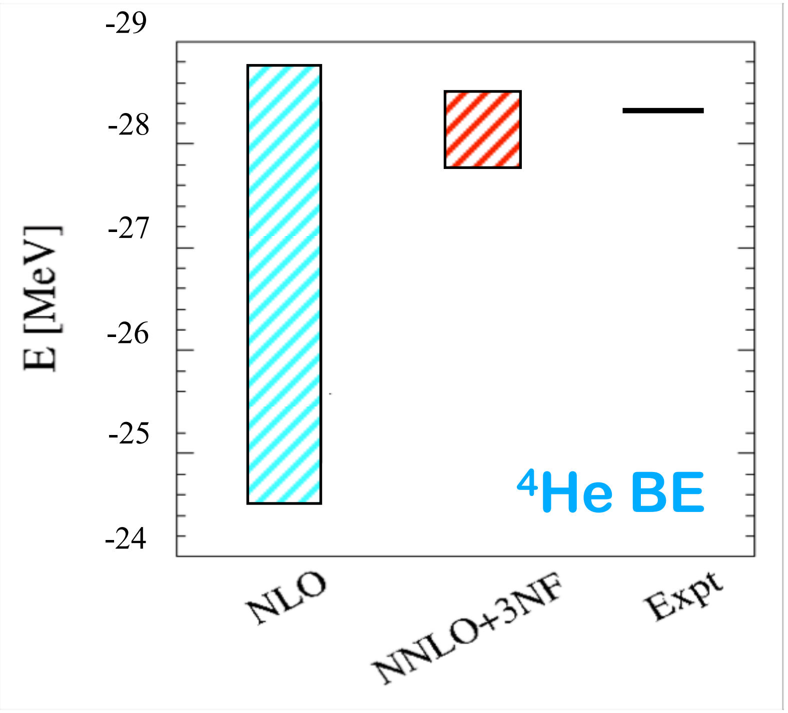


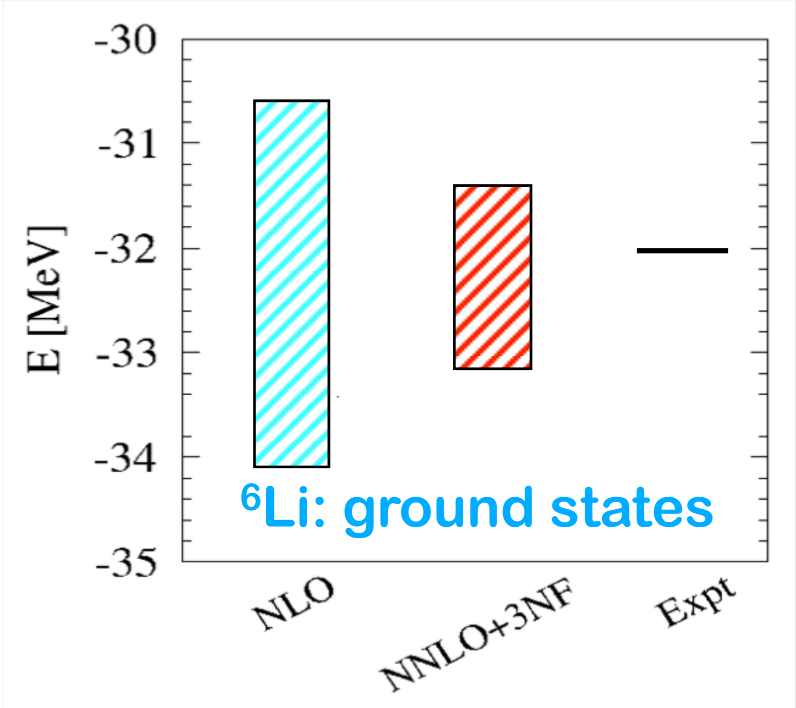
The LECs D,E are fixed to the ^3H BE and nd doublet scattering length

Light nuclei from chiral forces

Nogga et al.



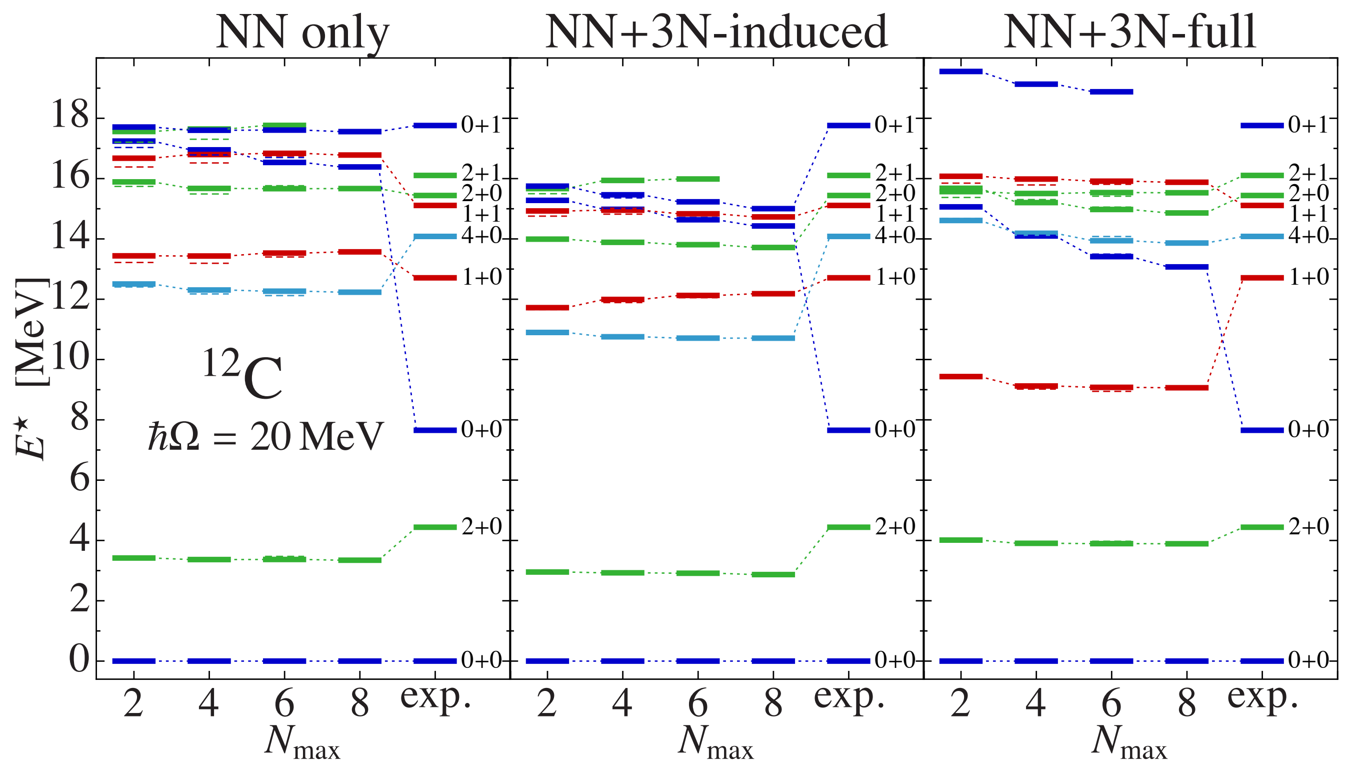




Nuclear structure with chiral forces

Importance truncated NCSM with SRG transformed chiral 2NF + 3NF

Roth, Langhammer, Calci, Binder, Navratil, PRL 107 (2011) 072501

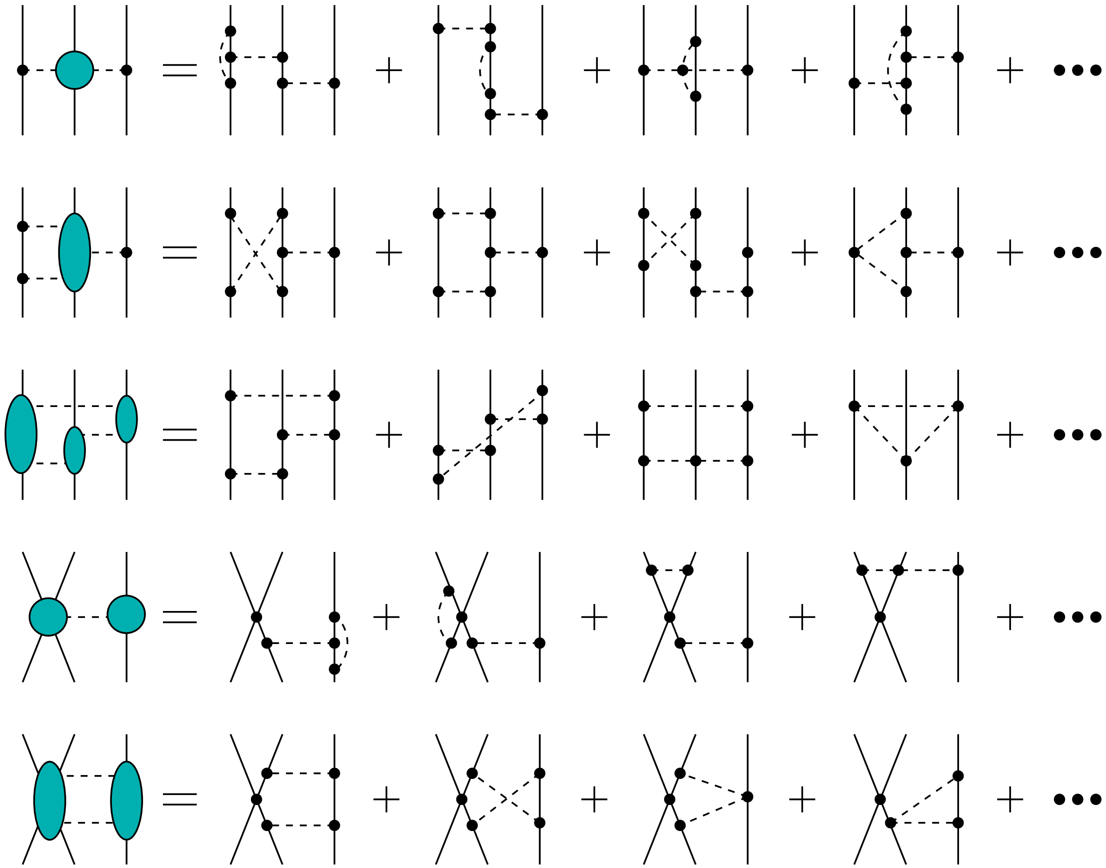


Chiral 3NF at N^2LO are also found to play important role in

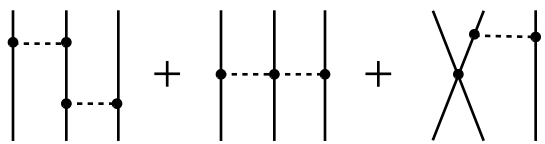
- explaining the long lifetime of ^{14}C Holt, Kaiser, Weise '10
- constraining the properties of neutron-rich matter & neutron star radii Hebeler et al.'10
- explaining the structure of Ca isotopes Holt, Otsuka, Schwenk, Suzuki '10

Chiral 3NF at N^{3}LO

Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); arXiv:1108.3816



1/m corrections to



no unknown LECs contribute → parameter-free!

Chiral 3NF at N^{3}LO

Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); arXiv:1108.3816

$$
\begin{aligned}
V_{2\pi-1\pi} \; = \; & \frac{\vec{\sigma}_1 \cdot \vec{q}_1}{q_1^2 + M_\pi^2} \Big[\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_1 \; (\vec{\sigma}_3 \cdot \vec{q}_2 \, \vec{q}_2 \cdot \vec{q}_1 \; F_1(q_2) + \vec{\sigma}_3 \cdot \vec{q}_2 \; F_2(q_2) + \vec{\sigma}_3 \cdot \vec{q}_1 \; F_3(q_2)) \\
& + \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \; (\vec{\sigma}_2 \cdot \vec{q}_2 \, \vec{q}_2 \cdot \vec{q}_1 \; F_4(q_2) + \vec{\sigma}_2 \cdot \vec{q}_1 \; F_5(q_2) + \vec{\sigma}_3 \cdot \vec{q}_2 \; F_6(q_2) \\
& \qquad + \vec{\sigma}_3 \cdot \vec{q}_1 \; F_7(q_2)) \\
& + \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \; \vec{\sigma}_2 \times \vec{\sigma}_3 \cdot \vec{q}_2 \; F_8(q_2) \Big]
\end{aligned}
$$

$$
\begin{aligned}
V_{\rm ring} \; = \; & \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_1 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_1 \; \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_2 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_2 \cdot \vec{q}_3 \; \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_3 \\
+ \; & \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_1 \; \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_4 + \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot \vec{q}_3 \; \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_5 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \; R_6 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_1 \; R_7 \\
+ \; & \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \; R_8 + \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot \vec{q}_1 \; R_9 + \vec{\sigma}_1 \cdot \vec{\sigma}_3 \; R_{10} + \vec{q}_1 \cdot \vec{q}_3 \times \vec{\sigma}_2 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3 \; R_{11} \\
+ \; & \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_1 + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_1 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_2 + \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_3 \cdot \vec{q}_1 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_3 \\
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+ \; & \vec{q}_1 \cdot \vec{q}_3 \times \vec{\sigma}_1 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3 \; S_7
\end{aligned}
$$

explicit expressions for all F_i, $R_i(q_1,q_2,q_3)$ and $S_i(q_1,q_2,q_3)$ are available in Bernard, EE, Krebs, Meißner, PRC77 (08) 064004

no unknown LECs contribute → parameter-free!

Chiral 3NF at N³LO

Ishikawa, Robilotta, PRC76 (07); Bernard, EE, Krebs, Meißner, PRC77 (08); arXiv:1108.3816

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$$+ \; \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \; (\vec{\sigma}_2 \cdot \vec{q}_2 \; \vec{q}_2 \cdot \vec{q}_1 \; F_4(q_2) + \vec{\sigma}_2 \cdot \vec{q}_1 \; F_5(q_2) \ldots$$

$$+ \; \vec{\sigma}_3 \cdot \vec{q}_1 \; F_7(q_2))$$

$$+ \; \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_1 \; \vec{\sigma}_2 \times \vec{\sigma}_3 \cdot \vec{q}_2 \; F \ldots$$

$$V_{\rm ring} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \; \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \; R_1 + \ldots$$

$$+ \; \vec{\sigma}_1 \cdot \vec{q}_3 \vec{\sigma}_2 \ldots + \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_1 \; R_7$$

$$+ \ldots \vec{q}_3 \times \vec{\sigma}_2 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3 \; R_{11}$$

$$\ldots \vec{\sigma}_3 \cdot \vec{q}_1 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_3$$

$$\ldots \vec{q}_3 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_5 + \vec{\sigma}_1 \cdot \vec{\sigma}_3 \; \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \; S_6$$

chiral 3NF at N³LO is ready to be used

Numerical implementation in progress, applications to appear...

Skibinsky et al., arXiv:1107.5163, to appear in PRC

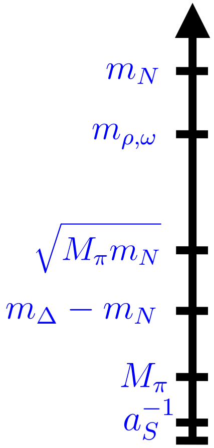
explicit expressions for all F_i, $R_i(q_1,q_2,q_3)$ and $S_i(q_1,q_2,q_3)$ are available in Bernard, EE, Krebs, Meißner, PRC77 (08) 064004

no unknown LECs contribute → parameter-free!

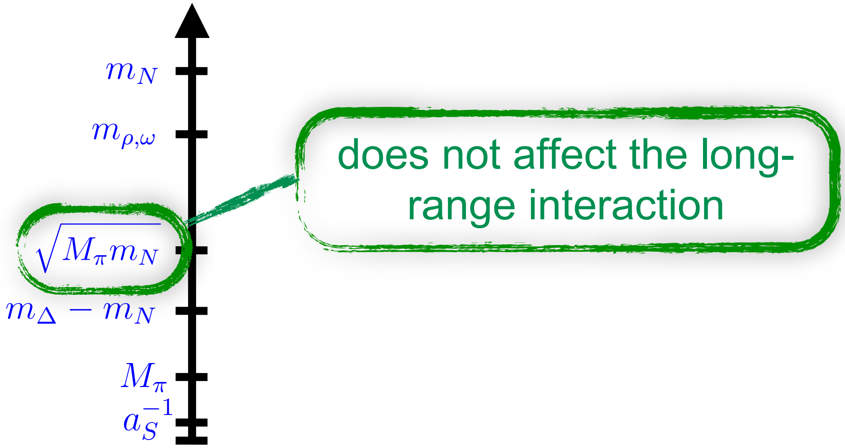
Chiral 3NF at N^3LO:
already converged?
Is it the end of the story?

Chiral 3NF at N^3LO:
already converged?
Is it the end of the story?

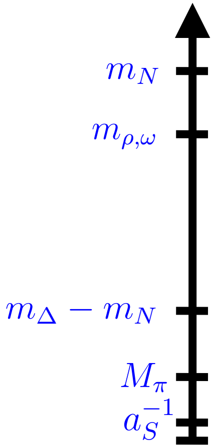
Chiral nuclear forces & the role of the Δ



Chiral nuclear forces & the role of the Δ

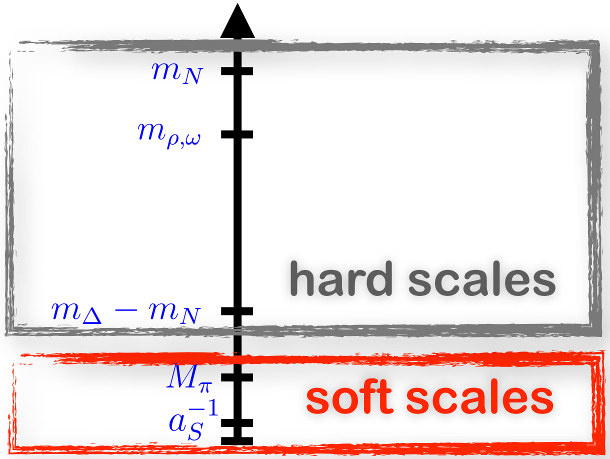


Chiral nuclear forces & the role of the Δ

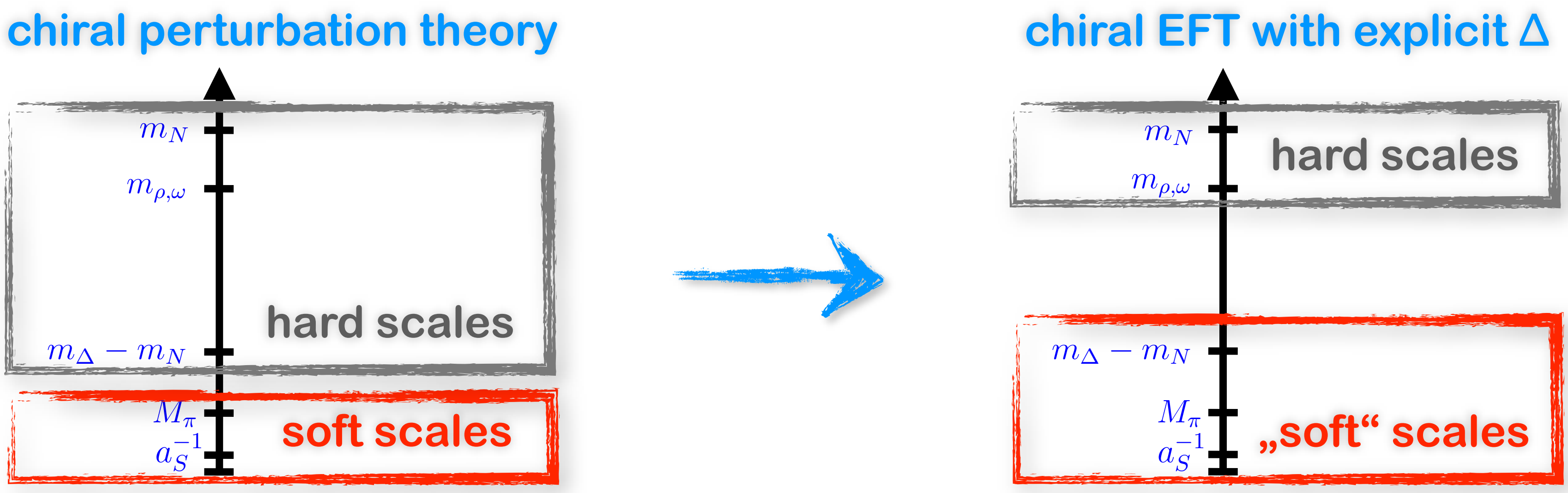


Chiral nuclear forces & the role of the Δ

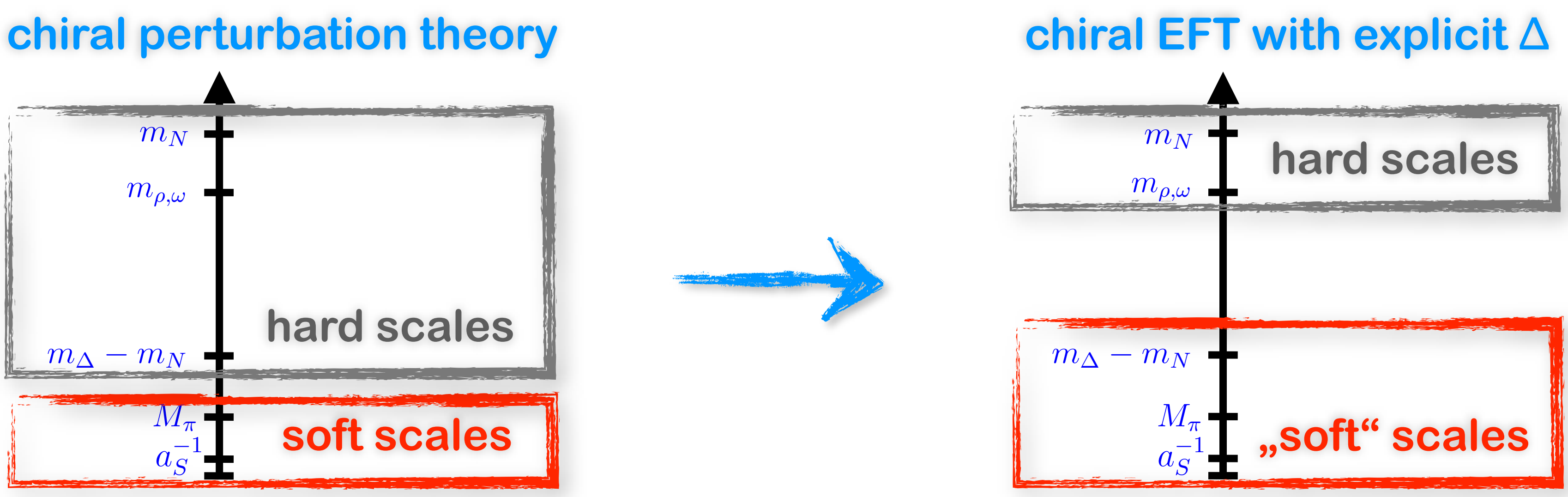
chiral perturbation theory



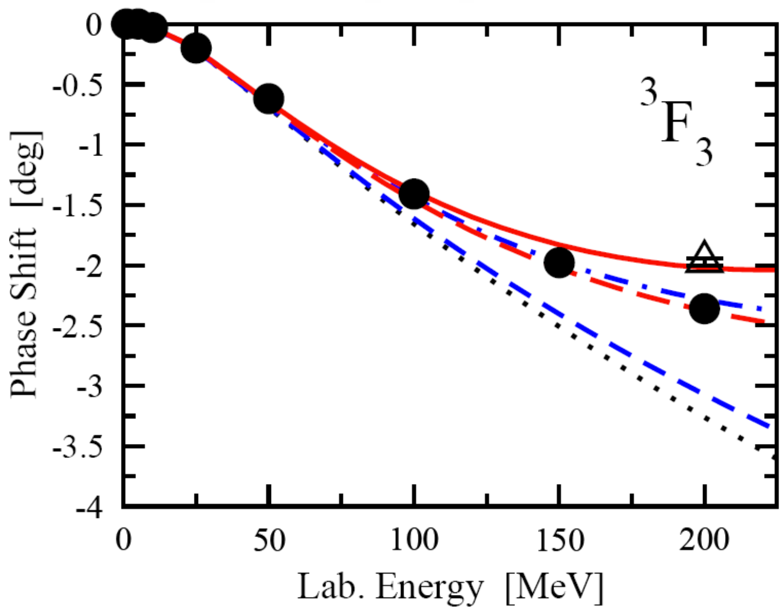
Chiral nuclear forces & the role of the Δ

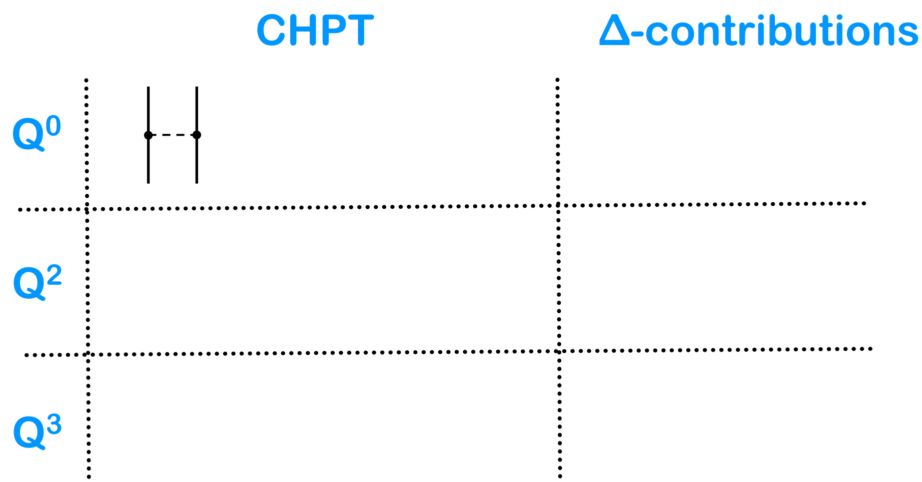


Chiral nuclear forces & the role of the Δ

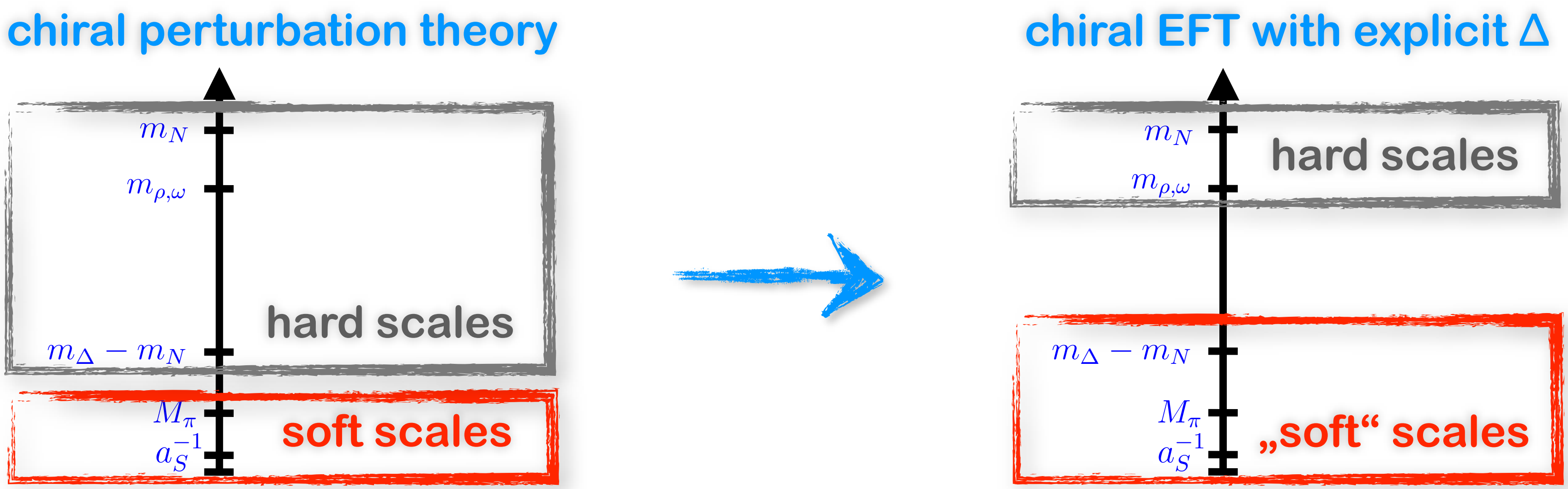


neutron-proton peripheral scattering

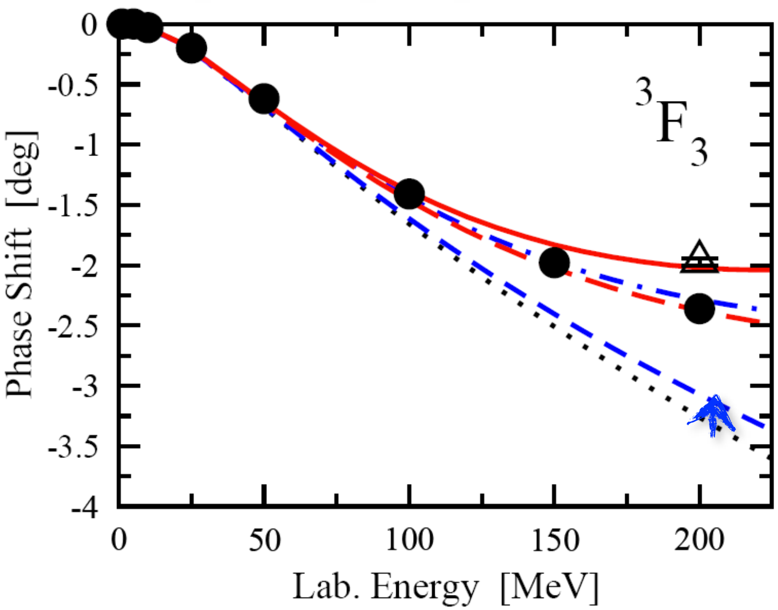


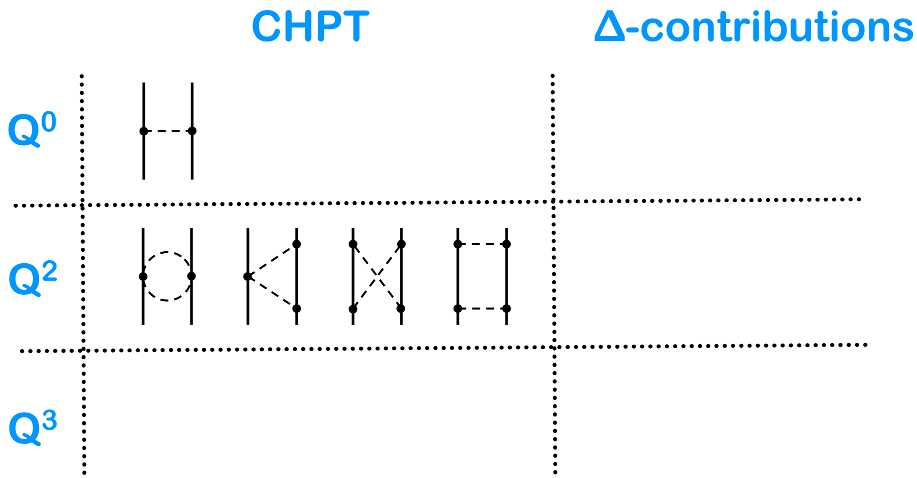


Chiral nuclear forces & the role of the Δ



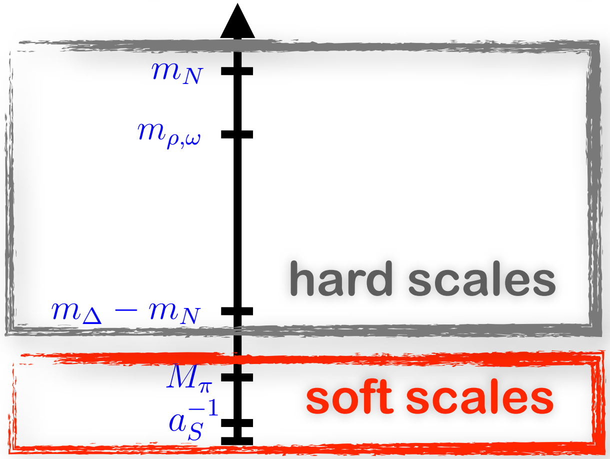
neutron-proton peripheral scattering

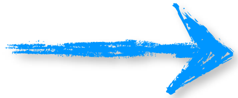




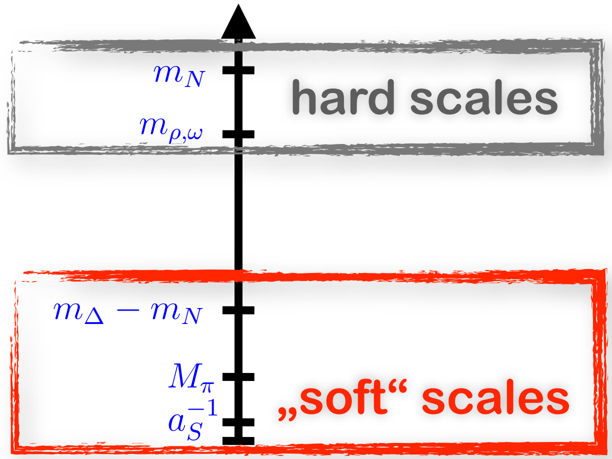
Chiral nuclear forces & the role of the Δ

chiral perturbation theory

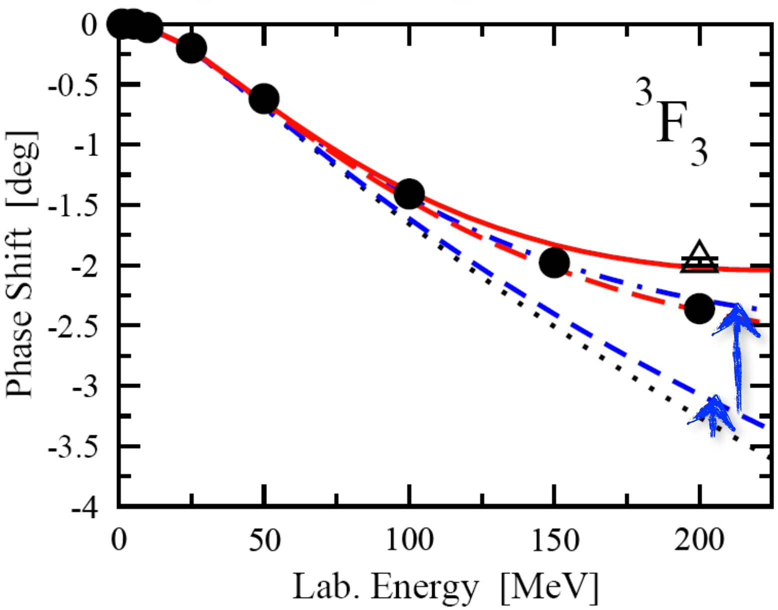


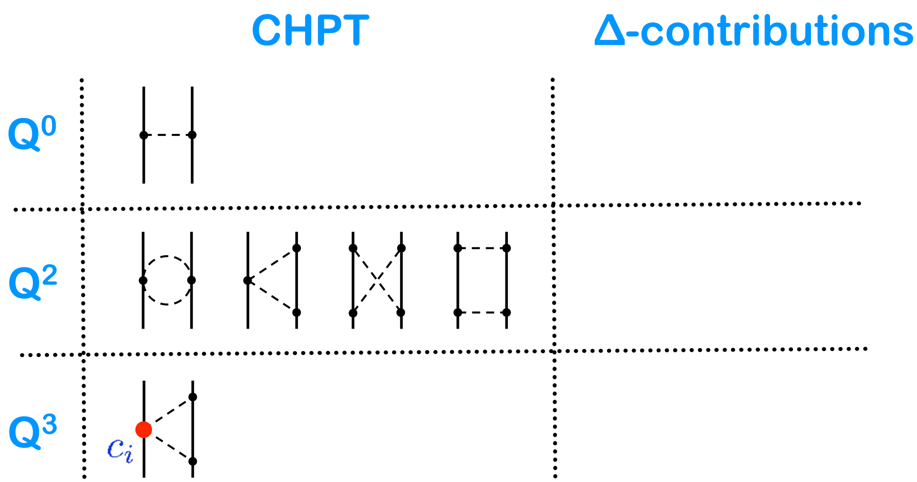


chiral EFT with explicit Δ

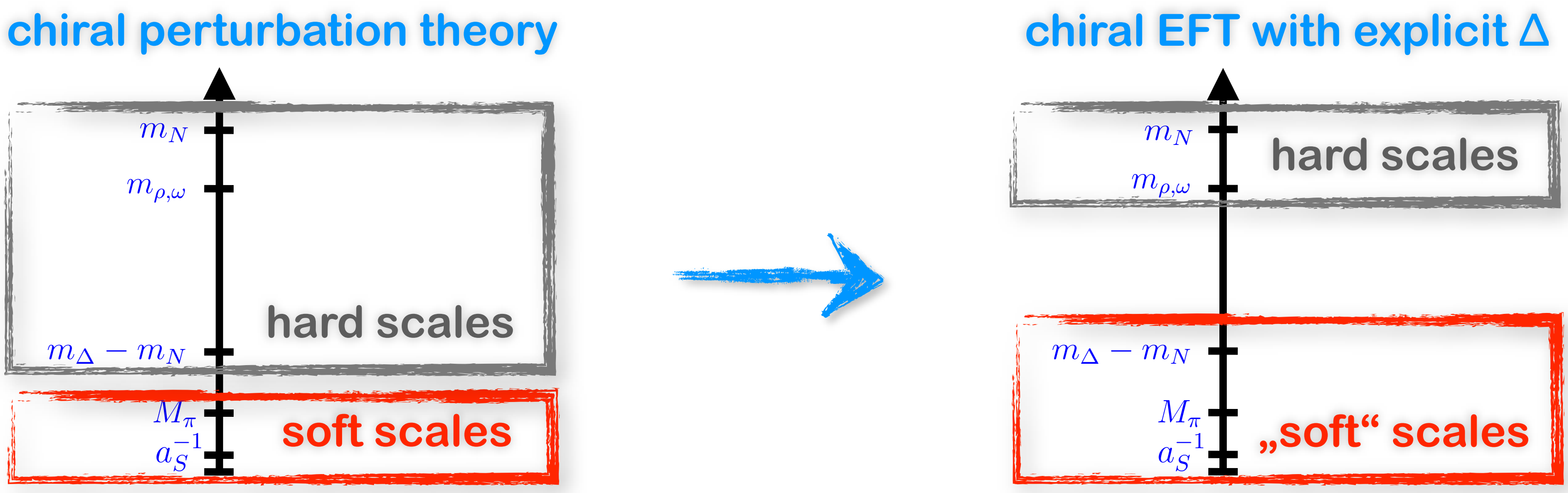


neutron-proton peripheral scattering

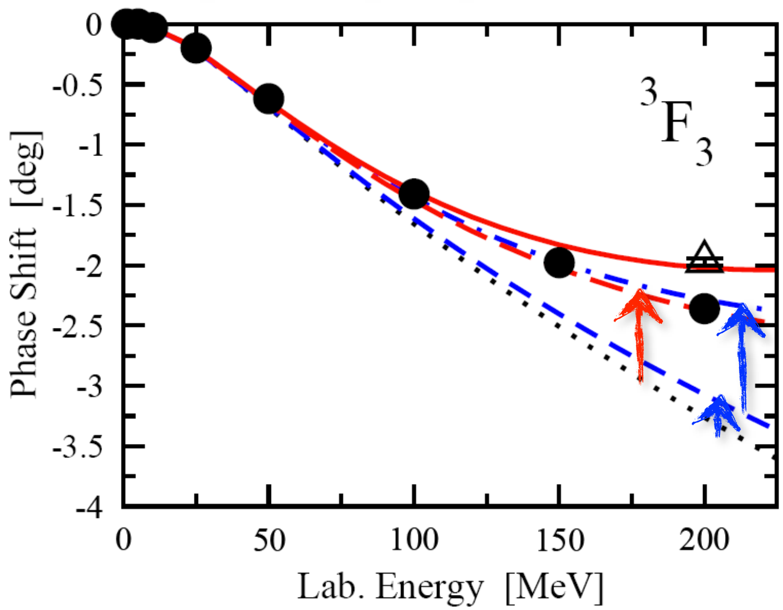


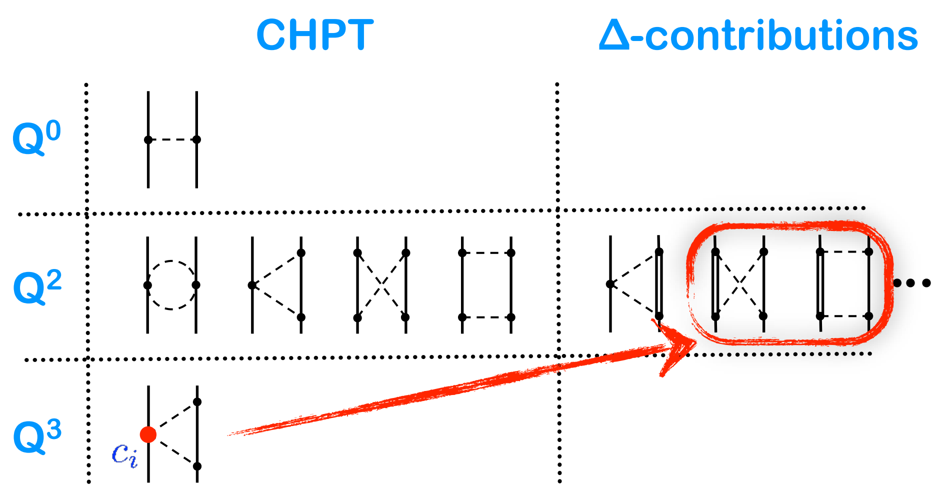


Chiral nuclear forces & the role of the Δ



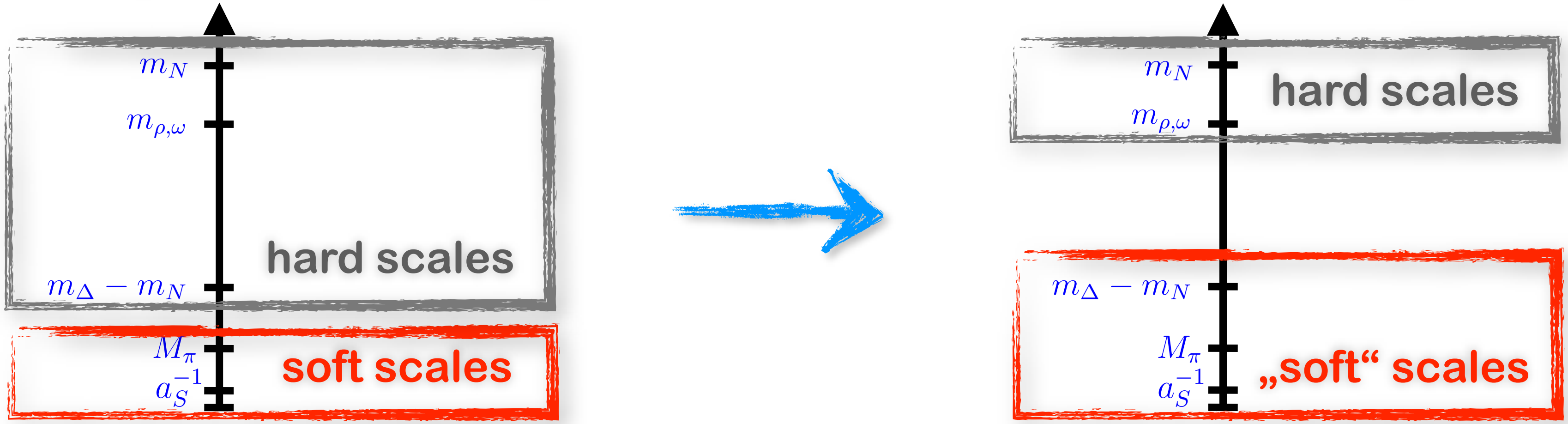
neutron-proton peripheral scattering



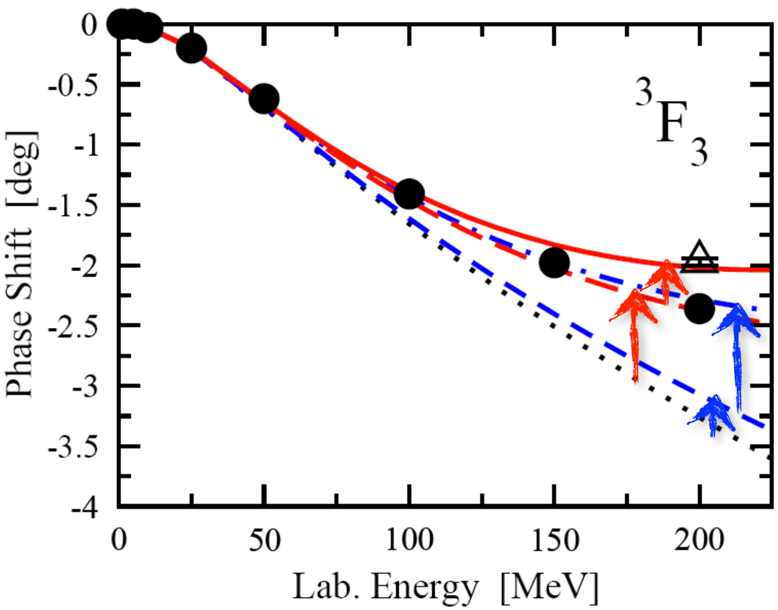


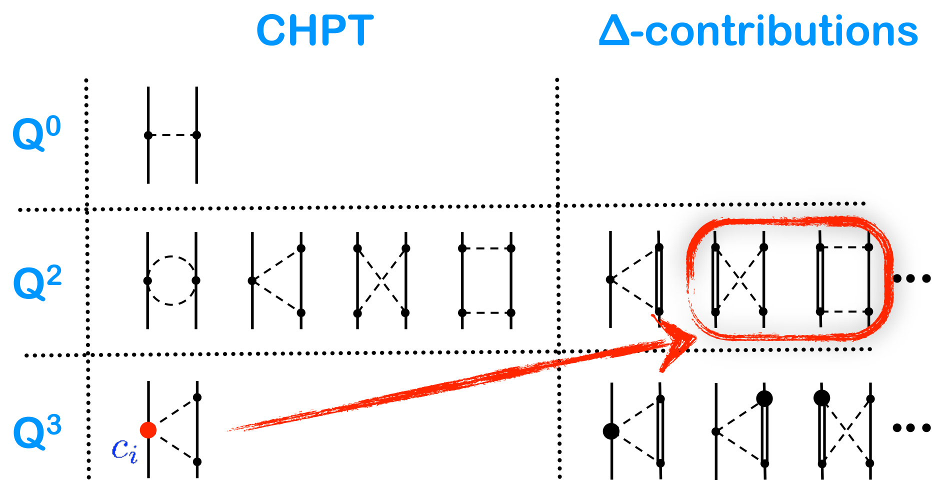
Chiral nuclear forces & the role of the Δ

chiral perturbation theory → **chiral EFT with explicit Δ**

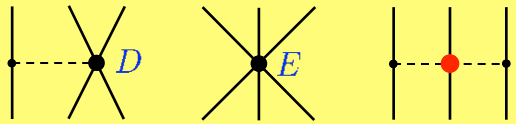
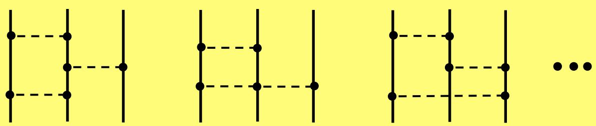


neutron-proton peripheral scattering

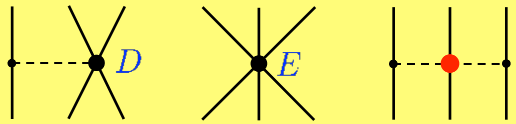
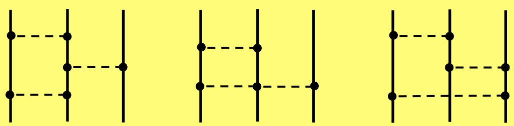
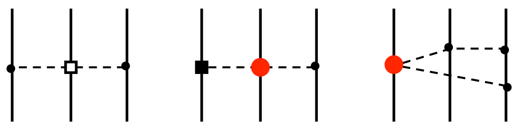




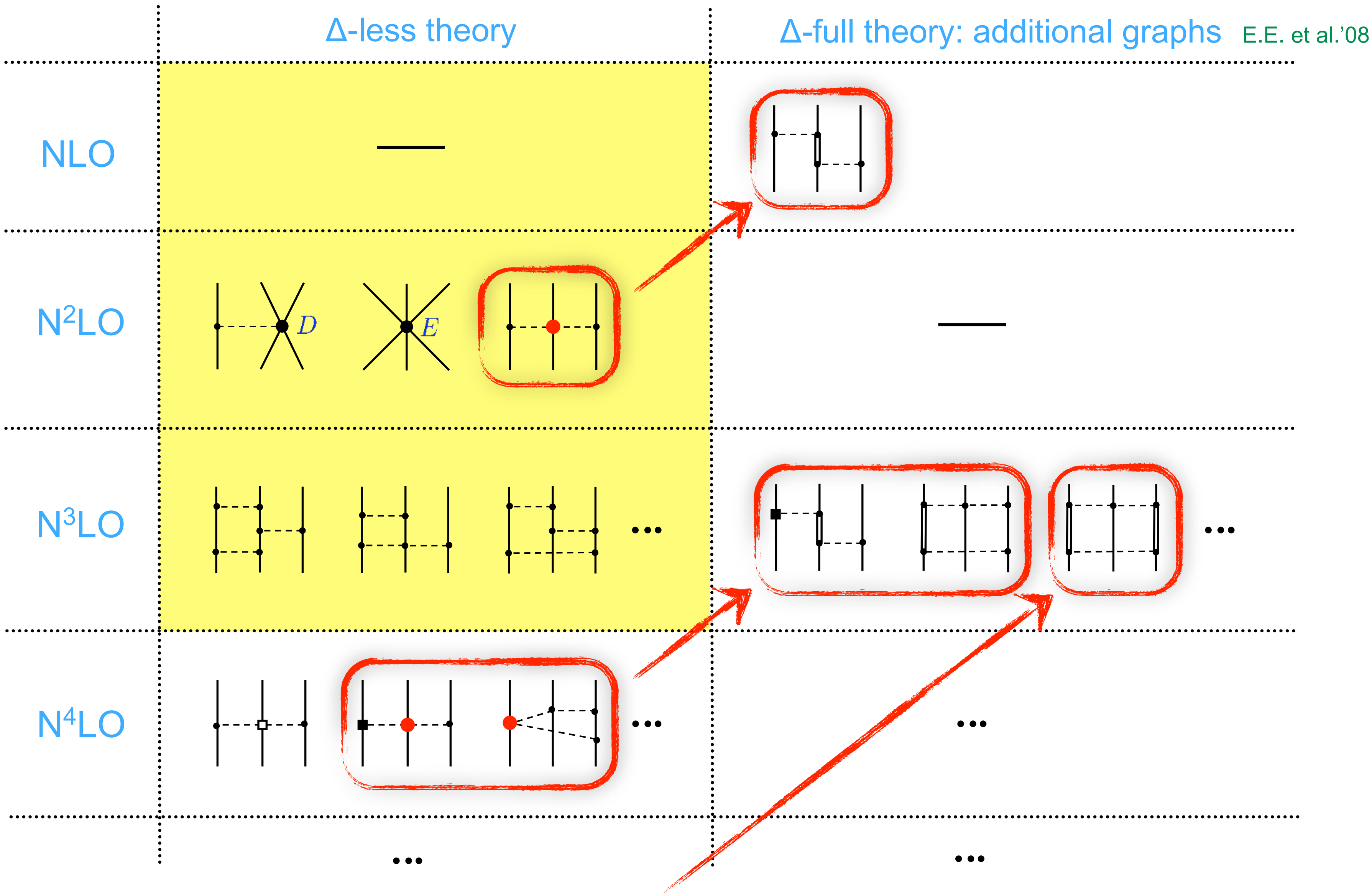
Chiral expansion of the 3NF

	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO		
N^3LO		

Chiral expansion of the 3NF

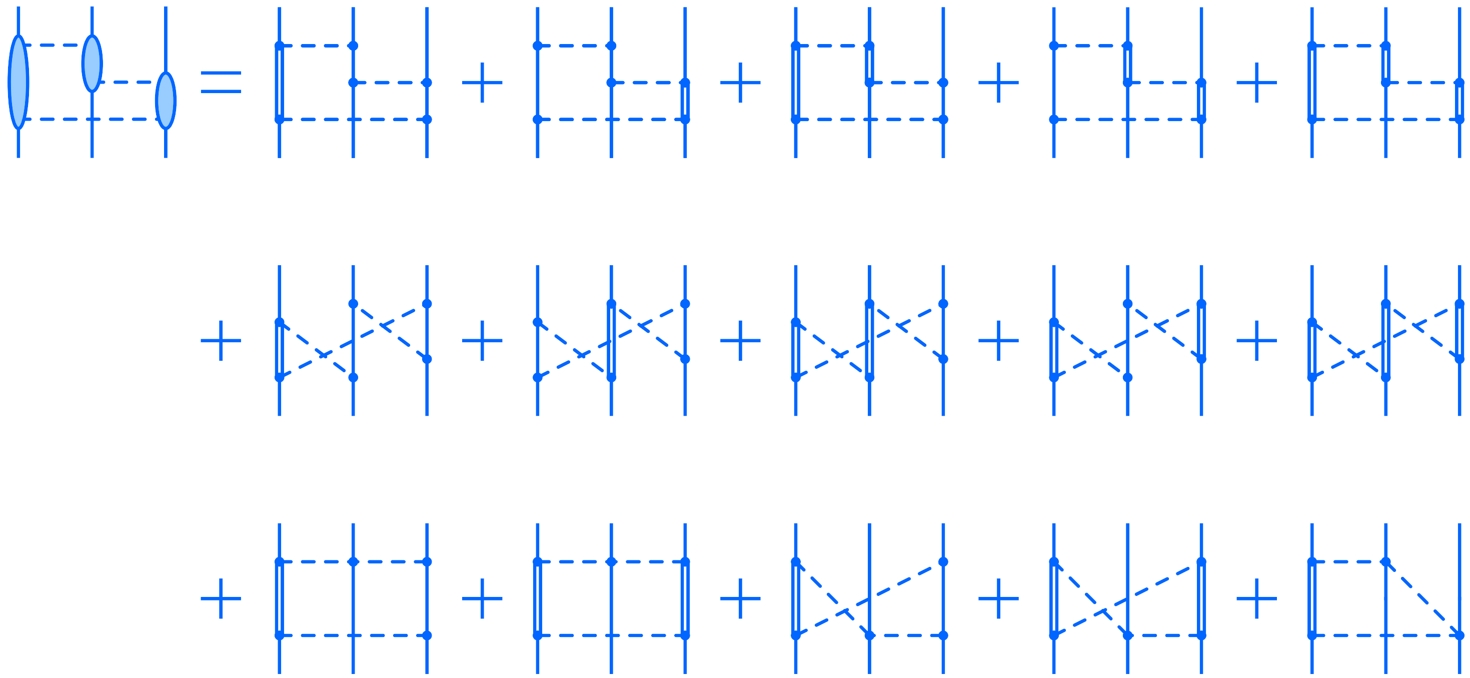
	Δ-less theory	Δ-full theory: additional graphs
NLO	—	
N^2LO		
N^3LO	 ...	
N^4LO	 ...	
	...	

Chiral expansion of the 3NF



Chiral EFT- Δ: ring topology

Krebs, EE, in progress...



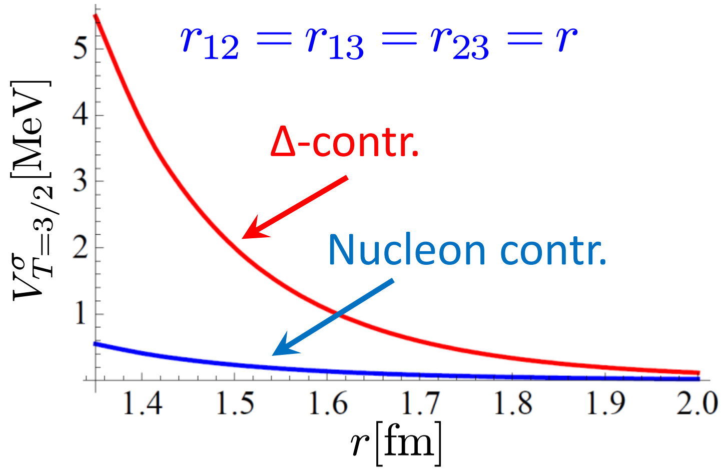
Notice:

- parameter-free,
- no suppression found for contributions from diagrams with two and three intermediate Δ...

Chiral EFT- Δ: ring topology (preliminary)

Krebs, EE, in progress...

Δ contributions to the ring topology at N³LO are considerably larger than the ones emerging in the Δ-less theory



$$V = \sum_i [\text{spin-space}]_i \times [\text{isospin}]_i \times f_i(r_{12}, r_{23}, r_{13})$$

„Form factors" for $r_{12} = r_{23} = r_{13} \sim M_\pi^{-1}$:

Δ-less theory			contributions of the Δ		
$\vec\sigma_2 \cdot \hat r_{12} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}$	$\times$	(0.47 MeV)	$\vec\sigma_2 \cdot \hat r_{12} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}$	$\times$	(4.01 MeV)
$\vec\sigma_1 \cdot \hat r_{13} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(−0.30 MeV)	1	$\times$	(−3.12 MeV)
$\vec\sigma_2 \cdot \hat r_{13} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}$	$\times$	(−0.27 MeV)	$\vec\sigma_2 \cdot \hat r_{13} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}$	$\times$	(−2.35 MeV)
$\vec\sigma_2 \cdot \hat r_{13}\, \vec\sigma_3 \cdot \hat r_{12}$	$\times$	(0.23 MeV)	$\vec\sigma_2 \cdot \hat r_{13}\, \vec\sigma_3 \cdot \hat r_{12}$	$\times$	(1.18 MeV)
$\vec\sigma_1 \cdot \hat r_{12} \times \hat r_{13}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(0.20 MeV)	$\vec\sigma_1 \cdot \vec\sigma_2$	$\times$	(1.07 MeV)
$\vec\sigma_1 \cdot \hat r_{12} \times \hat r_{13}\, \vec\sigma_3 \cdot \hat r_{12} \times \hat r_{13}\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(−0.18 MeV)	$\vec\sigma_2 \cdot \hat r_{12} \times \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(1.06 MeV)
$\vec\sigma_2 \cdot \vec\sigma_3$	$\times$	(0.15 MeV)	$\vec\sigma_1 \cdot \hat r_{12} \times \hat r_{13}\, \vec\sigma_3 \cdot \hat r_{13} \times \hat r_{23}\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(1.02 MeV)
$\vec\sigma_1 \cdot \vec\sigma_3\, \boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(0.14 MeV)	$\boldsymbol\tau_2 \cdot \boldsymbol\tau_3$	$\times$	(−0.86 MeV)
$\vec\sigma_1 \cdot \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{23}$	$\times$	(−0.13 MeV)	$\vec\sigma_2 \cdot \hat r_{23}\, \vec\sigma_3 \cdot \hat r_{23}$	$\times$	(−0.69 MeV)
...			...		

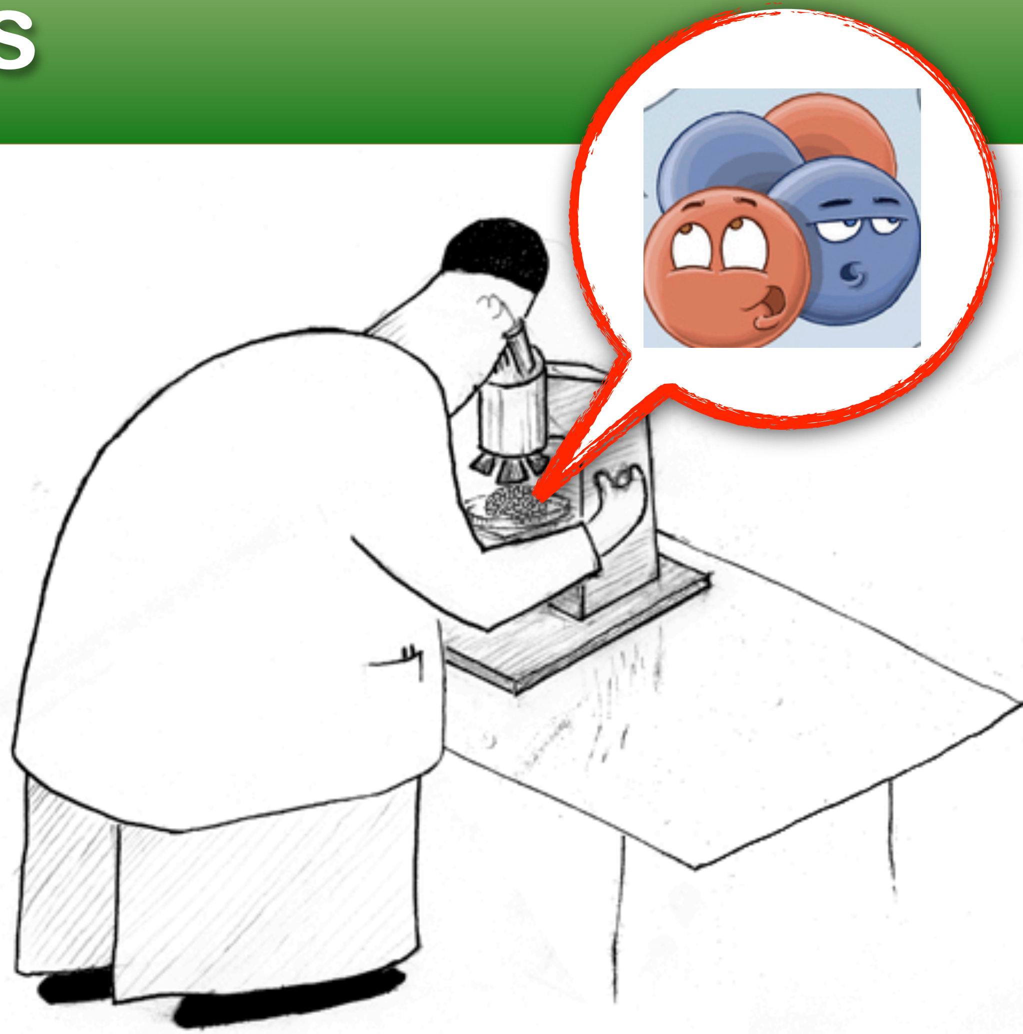
Chiral EFT- Δ: ring topology (preliminary)

Krebs, EE, in progress...

The new terms in the chiral 3NF will be tested in the deuteron breakup experiment in COSY (polarization observables, nucleon energy in the range 30...50 MeV)

Δ-less theory		contributions of the Δ	
$\vec{\sigma}_2 \cdot \hat{r}_{12} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}$	$\times$ (0.47 MeV)	$\vec{\sigma}_2 \cdot \hat{r}_{12} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}$	$\times$ (4.01 MeV)
$\vec{\sigma}_1 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (−0.30 MeV)	1	$\times$ (−3.12 MeV)
$\vec{\sigma}_2 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}$	$\times$ (−0.27 MeV)	$\vec{\sigma}_2 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}$	$\times$ (−2.35 MeV)
$\vec{\sigma}_2 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{12}$	$\times$ (0.23 MeV)	$\vec{\sigma}_2 \cdot \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{12}$	$\times$ (1.18 MeV)
$\vec{\sigma}_1 \cdot \hat{r}_{12} \times \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (0.20 MeV)	$\vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\times$ (1.07 MeV)
$\vec{\sigma}_1 \cdot \hat{r}_{12} \times \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{12} \times \hat{r}_{13}\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (−0.18 MeV)	$\vec{\sigma}_2 \cdot \hat{r}_{12} \times \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (1.06 MeV)
$\vec{\sigma}_2 \cdot \vec{\sigma}_3$	$\times$ (0.15 MeV)	$\vec{\sigma}_1 \cdot \hat{r}_{12} \times \hat{r}_{13}\, \vec{\sigma}_3 \cdot \hat{r}_{13} \times \hat{r}_{23}\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (1.02 MeV)
$\vec{\sigma}_1 \cdot \vec{\sigma}_3\, \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (0.14 MeV)	$\boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3$	$\times$ (−0.86 MeV)
$\vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{23}$	$\times$ (−0.13 MeV)	$\vec{\sigma}_2 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{23}$	$\times$ (−0.69 MeV)
$\ldots$		$\ldots$	

Few-N physics with external probes



External probes: π-deuteron scattering

Pion-nucleon amplitude at threshold (in the isospin limit): $T^{ba}_{\pi N} \propto \left[\delta^{ab} \boldsymbol{a}^+ + i\epsilon^{bac}\tau^c \boldsymbol{a}^-\right]$

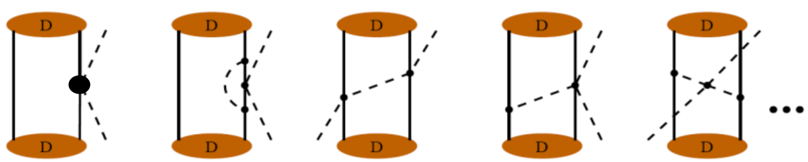
Recent data on hadronic atoms:

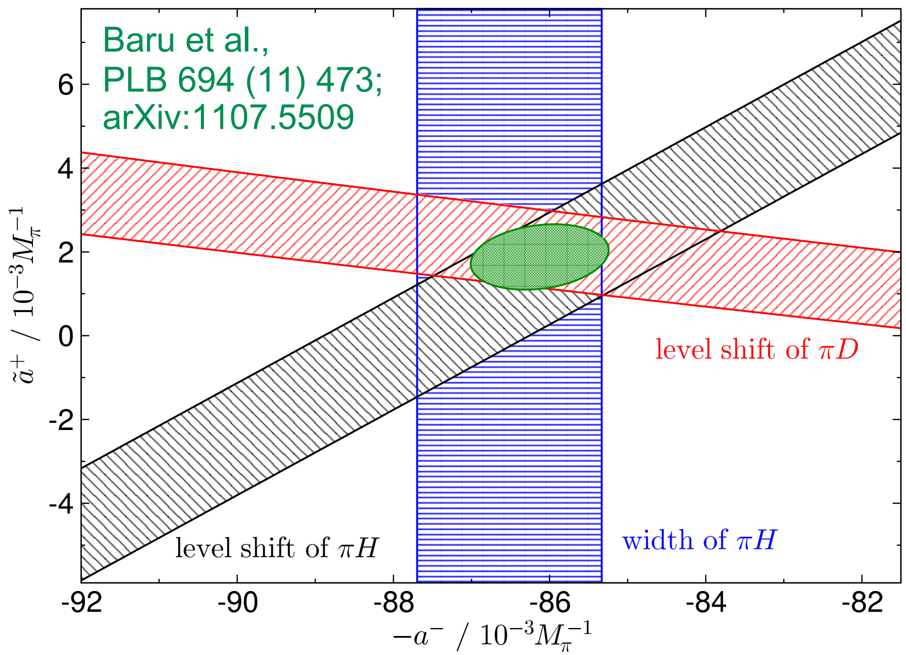
$\boldsymbol{\pi} H$: $\epsilon_{1s} = (-7.120 \pm 0.012)\,\mathrm{eV}$, $\Gamma_{1s} = (0.823 \pm 0.019)\,\mathrm{eV}$ Gotta et al., Lect. Notes. Phys. 745 (08) 165

$\boldsymbol{\pi} D$: $\epsilon^D_{1s} = (2.356 \pm 0.031)\,\mathrm{eV}$ Strauch et al., Eur. Phys. J A47 (11) 88

Use chiral EFT to extract information on a^+ and a^- from $a_{\pi d}$:

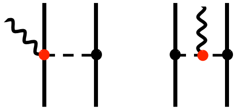
Weinberg '92; Beane et al.'98,'03; Liebig et al.'11; Meißner et al. '06; Baru et al. '04-'11;...

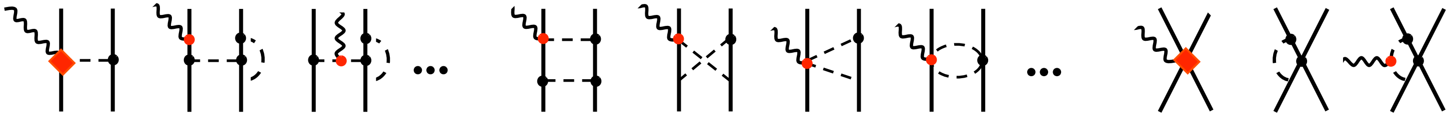




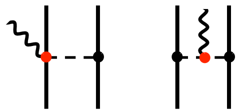
- careful analysis of IB effects
- radiative corrections included
- the scale $\sqrt{M_\pi m_N}$ must be taken into account (3-body singularity, dispersive corrections)

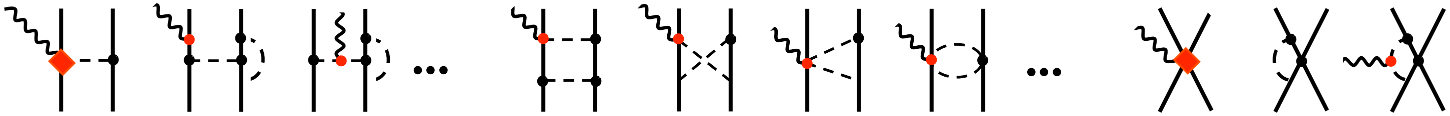
External probes: e.m. exchange currents

Order eQ^{-1} :  ← well known since decades Chemtob, Rho, Friar, Riska, Adam, …

Order eQ : 

External probes: e.m. exchange currents

Order eQ^{-1} :  well known since decades Chemtob, Rho, Friar, Riska, Adam, …

Order eQ : 

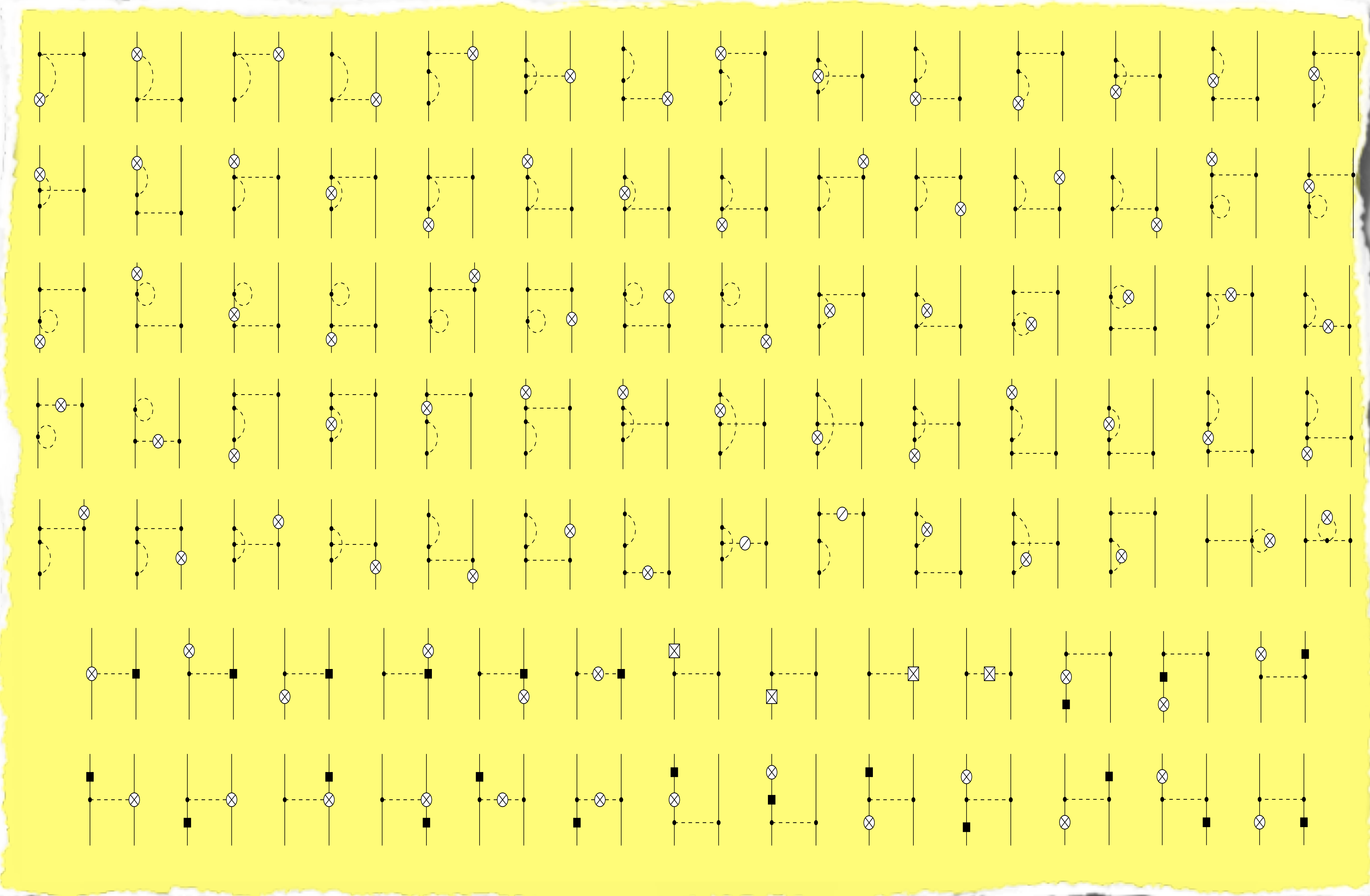
- **Threshold kinematics** Park, Min, Rho '95; Park, Kubodera, Min, Rho; Song, Lazauskas, Park, Min, ...

  Application to $np \to d\gamma$ at threshold: $\sigma_{1N} = 306.6 \text{ mb} \longrightarrow \boxed{\sigma_{1N+2N} = 334 \pm 3 \text{ mb}}$

  to be compared with $\sigma_{\text{exp}} = 334.2 \pm 0.5 \text{ mb}$

- **General kinematics** Pastore, Schiavilla, Girlanda, Viviani, '08-'11; Kölling, Krebs, EE, Meißner, '09-'11

External probes: e.m. exchange currents



External probes: e.m. exchange currents

$$
\begin{aligned}
\vec{J}_{1\pi} &= \frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\,[\vec{q}_1\times\vec{q}_2]\left[\tau_2^3\, f_1(k)+\vec{\tau}_1\cdot\vec{\tau}_2\, f_2(k)\right]+[\vec{\tau}_1\times\vec{\tau}_2]^3\,\frac{\vec{\sigma}_2\cdot\vec{q}_2}{q_2^2+M_\pi^2}\Big\{\vec{k}\times[\vec{q}_2\times\vec{\sigma}_1]\, f_3(k)\\
&+\ \vec{k}\times[\vec{q}_1\times\vec{\sigma}_1]\, f_4(k)+\vec{\sigma}_1\cdot\vec{q}_1\left(\frac{\vec{k}}{k^2}-\frac{\vec{q}_1}{q_1^2+M_\pi^2}\right) f_5(k)+\left[\frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\vec{q}_1-\vec{\sigma}_1\right] f_6(k)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
f_1(k) &= 2ie\frac{g_A}{F_\pi^2}\,\bar{d}_8\,,\quad f_2(k)=2ie\frac{g_A}{F_\pi^2}\,\bar{d}_9\,,\quad f_3(k)=-ie\frac{g_A}{64F_\pi^4\pi^2}\left[g_A^3\,(2L(k)-1)+32F_\pi^2\pi^2\bar{d}_{21}\right]\,,\\
f_4(k) &= -ie\frac{g_A}{4F_\pi^2}\,\bar{d}_{22}\,,\quad f_5(k)=-ie\frac{g_A^2}{384F_\pi^4\pi^2}\left[2(4M_\pi^2+k^2)L(k)+\left(6\,\bar{l}_6-\frac{5}{3}\right)k^2-8M_\pi^2\right]\,,\\
f_6(k) &= -ie\frac{g_A}{F_\pi^2}M_\pi^2\,\bar{d}_{18}\,,
\end{aligned}
$$

External probes: e.m. exchange currents

$$\vec{J}_{1\pi} = \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} [\vec{q}_1 \times \vec{q}_2] \left[\tau_2^3 \, f_1(k) + \vec{\tau}_1 \cdot \vec{\tau}_2 \, f_2(k)\right] + [\vec{\tau}_1 \times \vec{\tau}_2]^3 \frac{\vec{\sigma}_2 \cdot \vec{q}_2}{q_2^2 + M_\pi^2} \Big\{ \vec{k} \times [\vec{q}_2 \times \vec{\sigma}_1] \, f_3(k)$$

$$+ \vec{k} \times [\vec{q}_1 \times \vec{\sigma}_1] \, f_4(k) + \vec{\sigma}_1 \cdot \vec{q}_1 \left(\frac{\vec{k}}{k^2} - \frac{\vec{q}_1}{q_1^2 + M_\pi^2} \right) f_5(k) + \left[\frac{\vec{\sigma}_1 \cdot \vec{q}_1}{q_1^2 + M_\pi^2} \vec{q}_1 - \vec{\sigma}_1 \right] f_6(k) \Big\}$$

$$f_1(k) = 2ie \frac{g_A}{F_\pi^2} \bar{d}_8 \qquad f_2(k) = 2ie \frac{g_A}{F_\pi^2} \bar{d}_9 \qquad f_3(k) = -ie \frac{g_A}{64 F_\pi^4 \pi^2} \left[g_A^3 \, (2L(k) - 1) + 32 F_\pi^2 \pi^2 \bar{d}_{21} \right],$$

$$f_4(k) = -ie \frac{g_A}{4F_\pi^2} \bar{d}_{22}, \qquad f_5(k) = -ie \frac{g_A^2}{384 F_\pi^4 \pi^2} \left[2(4M_\pi^2 + k^2) L(k) + \left(6\bar{l}_6 - \frac{5}{3} \right) k^2 - 8 M_\pi^2 \right],$$

$$f_6(k) = -ie \frac{g_A}{F_\pi^2} M_\pi^2 \bar{d}_{18},$$

determined from other sources:
πN scattering, π photo-/electroproduction, ...

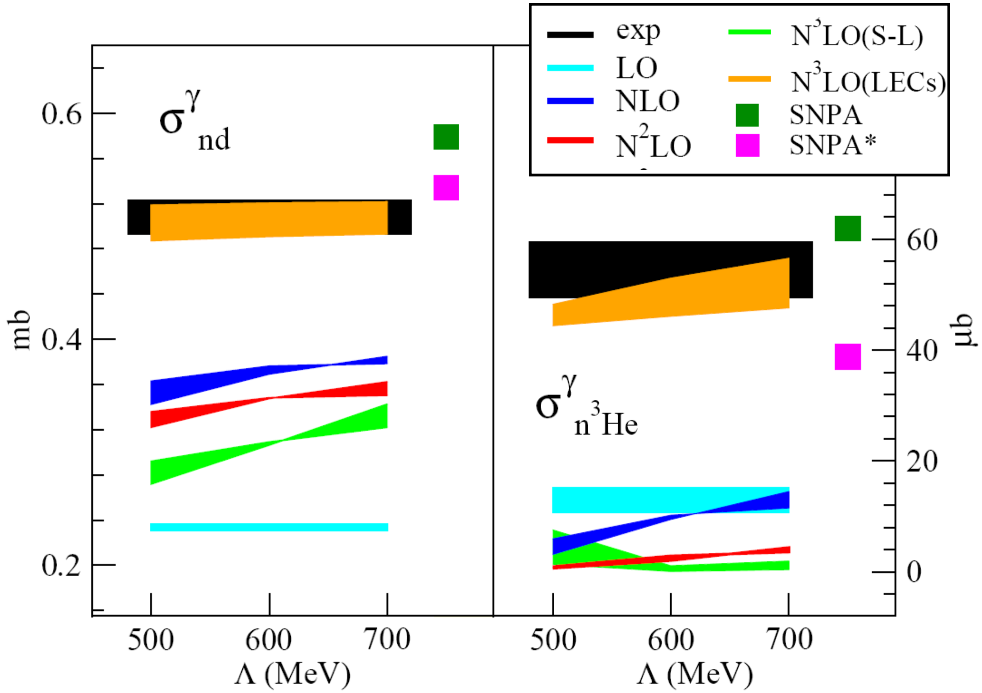
Explicit expressions for two-pion exchange and short-range currents and the corresponding charge densities are available as well.

- Kölling, Krebs, EE, Meißner, PRC80 (09); arXiv:1107.0602
- Pastore, Schiavilla, Goity PRC78 (08); Pastore, Girlanda, Schiavilla, Viviani et al., PRC80 (09); arXiv:1106.4539

External probes: e.m. exchange currents

Radiative capture of light nuclei

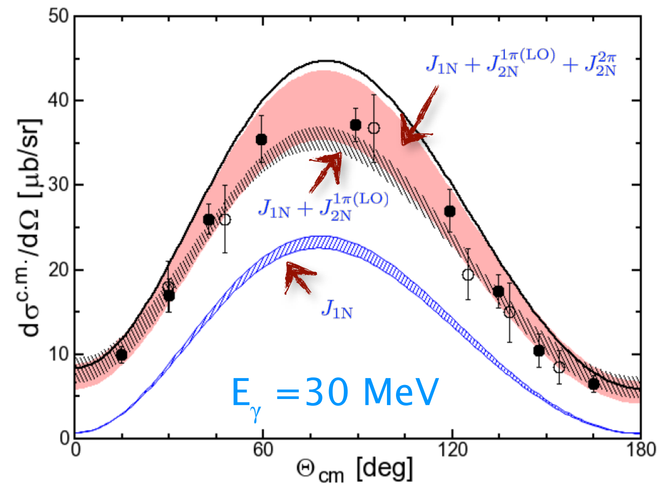
Girlanda et al., PRL 105 (10) 232502

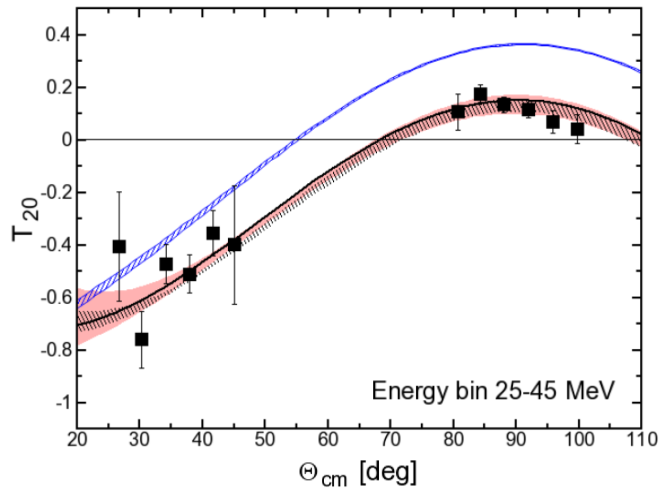


- LECs determined assuming Δ-dominance + magnetic moments of ^2H, ^3H, ^3He + σ^{γ}_{np}
- predictions for nd, n^3He radiative capture reactions for thermal neutrons (MEC dominated)
- related recent work: Lazauskas, Song, Park ‘09

Deuteron photodisintegration

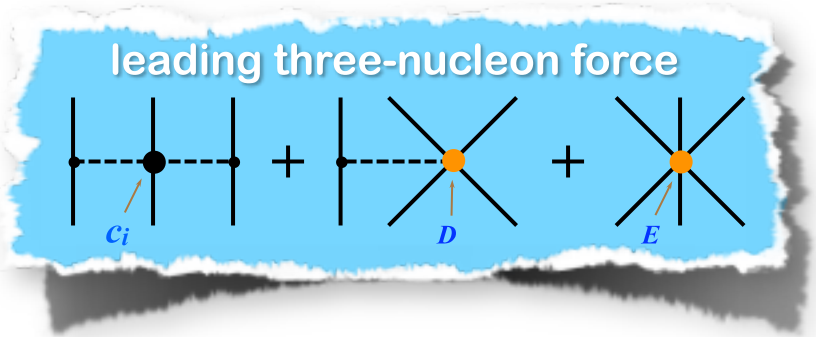
Rozpedzik et al., PRC 83 (11) 064004

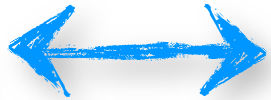


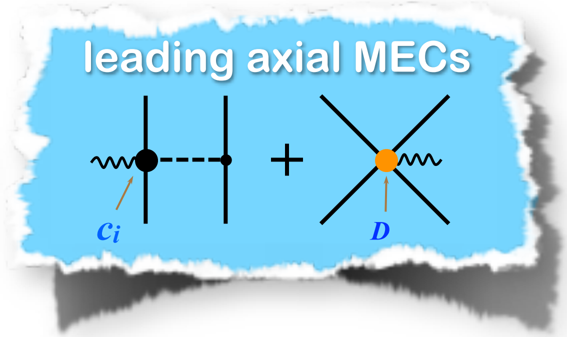


External probes: axial currents

Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502

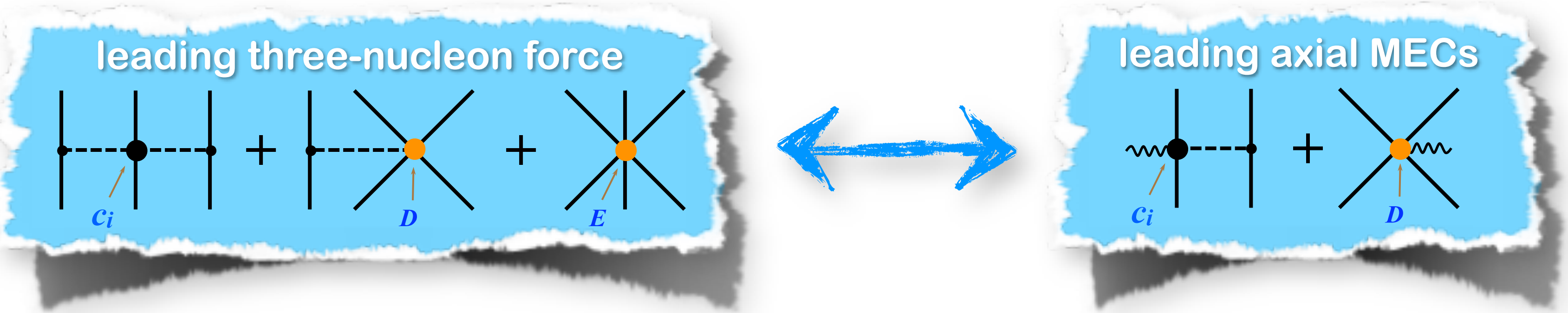


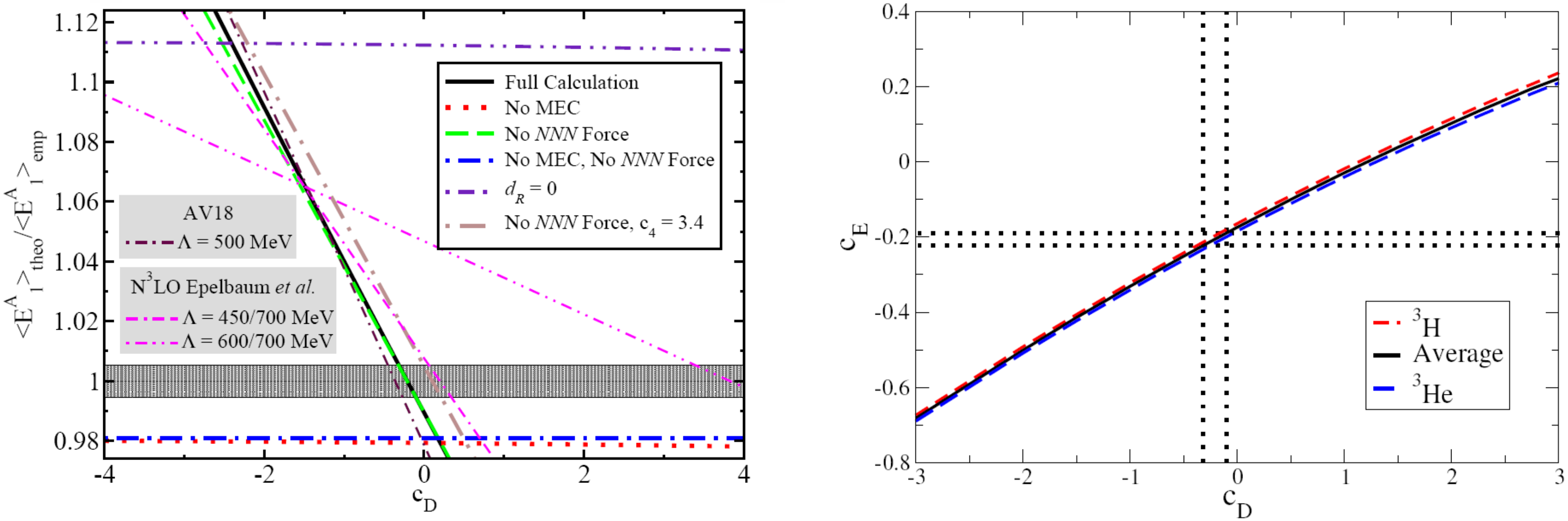




External probes: axial currents

Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502

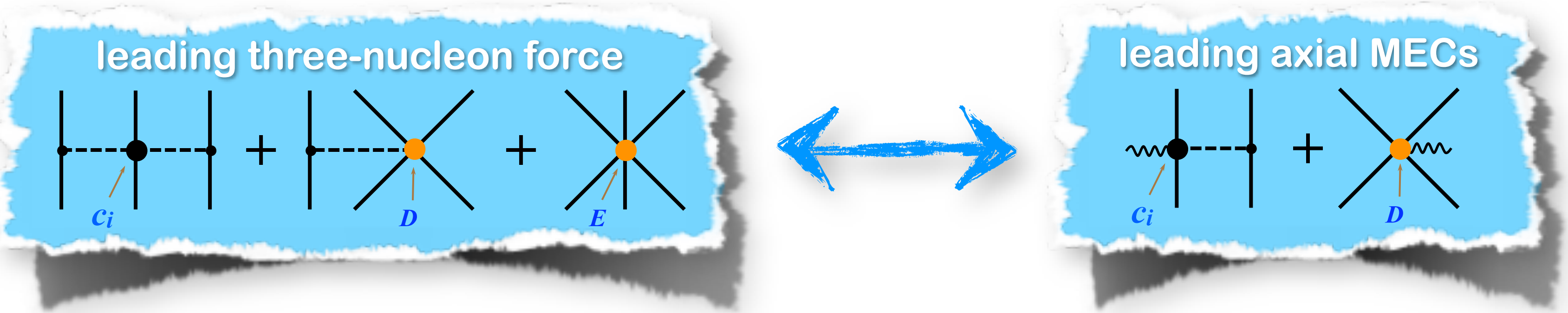


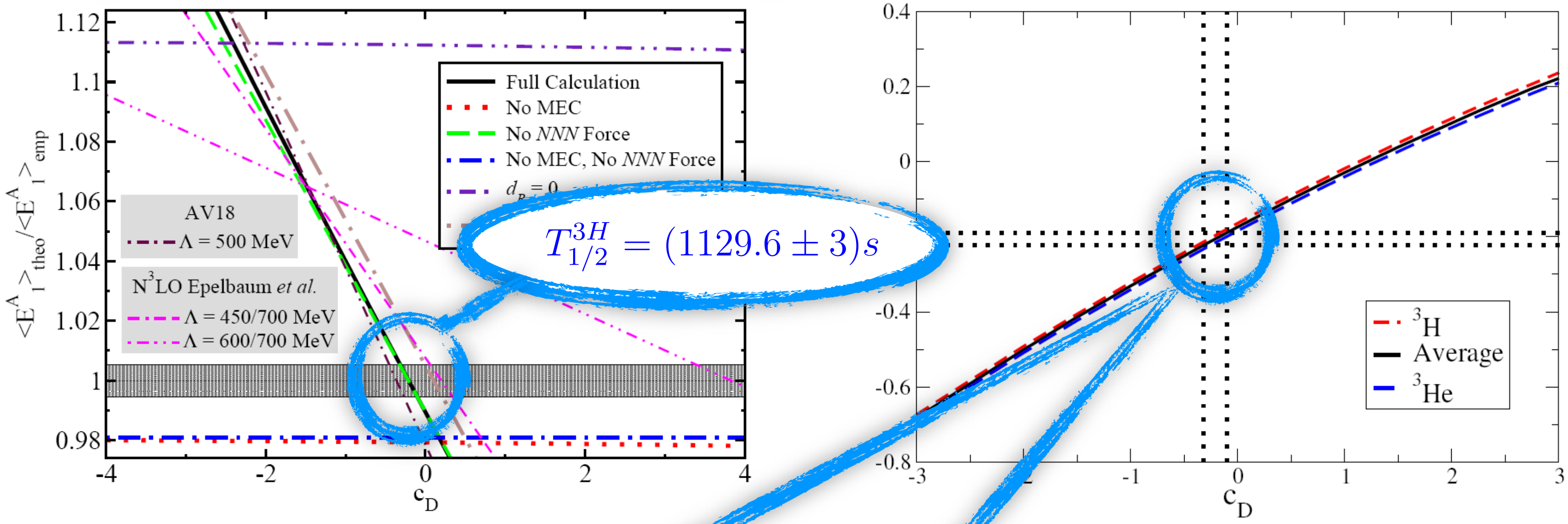


	^{3}H		^{3}He		^{4}He	
	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$
NN	−7.852(4)	1.651(5)	−7.124(4)	1.847(5)	−25.39(1)	1.515(2)
$NN+NNN$	−8.473(4)	1.605(5)	−7.727(4)	1.786(5)	−28.50(2)	1.461(2)
Expt.	−8.482	1.60	−7.718	1.77	−28.296	1.467(13)

External probes: axial currents

Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502





	³H		³He		⁴He	
	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2 \rangle^{1/2}$
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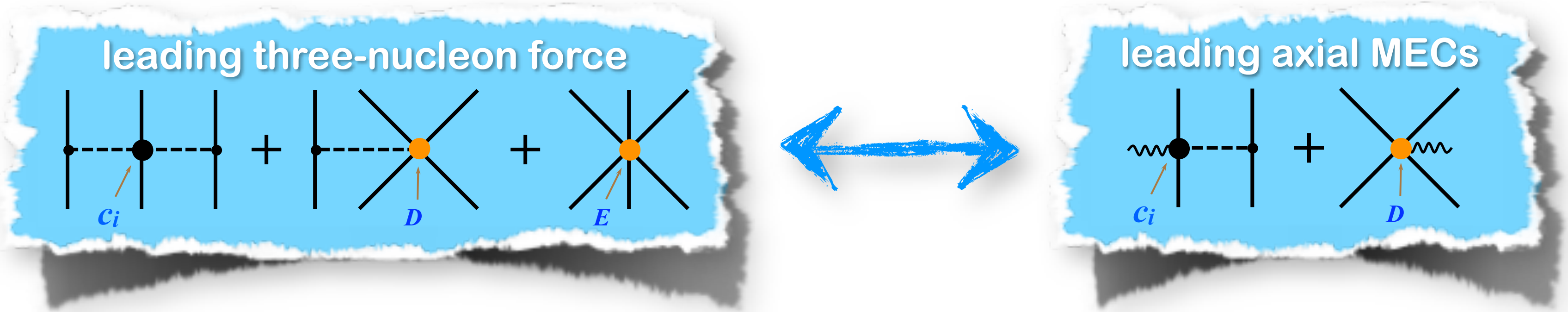
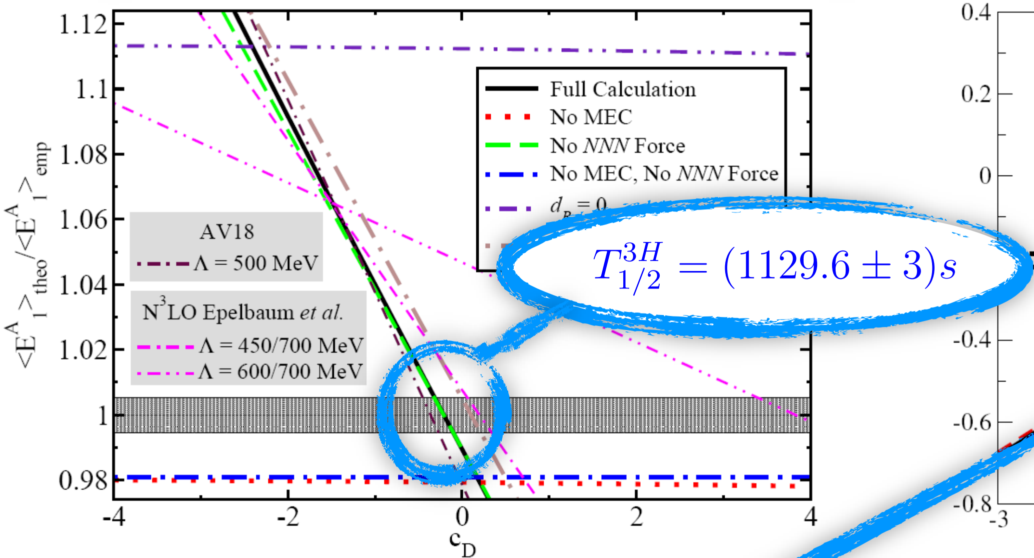
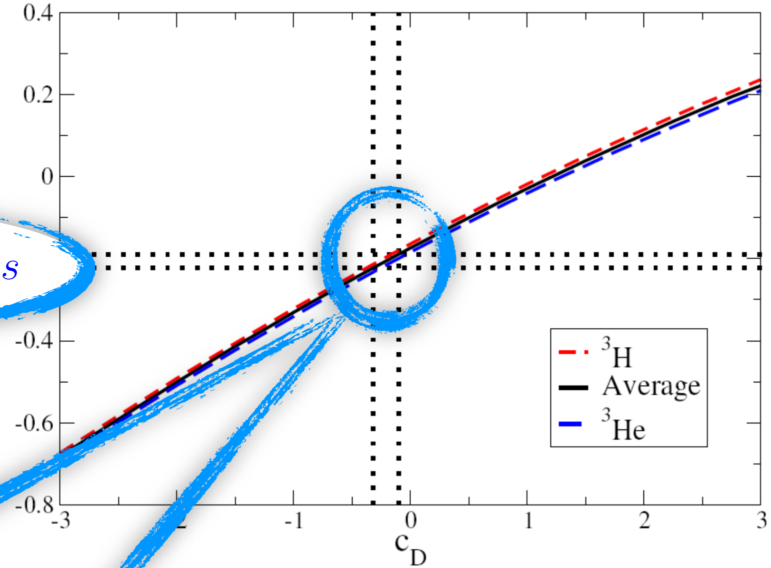
External probes: axial currents

Gazit, Quaglioni, Navratil, PRL 103 (2009) 102502







	^{3}H		^{3}He		^{4}He	
	$E_{g.s.}$	$\langle r_p^2\rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2\rangle^{1/2}$	$E_{g.s.}$	$\langle r_p^2\rangle^{1/2}$
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Expt.	−8.482	1.60	−7.718	1.77	−28.296	1.467(13)

External probes: axial currents

The determined value of D can be used to compute the muon doublet capture rate in

$$\mu^- + d \rightarrow n + n + \nu_\mu$$

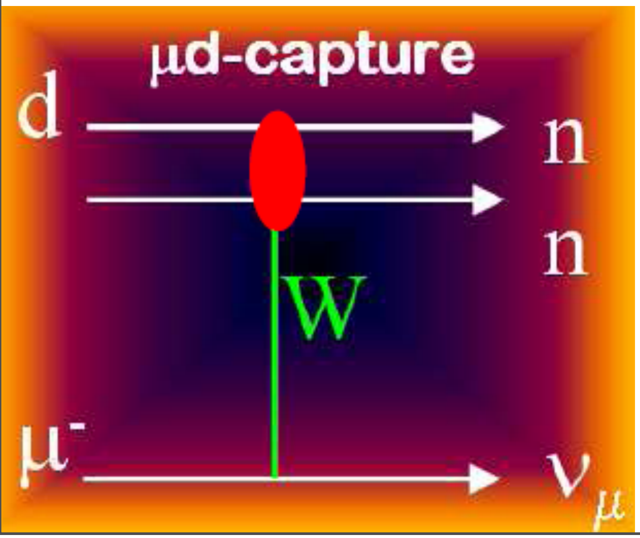
→ $\Lambda_{1/2} = (392.0 \pm 2.3) s^{-1}$ Marcucci et al., PRC 83 (11) 014002

(a somewhat different value reported in Adam et al., arXiv:1110.3183)

Exp: $\Lambda_{1/2} = (470.0 \pm 29) s^{-1}$ Martino '86

$\Lambda_{1/2} = (409.0 \pm 40) s^{-1}$ Cargnelli et al., '86, '87

New measurement planned by the MuSun Collaboration @ PSI: 1.5% accuracy for $\Lambda_{1/2}$



→
- Test chiral EFT
- Precision calculation of weak nuclear reactions

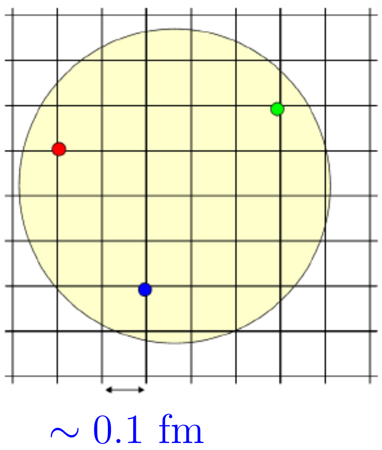
$$\begin{aligned} p + p &\rightarrow d + e^+ + \nu_e\,, \\ p + p + e^- &\rightarrow d + \nu_e\,, \\ p + {}^3He &\rightarrow {}^4He + e^+ + \nu_e\,, \\ {}^7Be + e^- &\rightarrow {}^7Li + \nu_e\,, \\ {}^8B &\rightarrow {}^8Be^* + e^+ + \nu_e \end{aligned}$$

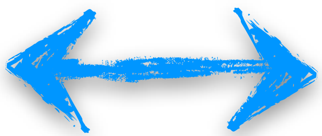
Few-N physics on the lattice

In collaboration with:
Dean Lee (North Carolina), Hermann Krebs (Bochum), Ulf-G. Meißner (Bonn/Jülich)

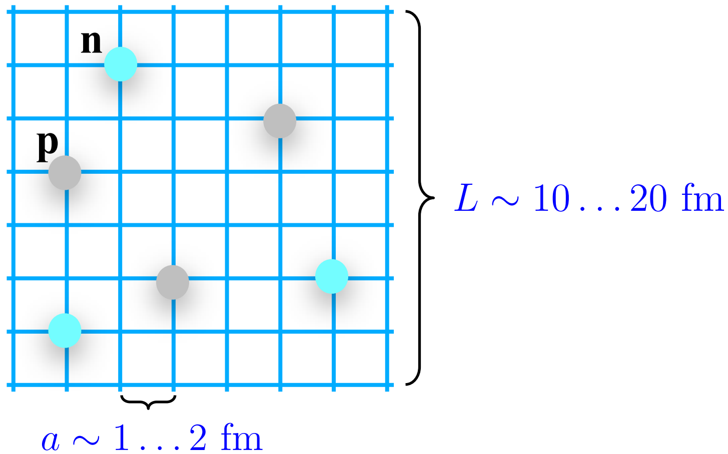
Borasoy, EE, Krebs, Lee, Meißner, EPJ A31 (07) 105; A34 (07) 185; A35 (08) 343; A35 (08) 357;
EE, Krebs, Lee, Meißner, EPJ A40 (09) 199; A41 (09) 125; A45 (10) 335; PRL 104 (10) 142501; PRL 106 (11) 192501

Lattice QCD





Chiral EFT on the lattice



- fundamental, the only parameters are m_q, α_{strong}
- hard to go beyond 1 hadron...

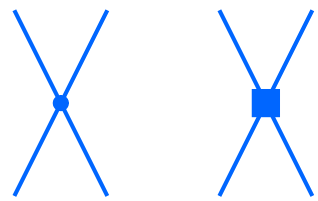
- effective hadronic description, LECs to be determined from the data/LQCD
- much more efficient for atomic nuclei

Calculation strategy

Lattice action (improved to minimize discr. errors, accurate to Q^3)

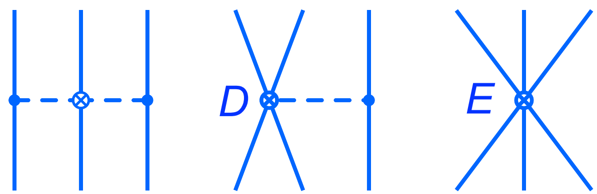
⇓ Lanczos method

Solve 2N Schröd. Eq. with the spherical wall boundary cond. ⟹ phase shifts ⟹ **fix** the LO and NLO (perturbatively) **contact terms**



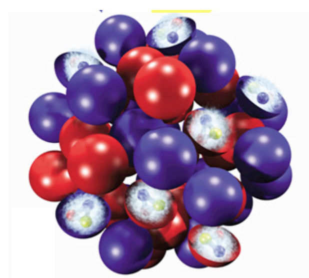
⇓ projection Monte Carlo (with auxiliary fields)

Determine the LECs D, E from ^{3}H and ^{4}He BEs ⟹ the nuclear Hamiltonian completely fixed up to NNLO (Q^3)



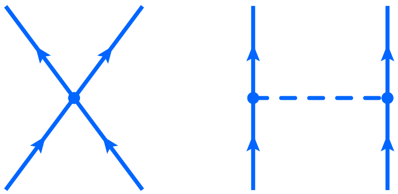
⇓ (Multi-channel) projection Monte Carlo with auxiliary fields

Simulate the ground (and excited) states of **light nuclei**



Lattice actions

Q^0

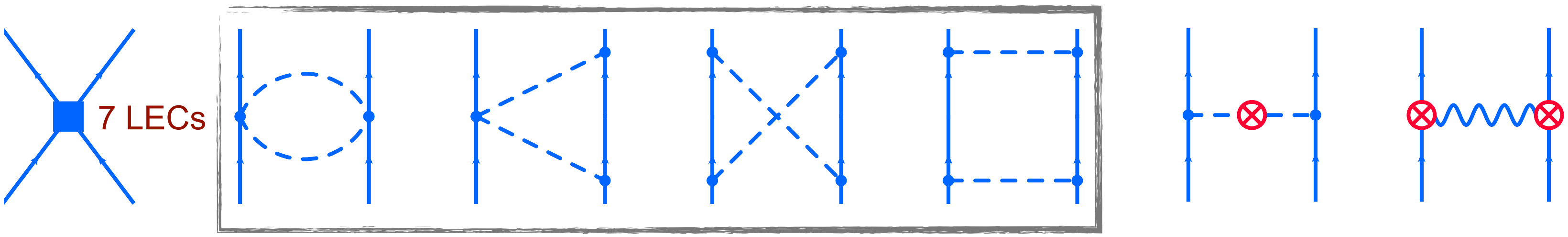


Different actions employed
LO1: no smearing,
LO2: smearing in all waves,
LO3: smearing in even-l waves

used in the simulation

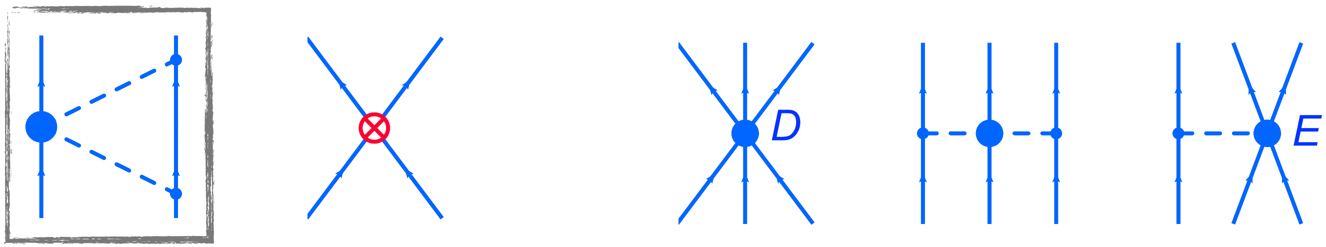
inserted perturbatively

Q^2

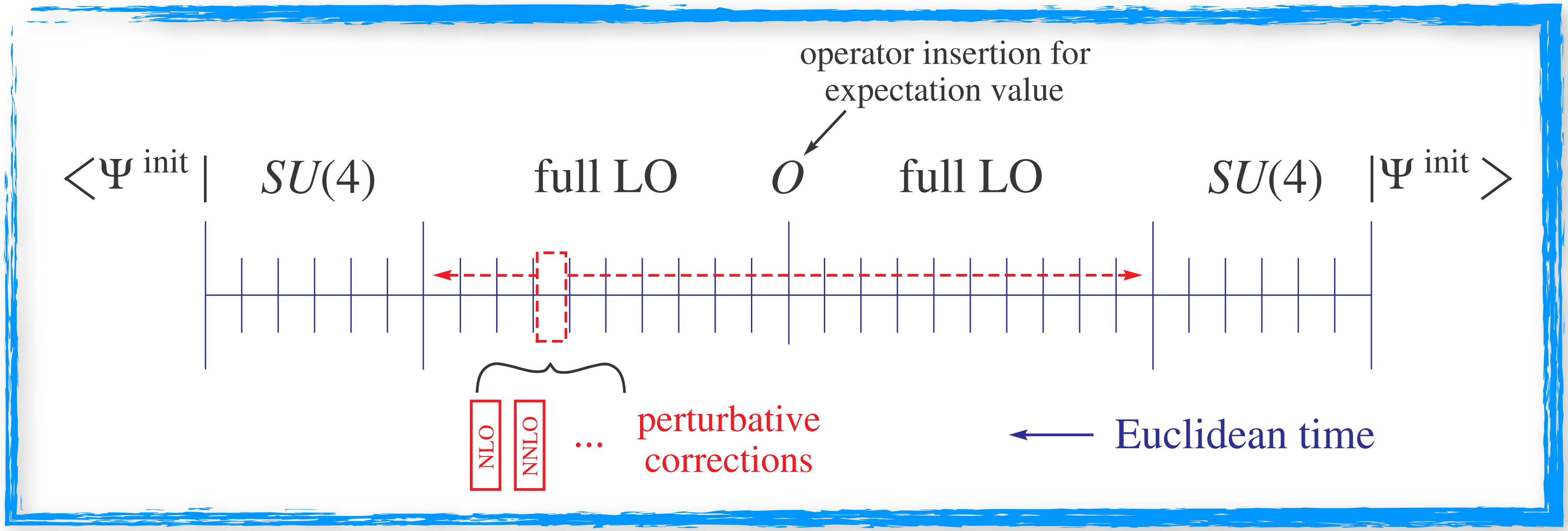


for a ~ 2 fm (Λ ~ 314 MeV) can well be represented by contact terms

Q^3



Ground state energies



Slater determinant of 1N states

- $|\Psi(t')\rangle = \left(M_{\mathrm{SU(4)}}\right)^{L_{t_0}} |\Psi^{\mathrm{init}}\rangle$ - cheap, no sign problem (even A) Lee'05,'07; Chen, Lee, Schäfer '04

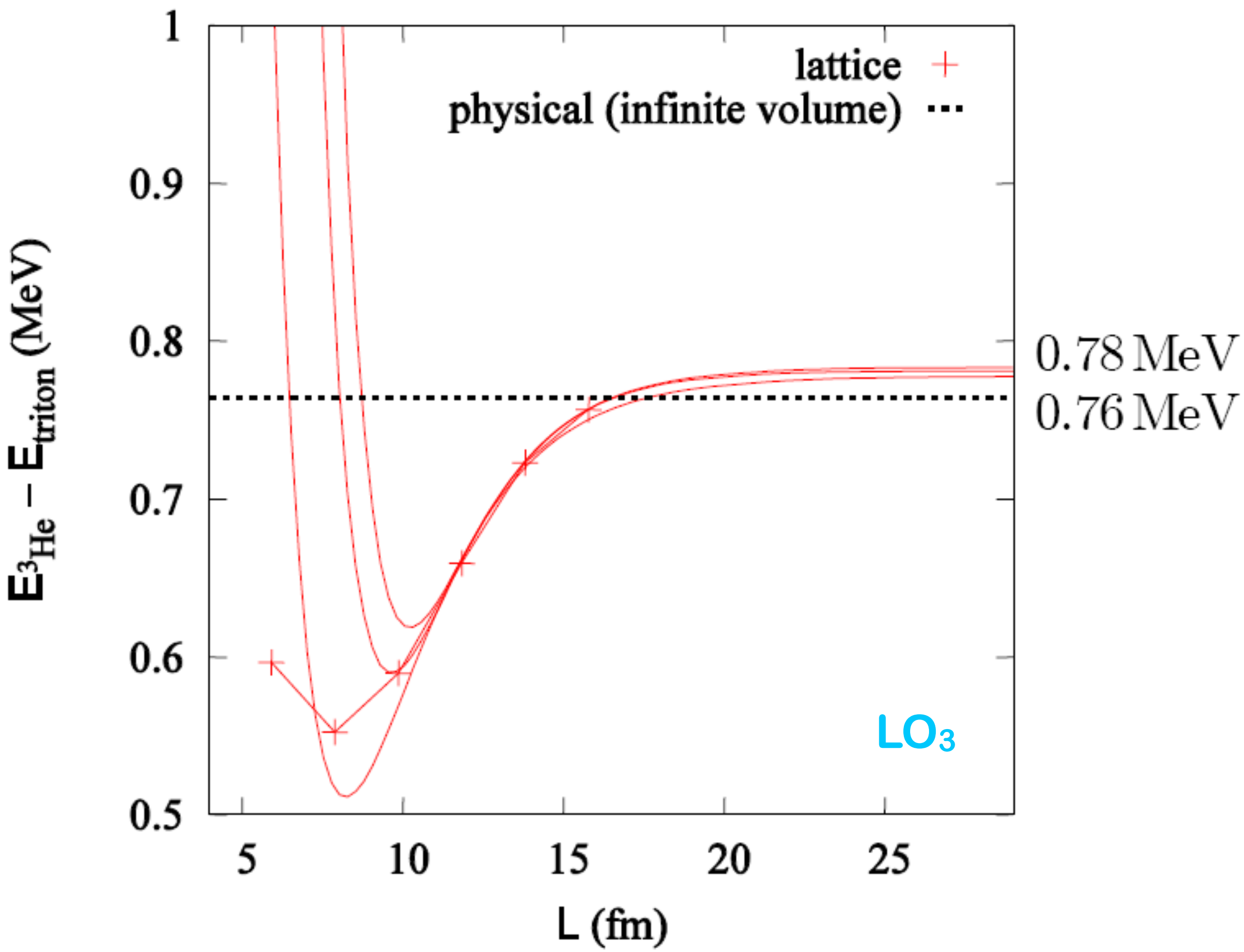
  $L_{t_0}\alpha_t$ transfer matrix (pion-less, SU(4)-inv.): $M_{\mathrm{SU(4)}} =: \exp(-H_{\mathrm{SU(4)}}\alpha_t)$:

- Transition amplitude: $Z_{\mathrm{LO}}(t) = \langle\Psi(t')| \left(M_{\mathrm{LO}}\right)^{L_t} |\Psi(t')\rangle$ with $M_{\mathrm{LO}} =: \exp\left(-H_{\mathrm{LO}}\alpha_t\right)$:

- Ground state energy: $\exp\left(-E_0^{\mathrm{LO}}\alpha_t\right) = \lim_{t\to\infty} Z(t+\alpha_t)/Z(t)$

^{3}H-^{3}He binding energy difference

E.E., Krebs, Lee, Meißner, PRL 104 (10) 142501



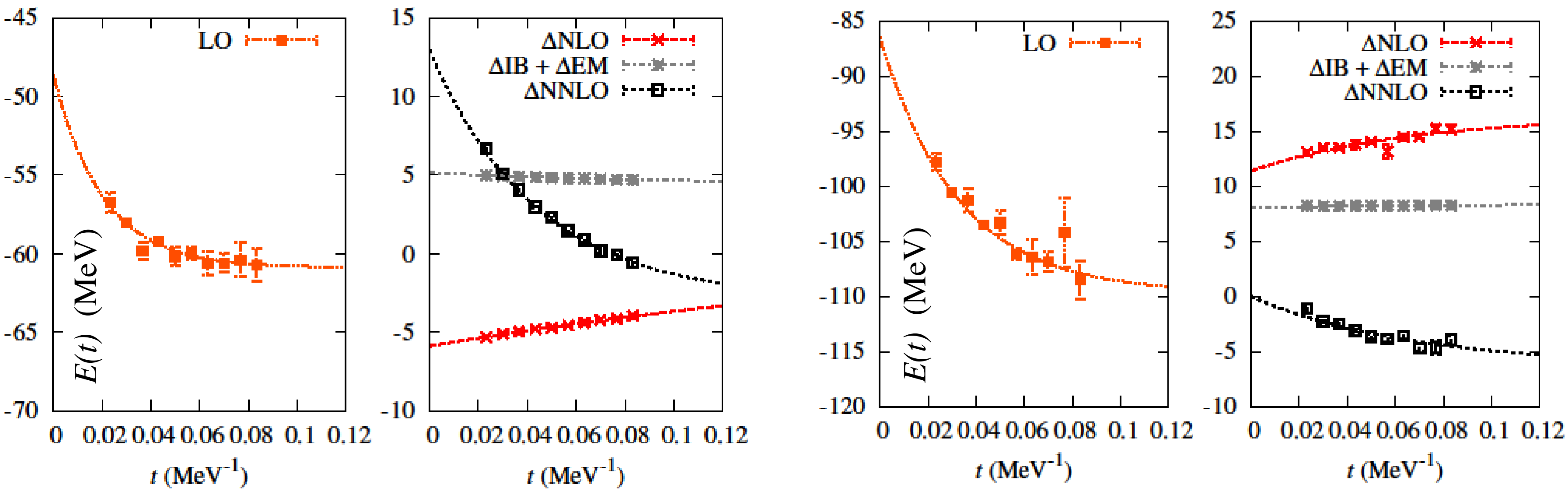
Infinite-volume extrapolations via: $E(L) = E(\infty) - \frac{C}{L} e^{-L/L_0} + \mathcal{O}\left(e^{-\sqrt{2}L/L_0}\right)$

Lüscher ’86

Ground states of ^{8}Be and ^{12}C

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501

Simulations of the ^{8}Be and ^{12}C GS energies (L=11.8 fm)

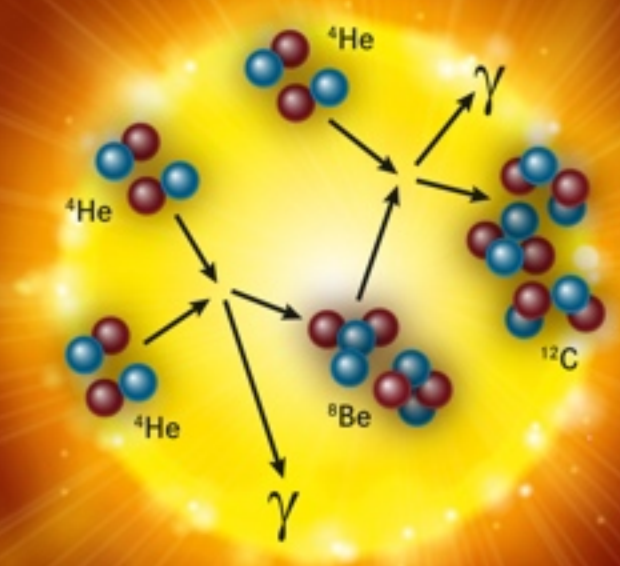


Various contributions to ^{4}He, ^{8}Be and ^{12}C GS energy

	^{4}He	^{8}Be	^{12}C
LO $[O(Q^0)]$	−24.8(2)	−60.9(7)	−110(2)
NLO $[O(Q^2)]$	−24.7(2)	−60(2)	−93(3)
IB + EM $[O(Q^2)]$	−23.8(2)	−55(2)	−85(3)
NNLO $[O(Q^3)]$	−28.4(3)	−58(2)	−91(3)
Experiment	−28.30	−56.50	−92.16

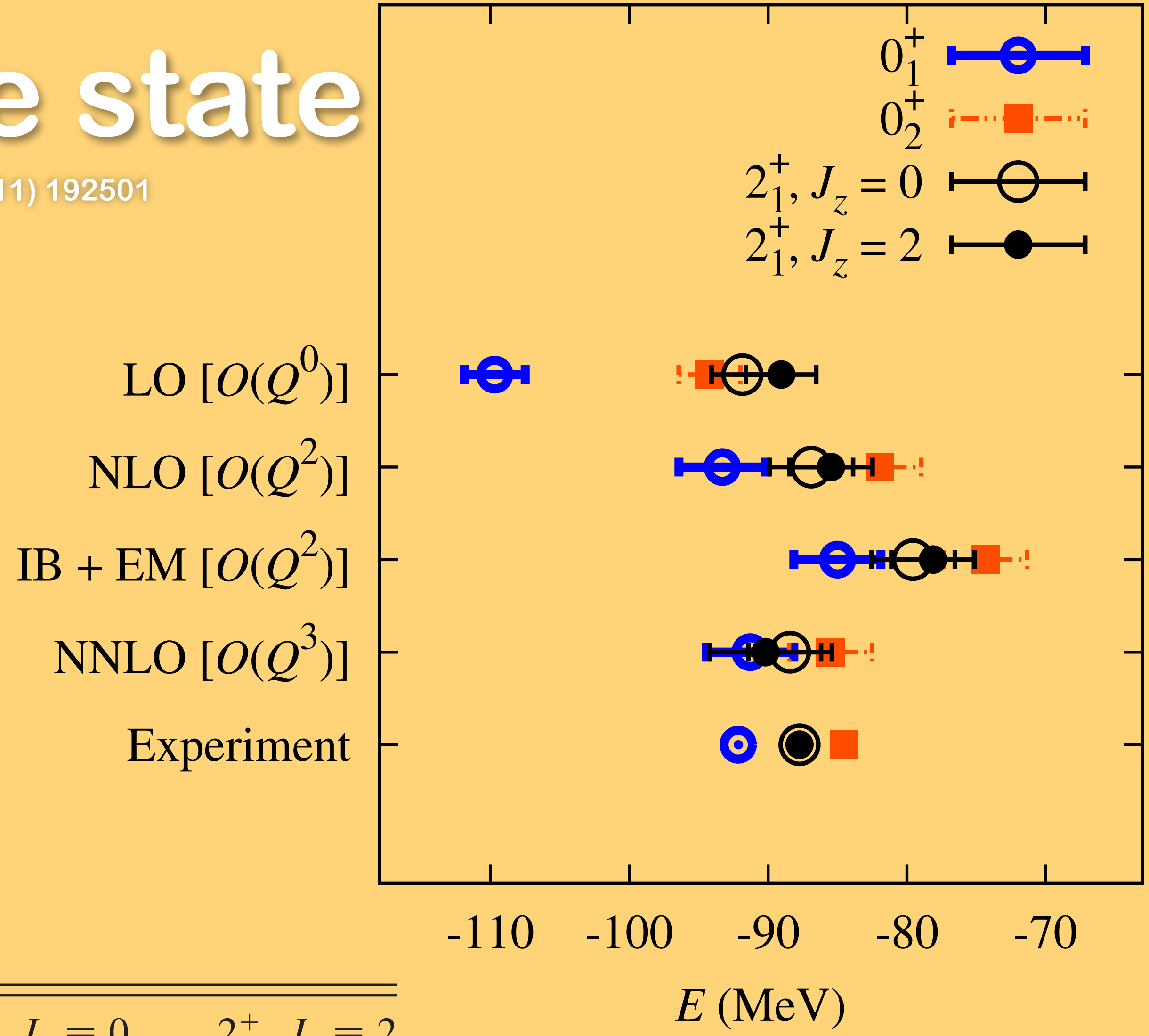
The Hoyle state

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501



The Hoyle state

E.E., Krebs, Lee, Meißner, PRL 106 (11) 192501



	0_2^+	$2_1^+, J_z = 0$	$2_1^+, J_z = 2$
LO $[O(Q^0)]$	$-94(2)$	$-92(2)$	$-89(2)$
NLO $[O(Q^2)]$	$-82(3)$	$-87(3)$	$-85(3)$
IB + EM $[O(Q^2)]$	$-74(3)$	$-80(3)$	$-78(3)$
NNLO $[O(Q^3)]$	$-85(3)$	$-88(3)$	$-90(4)$
Experiment	-84.51	-87.72	

Summary & outlook

Nuclear chiral EFT enters precision era:

accurate NN potentials at N^{3}LO, detailed analyses of MECs, high-precision determinations of πN scatt. lengths, precision calculations of the radiative/muon capture reactions, ...

Time to address unsolved problems:

e.g. the structure of the 3NF (work in progress...)

New trends/directions:

chiral EFT with explicit Δ, combining EFT with ab-initio many-body methods to provide access to spectra of light nuclei, bridging strong, weak and e.m. few-N reactions, ...

Dedicated experiments:

pd breakup @COSY, MuSun, experiments at MAMI, MAXlab, HIγS, ...

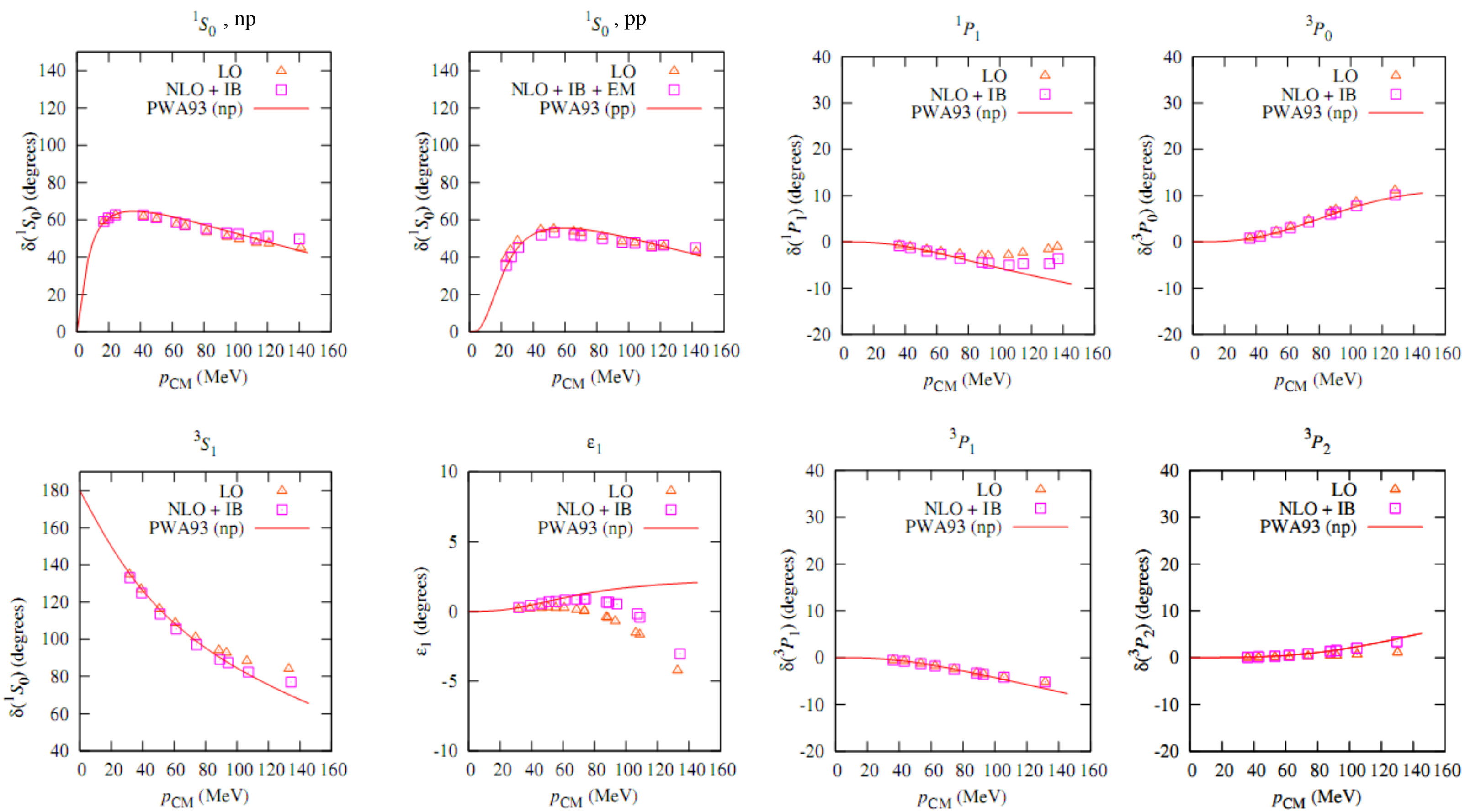
Further topics (not covered in the talk):

nuclear parity violation, hypernuclear physics, few-N systems and physics beyond the Standard Models (e.g. neutron EDM), ...

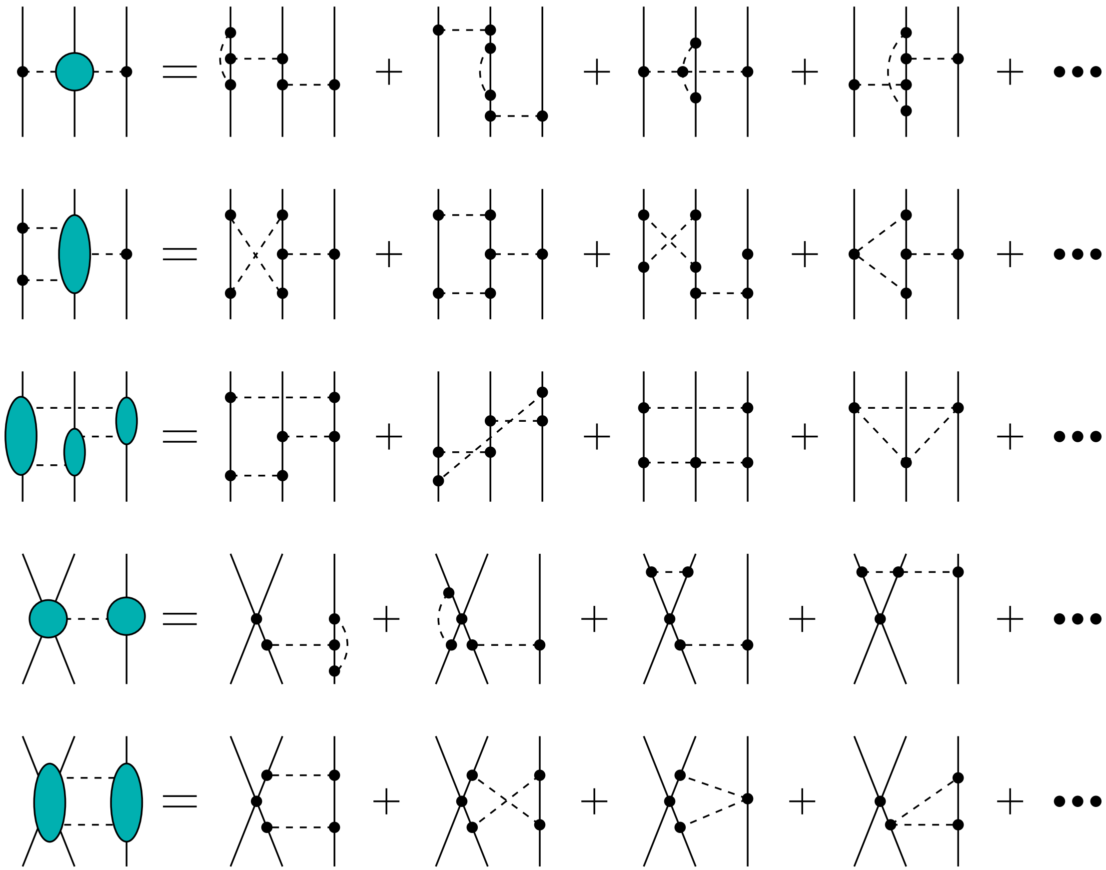
Two-nucleon phase shifts (LO_3)

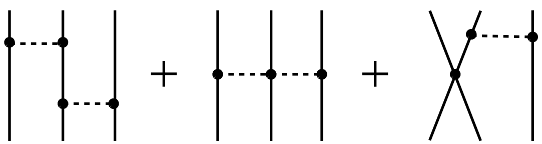
E.E., Krebs, Lee, Meißner, EPJA 45 (10) 335

- 9 LECs fitted to S- and P-waves and the deuteron quadrupole moment
- Coulomb repulsion and isospin-breaking effects taken into account
- Accurate results, deviations consistent with the expected size of higher-order terms



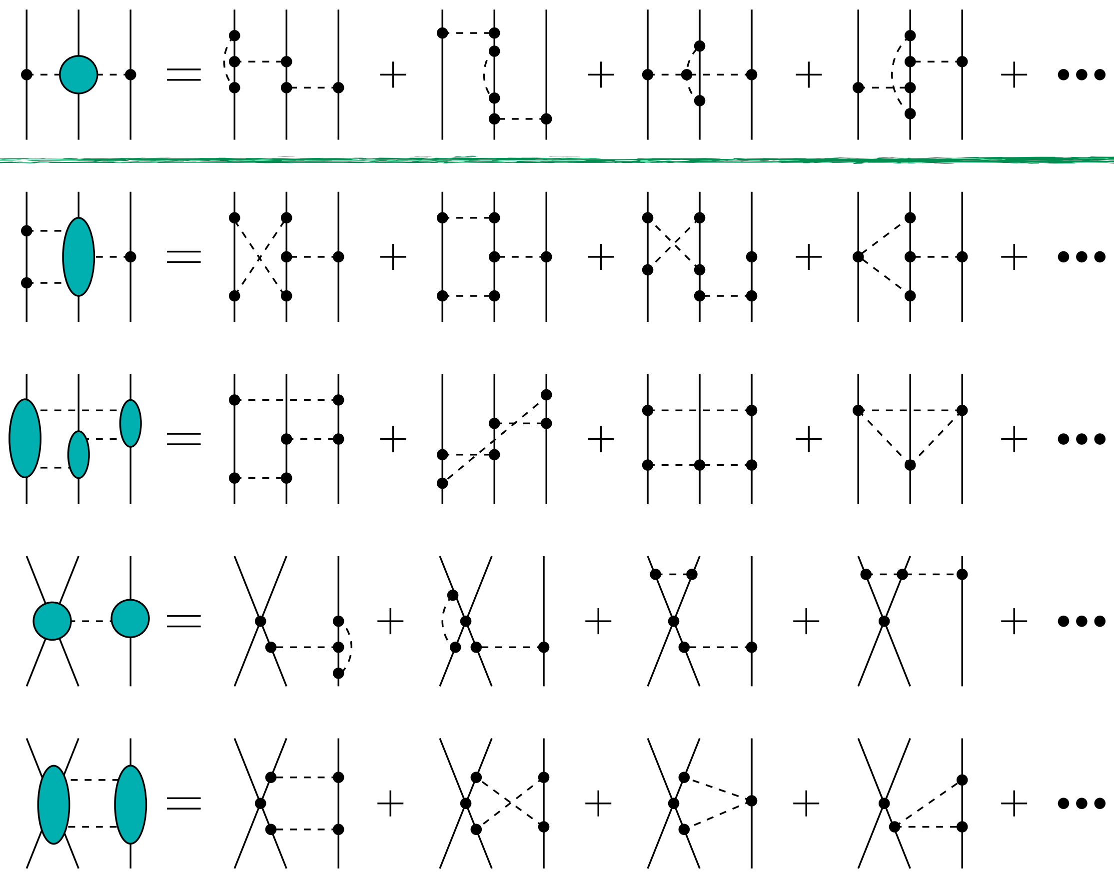
Chiral 3NF at N^3LO



1/m corrections to 

no unknown LECs contribute → parameter-free!

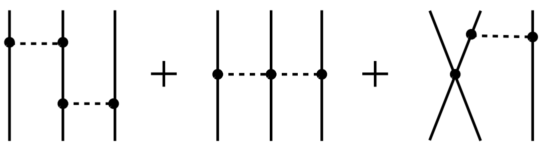
Chiral 3NF at N^3LO



Ishikawa, Robilotta, PRC76 (07)
Bernard, EE, Krebs, Meißner, PRC77 (08)

(mainly provide finite shifts to c_i)

1/m corrections to



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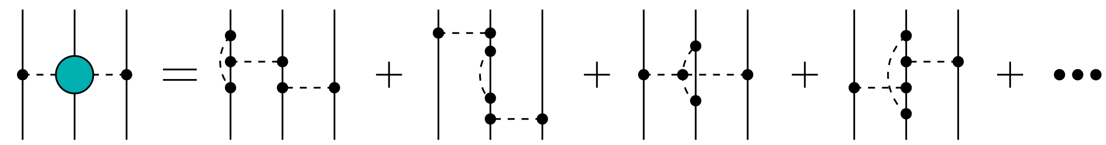
Chiral 3NF at N^3LO



Ishikawa, Robilotta, PRC76 (07)
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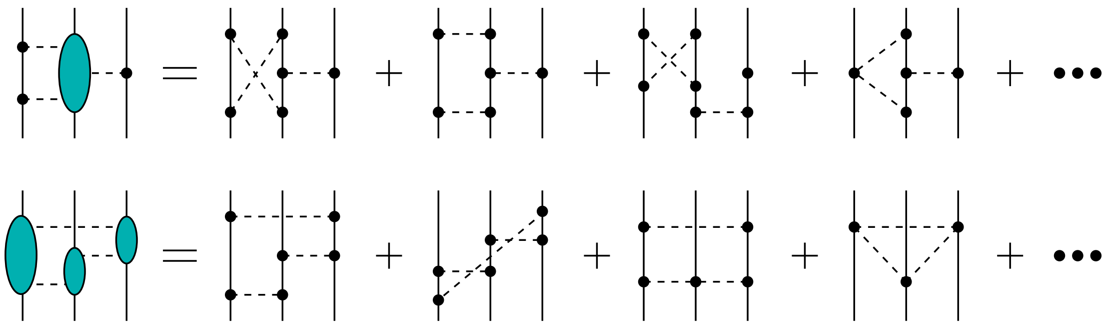
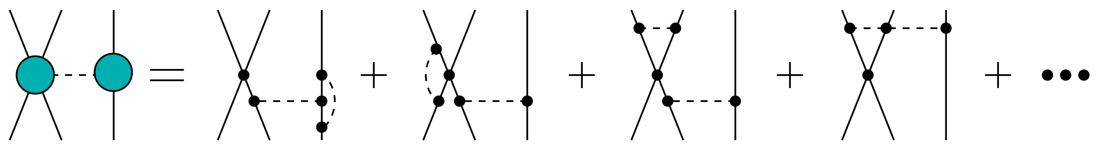
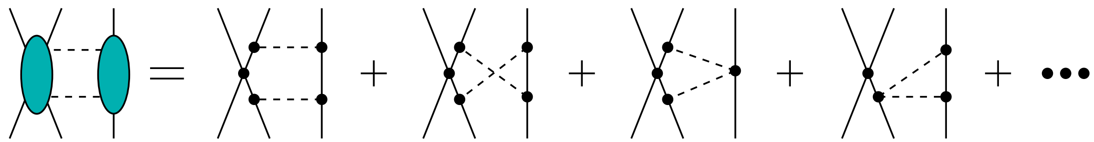


Bernard, EE, Krebs, Meißner, PRC77 (08)

- rich isospin/spin/momentum structures;
- involved expressions (ring)





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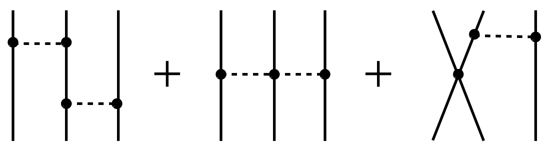


no unknown LECs contribute → parameter-free!

Chiral 3NF at N^{3}LO

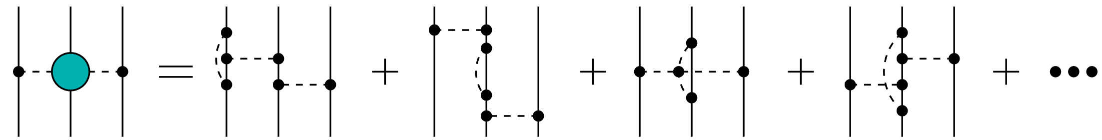
$$V_{2\pi-1\pi} = \frac{\vec{\sigma}_1\cdot\vec{q}_1}{q_1^2+M_\pi^2}\Big[\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_1\ (\vec{\sigma}_3\cdot\vec{q}_2\,\vec{q}_2\cdot\vec{q}_1\ F_1(q_2)+\vec{\sigma}_3\cdot\vec{q}_2\ F_2(q_2)+\vec{\sigma}_3\cdot\vec{q}_1\ F_3(q_2))$$
$$+\ \boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_1\ (\vec{\sigma}_2\cdot\vec{q}_2\,\vec{q}_2\cdot\vec{q}_1\ F_4(q_2)+\vec{\sigma}_2\cdot\vec{q}_1\ F_5(q_2)+\vec{\sigma}_3\cdot\vec{q}_2\ F_6(q_2)$$
$$+\ \vec{\sigma}_3\cdot\vec{q}_1\ F_7(q_2))$$
$$+\ \boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\cdot\boldsymbol{\tau}_1\ \vec{\sigma}_2\times\vec{\sigma}_3\cdot\vec{q}_2\ F_8(q_2)\Big]$$

$$\begin{aligned} V_{\rm ring} &= \vec{\sigma}_1\cdot\vec{\sigma}_2\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,R_1+\vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_2\cdot\vec{q}_1\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,R_2+\vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_2\cdot\vec{q}_3\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,R_3\\ &+ \vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_2\cdot\vec{q}_1\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,R_4+\vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_2\cdot\vec{q}_3\,\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3\,R_5+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3\,R_6+\vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_3\cdot\vec{q}_1\,R_7\\ &+ \vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_3\cdot\vec{q}_3\,R_8+\vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_3\cdot\vec{q}_1\,R_9+\vec{\sigma}_1\cdot\vec{\sigma}_3\,R_{10}+\vec{q}_1\cdot\vec{q}_3\times\vec{\sigma}_2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\,R_{11}\\ &+ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_1+\vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_3\cdot\vec{q}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_2+\vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_3\cdot\vec{q}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_3\\ &+ \vec{\sigma}_1\cdot\vec{q}_1\vec{\sigma}_3\cdot\vec{q}_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_4+\vec{\sigma}_1\cdot\vec{q}_3\vec{\sigma}_3\cdot\vec{q}_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_5+\vec{\sigma}_1\cdot\vec{\sigma}_3\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,S_6\\ &+ \vec{q}_1\cdot\vec{q}_3\times\vec{\sigma}_1\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\times\boldsymbol{\tau}_3\,S_7 \end{aligned}$$

explicit expressions for all F_i, $R_i(q_1,q_2,q_3)$ and $S_i(q_1,q_2,q_3)$ are available in Bernard, EE, Krebs, Meißner, PRC77 (08) 064004

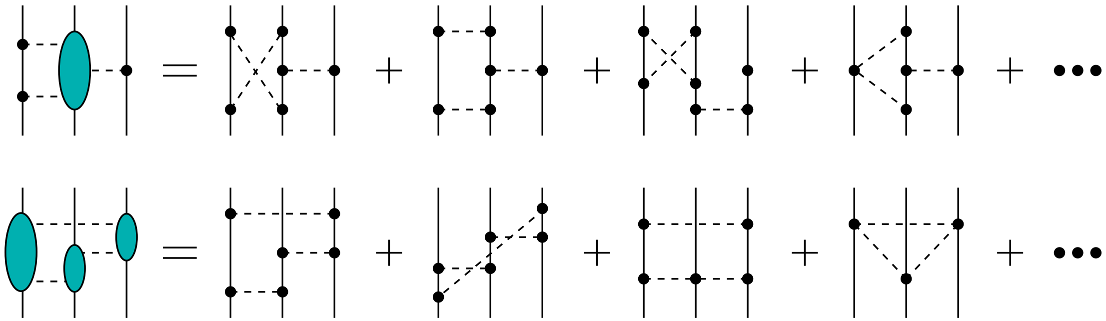
no unknown LECs contribute → parameter-free!

Chiral 3NF at N^3LO



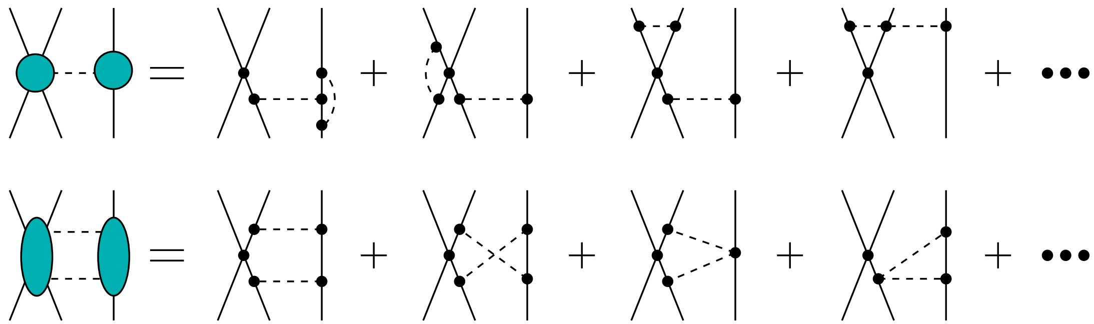
Ishikawa, Robilotta, PRC76 (07)
Bernard, EE, Krebs, Meißner, PRC77 (08)

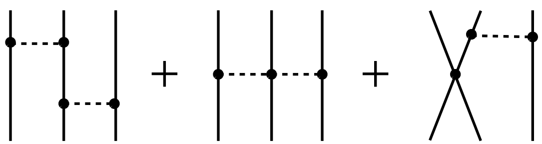
(mainly provide finite shifts to c_i)



Bernard, EE, Krebs, Meißner, PRC77 (08)

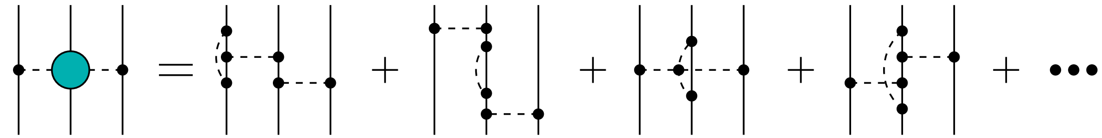
- rich isospin/spin/momentum structures;
- involved expressions (ring)



1/m corrections to 

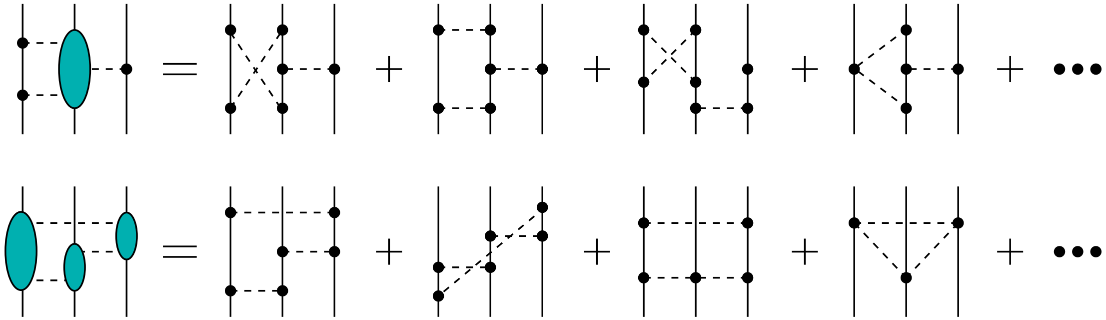
no unknown LECs contribute → parameter-free!

Chiral 3NF at N^3LO



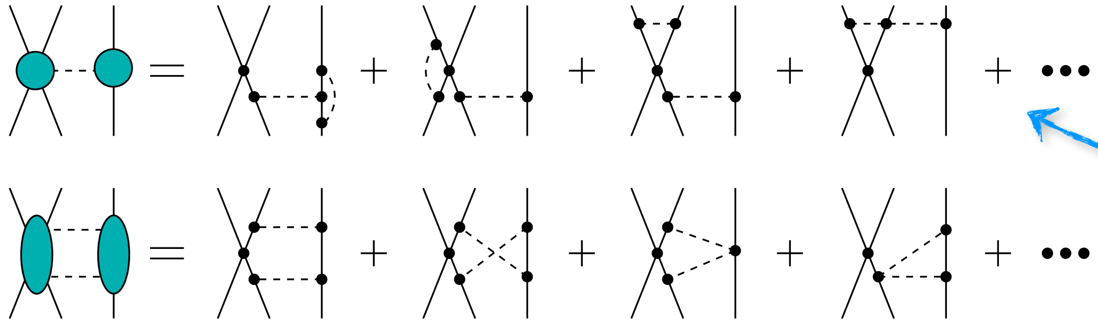
Ishikawa, Robilotta, PRC76 (07)
Bernard, EE, Krebs, Meißner, PRC77 (08)

(mainly provide finite shifts to c_i)



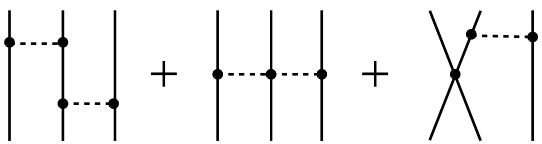
Bernard, EE, Krebs, Meißner, PRC77 (08)

- rich isospin/spin/momentum structures;
- involved expressions (ring)



Bernard, EE, Krebs, Meißner, arXiv:1108.3816

- total contribution vanishes completely (Pauli)
- 1/m corrections to two-pion also studied by: Friar, Coon, PRC49 (94)

1/m corrections to 

no unknown LECs contribute → parameter-free!

Chiral 3NF at N^3LO

no unknown LECs contribute → parameter-free!